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FREQUENCY-DOMAIN CONDITIONS FOR SYNTHESIS AND ANALYSIS OF ROBUST OUTPUT FEEDBACK CONTROL SYSTEMS

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Abstract: Frequency-domain conditions are derived for characterizing parameters of output dynamic controllers providing absolute stability and a relavant performance for various classes uncertain systems, such as nonlinear Lur'e systems, systems switched by an unknown law, linear systems with time-varying norm bounded parameters and some others.

Keywords: absolute stability, minimax techniques, output dynamic controllers, frequency-dependent characteristics

1. INTRODUCTION

The problem of designing output feedback controllers for nonlinear dynamic plants, whose mathematical models contain unknown subsystems, functions or parameters, is under study in the present paper. This branch of control theory has being developed actively within the framework of H_∞-optimal control (see, for example, (Doyle et al., 1989; Khargonekar et al., 1990; Savkin and Petersen, 1994; Battilotti, 2001) and references therein). In these papers, a class of control laws was derived, parameters of which are determined in terms of solutions to two Riccati nonlinear matrix equations of a minimax type, these solutions satisfying some additional constraint. Since up to the present, there aren't any necessary and sufficient conditions for the existence of such solutions, the problem of realization for these controllers is closely connected with a use of an appropriate software (for example, MATLAB).

In the given paper, we present frequency-domain conditions allowing one to establish if the closed-loop system with a given output feedback controller, chosen using a nominal model of the plant, will be absolutely stable for uncertainty from some class and if it satisfies a relevant robust performance requirement, without finding the abovementioned solutions to the Riccati equations. This situation is very similar to that of absolute stability theory, where finding an appropriate Lyapunov function is replaced by testing frequency-domain conditions.

The technique applied in this paper is taken over from the previous papers of the authors. It has been established in the papers of Kogan (1998b, 2001) that in the minimax-based robust control designs for various classes of uncertain systems (Lur'e systems with sector bounded nonlinearity, linear systems with time-varing norm bounded parameters and others), there is no necessity in solving nonlinear matrix equations or inequalities. Instead, there was indicated an alternative way for characterizing parameters of linear state feedbacks through testing their generalized return differences. This testing was shown to be reduced

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to checking availability of a real positive odd-multiple root for several polynomials, which is easy to do. On the other hand, in the papers (Brusin, 1996, 2000) a new class of output dynamic controllers with parameters expressed in terms of Riccati inequalities has been derived for nonlinear Lur'e systems whose uncertainties satisfy integral quadratic inequalities of some general type. A joint of these approaches allows to derive the frequency-domain conditions for characterizing output feedback controllers, providing various classes uncertain systems with a relevant performance. Note that the technique suggested can be generalized to some classes of infinite-dimensional systems.

2. PROBLEM STATEMENT AND PRELIMINARIES

The class of uncertain systems under consideration is described by the equations

$$\begin{aligned} \dot{x} &= Ax + B_1\xi + B_2 u \ , \\ y &= Lx \ , \end{aligned} \tag{1}$$

where $x \in R^n$ is a state, $u \in R^m$ is a control input, $y \in R^k$ is a measured, $\xi \in R^e$ is a dusturbance or/and an uncertainty input. The problem is to synthesize output feedback controllers of the form

$$\frac{d\hat{x}}{dt} = f(\hat{x}, y) \ , \quad u = g(\hat{x}) \ , \tag{2}$$

where $\hat{x}$ is a controller state, providing fulfillment of the relevant purposes in one of the following three cases:

$\mathbf{S_1}$. ξ is an external disturbance, and the purpose here is in attenuating the disturbance according to a criterion

$$\int_0^T F(x, u, \xi)\, dt \le C(x(0), \hat{x}(0)) \ , \quad \forall T \ge T_0 > 0; \tag{3}$$

