INTEGRAL CONTROLLER DESIGN BASED ON DISTURBANCE CANCELLATION VIA PARTIAL LTR

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Abstract: For non-minimum phase plants, integral controller design based on disturbance cancellation is discussed. A new partial loop transfer recovery (LTR) technique is proposed. The target of the output feedback design is a minimum phase state feedback controller including a disturbance estimator. It is shown that the feedback property of the target can be recovered by a formal procedure using a Riccati equation with a fictitious disturbance term. *Copyright* © 2002 *IFAC*

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1. INTRODUCTION

A method for designing integral controllers is to use a disturbance cancellation technique (*e.g.*, Franklin *et al.*, 1990). The standard loop transfer recovery (LTR) procedure can not be applied to this design problem since the extended system consisting of a plant and a disturbance model is not stabilizable. To overcome this difficulty, Guo *et al.* (1996a, b) have proposed a new LTR procedure for the discrete-time case. However, it is difficult to give clear system-theoretic meaning for the feedback property achieved for non-minimum phase plants.

For non-minimum phase plants, Moore and Xia (1987) have proposed a partial LTR technique which has clear system-theoretic meaning. Guo *et al.* (1995) and Ishihara (1995) have applied the partial LTR technique to design a class of discrete-time integral controllers discussed by Ishihara *et al.* (1992).

An application of the partial LTR technique to integral controller design based on disturbance cancellation is discussed in this paper. The target for the partial LTR is a controller including a disturbance estimator based on the measurement of minimum phase state. Although the target controller is fictitious, it has clear system-theoretic meaning. For the output feedback controller, a formal procedure using a Riccati equation is proposed to recover the target feedback property. A major difference from the conventional LTR procedure is that the Riccati equation used for the recovery contains a covariance matrix depending on the estimator gain matrix used in the target controller.

2. PRELIMINARIES

Consider a plant subject to step disturbances

$$\dot{x}(t) = Ax(t) + B[u(t) + d(t)],$$

$$y(t) = Cx(t),$$
(1)

where $x(t) \in R^n$ is a state vector, $u(t) \in R^m$ is a control input, $y(t) \in R^m$ is an output and $d(t) \in R^m$ is a step disturbance vector satisfying

$$\dot{d}(t) = 0. (2)$$

The following conditions are assumed:

C1: (A, B, C) is a minimal realization and $C(sI - A)^{-1}B$ is non-singular for almost all s.

C2: (A, B, C) is non-minimum phase.

C3: (A, B, C) has no zero at s = 0.

2.1 All-pass/minimum phase decomposition

Define

$$G(s) \triangleq C(sI - A)^{-1}B, \qquad (3)$$

then the matrix (3) can be factored as

$$G(s) = G_m(s)G_a(s), (4)$$

where $G_m(s)$ is a minimum phase part and $G_a(s)$ is an all-pass part including all the unstable zeros of G(s) and satisfying

$$G_a(s)G_a'(-s) = I. (5)$$

A minimal realization of $G_m(s)$ can be chosen as (A, B_m, C) . Let (A_a, B_a, C_a, D_a) denote a minimal realization of $G_a(s)$. Then the extended system consisting (1) and (2) can be written as

$$\dot{\xi}(t) = \Phi \xi(t) + \Gamma u(t),
y(t) = H \xi(t),$$
(6)

where

$$\xi(t) \triangleq \begin{bmatrix} x'_m(t) & x'_a(t) & d'(t) \end{bmatrix}', \tag{7}$$

$$\Phi \triangleq \begin{bmatrix}
A & B_{m}C_{a} & B_{m}D_{a} \\
0 & A_{a} & B_{a} \\
0 & 0 & 0
\end{bmatrix}, \Gamma \triangleq \begin{bmatrix}
B_{m}D_{a} \\
B_{a} \\
0
\end{bmatrix},$$

$$H \triangleq \begin{bmatrix} C & 0 & 0 \end{bmatrix}.$$
(8)

It can easily be checked by the PBH test that the pair (H, Φ) is detectable but (Φ, Γ) is not stabilizable.

2.2 Observer for the extended system

A full order observer for estimating state vector $\xi(t)$ of the extended system (6) is given by

$$\dot{\hat{\xi}}(t) = \Phi \hat{\xi}(t) + \Gamma u(t) + K[v(t) - H\hat{\xi}(t)], \qquad (9)$$

where $\hat{\xi}(t)$ is the estimate of $\xi(t)$ and K is an observer gain matrix. Define the partitions of the estimate and the gain matrix as

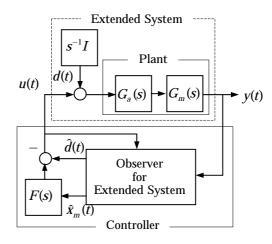


Fig. 1. The output feedback control system

$$\hat{\xi}(t) = \begin{bmatrix} \hat{x}'_m(t) & \hat{x}'_a(t) & \hat{d}'(t) \end{bmatrix}', \tag{10}$$

$$\mathbf{K} = \begin{bmatrix} K'_m & K'_a & K'_d \end{bmatrix}'. \tag{11}$$

Since the all-phase part is unobservable from the output, it follows that $K_a=0$. Then the three elements of $\hat{\xi}(t)$ can be expressed as

$$\dot{\hat{x}}_{m}(t) = A\hat{x}_{m}(t) + B_{m}G_{a}(s)[u(t) + \hat{d}(t)] + K_{m}[y(t) - C\hat{x}_{m}(t)],$$
(12)

$$\dot{\hat{x}}_a(t) = A_a \hat{x}(t) + B_a[u(t) + \hat{d}(t)],$$
 (13)

$$\hat{d}(t) = K_d[y(t) - C\hat{x}_m(t)].$$
 (14)

Remark: The parameter s in the time domain expressions $(12) \sim (14)$ should be interpreted as a differential operator with respect to t. Similar expressions will be used to simplify notations.