$\mathbf{S_2}$. ξ is an internal disturbance corresponding to a system uncertainty of the form

$$\xi(\cdot) = \varphi_t(x(\cdot), u(\cdot)) \ , \tag{4}$$

where φ_t is an unknown causal operator belonging to the class K given by the integral constraint

$$\int_0^T N(x(t), u(t), \xi(t))\, dt \ge 0, \forall T \ge T_0. \tag{5}$$

The purpose here is to provide the absolute stability of the closed-loop system around zero in the class K, i.e. the following conditions have to be satisfied for all admissible uncertainties:

$$\begin{aligned} |x(t)| + |\hat{x}(t)| + \int_0^\infty (|x(t)|^2 + |\hat{x}(t)|^2)\, dt \\ \le C(x(0), \hat{x}(0)), \\ |x(t)| \to 0, \quad |\hat{x}(t)| \to 0 \quad (t \to \infty) \ ; \end{aligned} \tag{6}$$

$\mathbf{S_3}$. ξ is a disturbance of the mixed kind, i.e.

$$\xi = \mathrm{col}(\xi_{ex}, \xi_{in}) \ , \quad B_1 = (B_1^{ex}, B_1^{in}) \ , \tag{7}$$

where $\xi_{ex}(t)$ is the external disturbance, and ξ_{in} is the internal disturbance defined in the similar way as in (4), i.e.

$$\xi_{in}(\cdot) = \varphi_t(x(\cdot), u(\cdot)) \ , \tag{8}$$

where the operator φ_t belongs to the class K given by the relation of the form (5). The purpose here consists in satisfying the following two conditions:

$\mathbf{G_1}$: for $\xi_{ex}(t) \equiv 0$, the closed-loop system is absolute stable in the class K, i.e. the conditions (6) hold;

$\mathbf{G_2}$: for $\xi_{ex}(t) \not\equiv 0$, the external disturbances are to be attenuated according to the criterion

$$\sup_{\varphi_t \in K} \int_0^T F_0(x, u, \xi_{ex})\, dt \le C(x(0), \hat{x}(0)) \ , \quad \forall T > T_0 \ . \tag{9}$$

In the above relations (3), (5) and (9), the integrands are of indefinite sign quadratic forms of a general form

$$\begin{aligned} & x^T M_{11} x + 2x^T M_{12} u + 2x^T M_{13} \xi \\ & + u^T M_{22} u + 2u^T M_{23} \xi - \xi^T M_{33} \xi \ , \\ & M_{ii}^T = M_{ii} > 0 \ , \quad i = 1, 2, 3 \ , \end{aligned} \tag{10}$$

and $C(x(0), \hat{x}(0))$ are constants converting to zero at $x(0) = \hat{x}(0) = 0$.

The subsequent synthesis of the controllers guaranteeing for the closed-loop system (1), (2) the fulfillment of the purposes (3), (6) or (9) with functions $F(x, u, \xi)$, $N(x, u, \xi)$ or $F_0(x, u, \xi)$ of the form (10) is based on the well-known Riccati matrix equation:

$$\begin{aligned} & PA + A^T P + M_{11} - \Theta_*^T M_{22} \Theta_* \\ & + 2\Theta_*^T M_{23} \Pi_* + \Pi_*^T M_{33} \Pi_* = 0 \ , \end{aligned} \tag{11}$$

where

$$
\begin{aligned}
\Theta_* &= (M_{22} + M_{23}M_{33}^{-1}M_{23}^T)^{-1} \\
&[PB_2 + M_{12} + (PB_1 + M_{13})M_{33}^{-1}M_{23}^T]^T , \\
\Pi_* &= (M_{33} - M_{23}^T M_{22}^{-1} M_{23})^{-1} \\
&[PB_1 + M_{13} - (PB_2 + M_{12})M_{22}^{-1}M_{23}]^T .
\end{aligned}
\tag{12}
$$

The main property of this equation used below consists in the fact that, for any solution to the system (1), the following relation holds:

$$
\begin{aligned}
&\frac{d}{dt} x^T P x + F(x,u,\xi) \\
&= \tilde{u}^T M_{22} \tilde{u} + 2\tilde{u}^T M_{23} \tilde{\xi} - \tilde{\xi}^T M_{33} \tilde{\xi} ,
\end{aligned}
\tag{13}
$$

where

$$
\begin{aligned}
\tilde{u} = u - u_* , &\quad \tilde{\xi} = \xi - \xi_* , \\
u_* = -\Theta_* x , &\quad \xi_* = \Pi_* x .
\end{aligned}
\tag{14}
$$

The equation (13) is a well-known Hamilton-Jacobi equation for an appropriate minimax problem, $V(x) = x^T P x$ being a Bellman function.

Below, instead of the equation (11), we will apply the corresponding Riccati inequality

$$
\begin{aligned}
&PA + A^T P + M_{11} - \Theta_*^T M_{22} \Theta_* \\
&+ 2\Theta_*^T M_{23} \Pi_* + \Pi_*^T M_{33} \Pi_* \le 0 ,
\end{aligned}
\tag{15}
$$

therefore, the inequality obtained from (13) after replacing $=$ by $\le$ will hold. The matrix P is known to be a stabilizing solution to this inequality, if matrix $A_c = A + B_1 \Pi_* - B_2 \Theta_*$ is Hurwitz.

3. SOME CONTROL PROBLEMS LEADING TO THE ABOVE SCHEME

We will show here that many well-known control problems such as designs of absolutely stabilizing and robust controllers for various classes of uncertain systems are in the framework of the above problem statement.

3.1 *Absolute stabilizability*

For the nonlinear Lur'e system

$$
\begin{aligned}
\dot{x} &= Ax + B_1 \xi + B_2 u , \\
y &= Lx , \\
z &= \begin{pmatrix} C_0 \\ 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ D_0 \end{pmatrix} u ,
\end{aligned}
\tag{16}
$$

where $\xi = \varphi(y,t)$ is an unknown continuous function of y и t, $\varphi(0,t) \equiv 0$, each component of which satisfies the sector constraint

$$
h_-^{(i)} \le \frac{\varphi_i(y_i,t)}{y_i} \le h_+^{(i)} , \quad i = 1, 2, \dots, l , \tag{17}
$$

an absolute stabilization problem consists in designing an output feedback controller so that the closed-loop system would be absolutely stable in the class K in the sense of the conditions (6).

From (17) immediately follows that the nonlinearities at hand satisfy

$$
(\xi - H_- y)^T \Gamma (H_+ y - \xi) \ge 0 ,
$$

where

$$
\begin{aligned}
H_- &= \mathrm{diag}\left(h_-^{(1)}, \dots, h_-^{(l)}\right) , \\
H_+ &= \mathrm{diag}\left(h_+^{(1)}, \dots, h_+^{(l)}\right) , \\
\Gamma &= \mathrm{diag}\left(\gamma_1^2, \dots, \gamma_l^2\right) > 0 ,
\end{aligned}
$$

which corresponds to relation (5) for

$$
N(x,\xi) = (\xi - H_- L x)^T \Gamma (H_+ L x - \xi) . \tag{18}
$$

3.2 *A switched system*

Suppose that the disturbance input in the system (16) is defined as $\xi = C(t)x$, where the variable matrix C takes one of two possible values, C_1 or C_2. A switching law of the matrix values is assumed to be arbitrary, but is such that only a finite number of switching may occur during a finite time interval (see (Liberson and Morse, 1999) for details). The robust control problem for this system is to synthesize a controller under which the conditions (6) hold for any admissible switching law.

As it can immediately be checked, the nonlinearities of such a kind satisfy the condition (5) with

$$
\begin{aligned}
N(x,\xi) \equiv\ & (\xi - C_1 x)^T (D_1 x + a\xi) \\
&+ (\xi - C_2 x)^T (D_2 x + b\xi) \\
&- \delta^2 |(C_1 - C_2)x|^2 \ge 0 ,
\end{aligned}
\tag{19}
$$

$D_1 = (\alpha^2 - a)C_2 - \alpha^2 C_1$, $D_2 = (\beta^2 - b)C_1 - \beta^2 C_2$, $\delta^2 = \min(\alpha^2, \beta^2)$, α, β, a, b are any scalars such that $a + b < 0$.

3.3 *Robust H_∞-control*

Let an uncertain system be descibed by the equation

$$
\begin{aligned}
\dot{x} = & [A + F_1 \Omega_1(t) E_1]x + [B + F_2 \Omega_2(t) E_2]u \\
& + [G + F_3 \Omega_3(t) E_3]w, \quad x(0) = x_0 , \\
y &= Lx , \\
z &= \begin{pmatrix} C_0 \\ 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ D_0 \end{pmatrix} u ,
\end{aligned}
\tag{20}
$$

where $\Omega_i(t), i = 1,2,3$ are unknown matrices satisfying the inequalities

$$\Omega_i^T(t)\,\Omega_i(t) \le I, \quad i = 1,2,3, \quad \forall t \;, \quad (21)$$

and A, B, G, F_i, E_i, $i = 1,2,3$ are given matrices of compatible orders. The problem is in constructing a robust output feedback H_∞ controller.

Rewriting the equation (20) in the form

$$\dot{x} = Ax + Bu + G\xi_{ex} + (F_1, F_2, F_3)\xi_{in} \;,$$
$$\xi_{ex} = w \;,$$
$$\xi_{in} = \mathrm{col}(\Omega_1(t)E_1 x \;, \Omega_2(t)E_2 u \;, \Omega_3(t)E_3 w)$$

and observing that from (21) it follows that the condition (5) holds, where

$$N(x,u,\xi) = |E_1 x|^2 + |E_2 u|^2 + |E_3 \xi_{ex}|^2 - |\xi_{in}|^2,$$

one can see the control purpose will be attained if the inequality (9) holds with

$$F_0(x,u,\xi_{ex}) = x^T C_0^T C_0 x + u^T D_0^T D_0 u - \gamma^2 |\xi_{ex}|^2.$$

4. SYNTHESIS ON THE BASIS OF MATRIX RELATIONS

Consider a class of linear dynamic controllers defined by the equations

$$\frac{d\hat{x}}{dt} = A\hat{x} + B_1\hat{\xi} + B_2 u + Z(L\hat{x} - y) \qquad (22)$$
$$u = -\Theta_* \hat{x} \;, \quad \hat{\xi} = \Pi_* \hat{x} \;,$$

where Θ_*, Π_* are given in (12), and Z is any matrix of an appropriate order. The following result is in order.

Theorem 1. Let $F(x,u,\xi)$ be of the form (10) with the matrix coefficients M_{ij}, $P > 0$ be a solution to the appropriate inequality (15), in which Θ_* and Π_* are given in (12), and the matrix Z be such that $A_Z = A + B_1\Pi_* + ZL$ is Hurwitz. Suppose that the following frequency-domain condition holds for some $\epsilon > 0$:

$$\Xi(i\,\omega) = K_*(-i\,\omega)^T M_{22} K_*(i\,\omega)$$
$$+2Re\,K_*(-i\,\omega)^T M_{23} - M_{33} \le -\epsilon I \;, \forall\omega, \qquad (23)$$

where $K_*(p) = \Theta_*(pI - A_Z)^{-1}B_1$ and $i = \sqrt{-1}$. Then, for all solutions of the closed-loop system (1), (22), the following statement is true:

$$\int_0^T [F(x,u,\xi) + \epsilon_1|\tilde{\xi}|^2]\,dt \le C(x(0),\hat{x}(0)), \qquad (24)$$
$$0 < \epsilon_1 < \epsilon \;, \quad \forall T > 0 \;,$$

where $\tilde{\xi} = \xi - \Pi_* x$, and $C(x(0),\hat{x}(0))$ is a constant depending on the initial conditions of the both plant and controller, $C(0,0) = 0$.

As it is apparent from (24), Theorem 1 gives a solution to the problem at hand in the case $\mathbf{S_1}$. The proof of this theorem is based upon the below lemma. Before formulating it, let us denote $\tilde{x} = x - \hat{x}$ and derive, from the equations (1), (22), in view of (14), the following equations for the closed-loop variables:

$$\frac{d\tilde{x}}{dt} = A_Z\tilde{x} + B_1\tilde{\xi} \qquad (25)$$
$$\tilde{u} = \Theta_*\tilde{x} \;.$$

Here, $\tilde{\xi}$ и $\tilde{u}$ can be considered as an input and an output, respectively, for a stable (by virtue of the theorem assumption) system with $\tilde{x}$ being the state and $K_*(p)$ the transfer function.

Lemma 1. Let A_Z be Hurwitz and the frequency-domain condition (23) be held. Then, for any $T > 0$, there exists such $\epsilon_1 > 0$ that

$$I(T) = \int_0^T (\tilde{u}^T M_{22}\tilde{u} + 2\tilde{u}^T M_{23}\tilde{\xi} - \tilde{\xi}^T M_{33}\tilde{\xi})\,dt \qquad (26)$$
$$\le -\epsilon_1 \int_0^T |\tilde{\xi}|^2\,dt + C_1(\tilde{x}(0)) \;,$$

where $C_1(0) = 0$, holds for solutions of the system (1), (22).

For the case $\mathbf{S_2}$, where the nonlinear system is obtained from (1) by introducing an additional feedback from x and u to ξ, the following statement is in order.

Theorem 2. Suppose that all the assumptions of the Theorem 1 hold for the function $F(x,u,\xi) = N(x,u,\xi) + \rho^2|u|^2$, where ρ is a scalar, and, additionally, let P be a stabilizing solution to the corresponding Riccati inequality. Then the closed-loop system (1), (4), (22) will be absolutely stable for all uncertainties belonging to the class K defined by the condition (5).

Finally, in the case $\mathbf{S_3}$, when both external and internal disturbances are available, we have the following result.

Theorem 3. Suppose that for the system (1), (7) and the function

$$F(x,u,\xi) = F_0(x,u,\xi_{ex}) + N(x,u,\xi) + \rho^2|u|^2,$$

where $\xi = \mathrm{col}(\xi_{ex}, \xi_{in})$ and $F_0(x,u,0) \ge 0$, all the assumptions of the Theorem 1 holds and P is a stabilizing solution to the corresponding Riccati inequality. Then, all the processes of the closed-loop system (1), (7), (22) satisfy the purpose conditions $\mathbf{G_1}$ and $\mathbf{G_2}$.

5. SYNTHESIS ON THE BASIS OF FREQUENCY-DOMAIN CONDITIONS

According to the above-mentioned, for robust controller designs it is required to determine a stabilizing solution $P > 0$, if any, of the Riccati inequality (15), to put it into the formula (12) for calculating Θ_* and Π_* and, finally, to form the controller (22), choosing for it a matrix Z so that $A_Z = A + B_1\Pi_* + ZL$ would be Huirwitz and the frequency-domain condition (23) would be held. In what follows, we will show that solving the Riccati inequality and the matrix P itself might be excluded from the above procedure. Instead, using an ideology of inverse variation problems and, in particular, the inverse problem for differential games (Kogan, 1998a) as well as the Kalman-Yakubovich-Popov lemma (Gelig et al, 1978), we will derive frequency-domain conditions, expressed in the terms of Θ and Π immediately, for a controller of the form

$$\begin{aligned} \frac{d\hat{x}}{dt} &= A\hat{x} + B_1\hat{\xi} + B_2u + Z(L\hat{x} - y) \\ u &= -\Theta\hat{x}\ , \quad \hat{\xi} = \Pi\hat{x} \end{aligned} \tag{27}$$

to be desired. It is worth noticing that these conditions will cover all the parameters Θ_* and Π_* which could be computed by the formulas (12), (15).

For the sake of simplicity, let $M_{23} = 0$ in (10), and then the matrices Θ_* and Π_* given in (11) will be of the form

$$\begin{aligned} \Theta_* &= M_{22}^{-1}(PB_2 + M_{12})^T\ , \\ \Pi_* &= M_{33}^{-1}(PB_1 + M_{13})^T\ , \end{aligned} \tag{28}$$

and the Riccati inequality (15) takes the form

$$\begin{aligned} &PA + A^TP + M_{11} \\ &-\Theta_*^T M_{22}\Theta_* + \Pi_*^T M_{33}\Pi_* \leq 0\ . \end{aligned} \tag{29}$$

If $M_{23} \neq 0$, then after the change of variables $\xi = \eta + M_{33}^{-1}M_{23}^T u$ one may easy to pass to a new equation and a new quadratic form with a zero required block.

With reference to the purpose stated, at first note that the equations (22) define a whole class of robust controllers with the matrices Θ_* and Π_* being computed according to the formulas (28), where $P > 0$ is some solution of the inequality (29). This suggests an idea to consider an inverse, in some sense, approach to the synthesis, consisting in assigning the parameters Θ and Π for a controller of the form (27) and in checking some test for their suitability. As it will be seen in the sequel, this approach does not need in finding any solution of the inequality (29), establishing, at the same time, the fact of its solvability. (This makes it related to the approach of absolute stability theory.)

The rigorous formulating of the present inverse approach leads us to the following problem: *given the matrices Θ and Π such that $A_\Theta = A - B_2\Theta$ and $A_c = A - B_2\Theta + B_1\Pi$ are Hurwitz, find necessary and sufficient conditions for the existence of a matrix $P = P^T > 0$, satisfying both the equations*

$$\begin{aligned} M_{22}^{-1}(PB_2 + M_{12})^T &= \Theta\ , \\ M_{33}^{-1}(PB_1 + M_{13})^T &= \Pi \end{aligned} \tag{30}$$

and the inequality

$$\begin{aligned} &PA + A^TP + M_{11} - \Theta^T M_{22}\Theta \\ &+\Pi^T M_{33}\Pi \leq 0. \end{aligned} \tag{31}$$

The next lemma gives a solution to the above problem in the terms of the following transfer function matrix of the system (1) from the inputs $\zeta = \mathrm{col}(u, \xi)$ to the outputs $\hat{y} = \mathrm{col}(y_1, y_2)$, where $y_1 = u + \Theta x$ and $y_2 = \xi - \Pi x$:

$$\begin{aligned} W(p)^T &= (W_u(p)^T\ , W_\xi(p)^T)\ , \\ W_u(p) &= (I, 0) + \Theta H(p)\ , \\ W_\xi(p) &= (0, I) - \Pi H(p)\ , \\ H(p) &= (pI - A)^{-1}(B_2, B_1)\ . \end{aligned} \tag{32}$$

Note that for the disturbance-free system ($B_1 = 0$), the transfer matrix $W_u(p)$ passes to the return difference matrix for the control law $u = -\Theta x$, and for the uncontrolled system ($B_2 = 0$), the transfer matrix $W_\xi(p)$ moves to the return difference matrix for the disturbance law $\xi = \Pi x$.

Lemma 2. Suppose that all the eigenvalues of A do not lie on the imaginary axis and the pair $(A, \bar{B})$ is controllable,where $\bar{B} = (B_1, B_2)$. Let Θ и Π be chosen in such a way that, first, $A_\Theta = A - B_2\Theta$ and $A_c = A + B_1\Pi - B_2\Theta$ are Huzwitz, second, the following inequality is satisfied for a function $F(x, u, \xi)$ of the form (10) with $M_{23} = 0$:

$$\begin{aligned} &M_{11} - M_{12}M_{22}^{-1}M_{12}^T + \\ &+ \left(\Theta - M_{22}^{-1}M_{12}^T\right)^T M_{22}\left(\Theta - M_{22}^{-1}M_{12}^T\right) \\ &+\Pi^T M_{33}\Pi = C_{\Theta\Pi}^T C_{\Theta\Pi} \geq 0\ , \end{aligned} \tag{33}$$

and, third, the pair $(A_\Theta, C_{\Theta\Pi})$ is observable. Then, there exists $P = P^T > 0$ satisfying both the equations (30) and the inequality (31) if and only if the following frequency-domain condition holds $\forall\ \omega \in (-\infty, \infty)$ and $\forall$ complex ζ:

$$\zeta^* W(-i\,\omega)^T \begin{pmatrix} M_{22} & 0 \\ 0 & -M_{33} \end{pmatrix} W(i\,\omega)\zeta \geq F\left[H\left(i\,\omega\right)\zeta, \zeta\right] \ , \tag{34}$$

where $W(p)$ is given in (32), $*$ denotes the complex cojugate transpose, and $F(x,\zeta)$, where $\zeta = \mathrm{col}(u,\xi)$, is of the form (according to (10))

$$F(x,\zeta) = x^* M_{11} x + 2\mathrm{Re}\, x^*(M_{12}\, M_{13})\zeta + \zeta^* \mathrm{diag}(M_{22},\ -M_{33})\zeta\ . \tag{35}$$

Combining the results of Theorems 1,2,3 with those of Lemma 2, we come to our main contribution of this paper, stated separately in the following theorems for each of the cases $\mathbf{S_1}, \mathbf{S_2}, \mathbf{S_3}$.

Theorem 4. Suppose that all the eigenvalues of A do not lie on the imaginary axis and the pair $(A, \bar{B})$ is controllable,where $\bar{B} = (B_2, B_1)$. Let Θ, Π and Z be chosen in such a way that:

(1) $A_\Theta = A - B_2\Theta$, $A_c = A + B_1\Pi - B_2\Theta$ and $A_Z = A + B_1\Pi + ZL$ are Hurwitz.
(2) Given $F(x,u,\xi)$ of the form (10) with $M_{23} = 0$, the condition (33) holds.
(3) $(A_\Theta, C_{\Theta\Pi})$ is observable.
(4) For $W(p)$ given in (32), the frequency-domain condition (34) holds.
(5) For $K_Z(p) = \Theta(pI - A_Z)^{-1}B_1$, the following frequency-domain condition holds:

$$K_Z(-i\,\omega)^T M_{22} K_Z(i\,\omega) + 2Re\, K_Z(-i\,\omega)^T M_{23} - M_{33} \leq -\epsilon I, \forall \omega. \tag{36}$$

Then the controller (27) provides the fulfillment of the purpose (24) for the system (1).

Theorem 5. Let all the assumptions of Theorem 4 be satisfied with respect to $F(x,u,\xi) = N(x,u,\xi) + \rho^2|u|^2$. Then the closed-loop system (1), (4), (27) is absolutely stable in the class K defined by the condition (5).

Theorem 6. Let all the assumptions of Theorem 4 be satisfied for the system (1), (7) and $F(x,u,\xi) = F_0(x,u,\xi_{ex}) + N(x,u,\xi) + \rho^2|u|^2$, where $F_0(x,u,0) \geq 0$. Then, the purpose conditions $\mathbf{G_1}$ and $\mathbf{G_2}$ hold for all the processes in the closed-loop system (1), (7), (22).

By applying the above results to the control problems listed in the section 3, frequency-domain conditions for parameters Θ, Π and Z of the appropriate output feedback controllers (27)are derived immediately.

6. CONCLUSION

A robust control design problem for some classes uncertain systems such as nonlinear Lur'e systems, systems switched by an unknown law, linear systems with time-varying norm bounded parameters has been considered in some general framework. It has been shown that robust output feedback controllers for such systems can be characterized immediately in terms of frequency-domain conditions, avoiding thus the necessity of solving matrix equations or inequalities.

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