Remark: For the standard estimation problem for non-minimum phase plants, it is very easy to guarantee the stability of the observer when the observer gain matrix for the all-pass part is set to zero (Moore and Xia, 1987). For the present case, the problem of characterizing the gain matrices K_m and K_d that guarantee the stability of the observer when $K_a = 0$ is not simple. A method for determining K_m and K_d that stabilize the observer with $K_a = 0$ is given in Section 4.

2.3 Disturbance cancellation by output feedback controller

Assume that K_m and K_d that stabilize the observer (12) ~ (14) can be found. The observer is used to construct a controller cancelling the effect of the disturbance by its estimate. The control input is given by

$$u(t) = -F \begin{bmatrix} \hat{x}_m(t) \\ \hat{x}_a(t) \end{bmatrix} - \hat{d}(t) , \qquad (15)$$

the matrix F is a feedback gain matrix that makes the matrix

$$\begin{bmatrix} A & B_m C_a \\ 0 & A_a \end{bmatrix} - \begin{bmatrix} B_m D_a \\ B_m \end{bmatrix} F \tag{16}$$

stable. Define the partition of the matrix F as

$$F = \begin{bmatrix} F_m & F_a \end{bmatrix}. \tag{17}$$

The control law (15) can be expressed as

$$u(t) = -F(s)\hat{x}_m(t) - \hat{d}(t),$$
 (18)

where

$$F(s) \triangleq [I + F_a(sI - A_a)^{-1}B_a]^{-1}F_m.$$
 (19)

The above result suggests that the control law (15) can be realized by the estimate feedback of the minimum phase state with the frequency-shaped feedback gain matrix F(s).

Guo *et al.* (1992, 1995; 1996a, b) and Ishihara (1995) have used factorizations of sensitivity matrices instead of loop transfer matrices to discuss LTR methods. In the following discussion, this approach is adopted.

From straightforward matrix calculations using $(12) \sim (14)$ and (18), the sensitivity matrix at the input of the plant can be expressed as follows.

Proposition 1: Consider the control system consisting of the plant (1) and the output feedback controller (18). Then the sensitivity matrix at the plant input side can be factored as

$$\Sigma(s) = s\Sigma_{F}(s)[I + F(s)(sI - A + K_{m}C)^{-1}$$

$$B_{m}G_{a}(s)][sI + K_{d}C(sI - A + K_{m}C)^{-1}$$

$$B_{m}G_{a}(s)]^{-1},$$
(20)

where

$$\Sigma_{F}(s) \triangleq [I + F(s)(sI - A)^{-1}B_{m}G_{a}(z)]^{-1}$$
 (21)

is the sensitivity matrix for the minimum phase state feedback regulator with the frequency-shaped gain matrix F(s) defined in (19).

Remark: The zero at s = 0 in the expression (20) explicitly shows that the controller introduces the integral action.

Remark: Since the disturbance rejection by feedback is primary concern of this paper, a reference input is not included in the control law (15). A reference input can easily be introduced by the standard techniques.

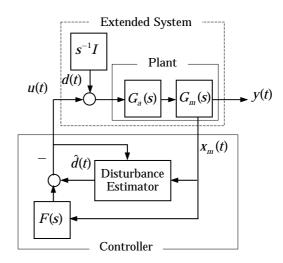


Fig. 2. The target control system

3. TARGET CONTROL SYSTEM

The target of the conventional partial LTR technique (Moore and Xia, 1987) is a partial state feedback controller using only the minimum phase state variables. For the present problem, a partial state feedback controller including a disturbance estimator is considered as a target.

The target is derived on the assumption that the state vector $x_m(t)$ of the minimum phase part can be measured perfectly. This assumption is impractical but has a clear system-theoretic meaning.

The target controller generates the control input

$$u(t) = -F(s)x_{m}(t) - \hat{d}(t),$$
 (22)

where F(s) is a frequency-shaped feedback gain matrix defined in (19). An estimator generating the disturbance estimate $\hat{d}(t)$ in (22) can be constructed as follows.

3.1 Disturbance estimator

Since the state $x_m(t)$ for the minimum phase part is measurable, we can write the observation relation as

$$\dot{\xi}_{a}(t) = \Phi_{a}(s)\xi_{a}(t) + \Gamma_{a}u(t),
\dot{x}_{m}(t) - Ax_{m}(t) - B_{m}D_{a}u(t) = H_{a}\xi_{a}(t),$$
(23)

where

$$\xi_{a}(t) \triangleq \begin{bmatrix} x'_{a}(t) & d'(t) \end{bmatrix}',$$

$$H_{a} \triangleq \begin{bmatrix} B_{m}C_{a} & B_{m}D_{a} \end{bmatrix}, \qquad (24)$$

$$\Phi_{a} \triangleq \begin{bmatrix} A_{a} & B_{a} \\ 0 & 0 \end{bmatrix}, \Gamma_{a} \triangleq \begin{bmatrix} B_{a} \\ 0 \end{bmatrix}.$$

From (23), an observer for estimating the state $\xi_a(t)$ can be constructed as

$$\dot{\hat{\xi}}_{a}(t) = \Phi_{a}\hat{\xi}_{a}(t) + \Gamma_{a}u(t) + K^{*}[\dot{x}_{m}(t) - Ax_{m}(t) - B_{m}D_{a}u(t) - H_{a}\hat{\xi}_{a}(t)],$$
(25)

where

$$\hat{\xi}_a(t) \triangleq \left[\hat{x}_a'(t) \quad \hat{d}'(t) \right]' \tag{26}$$

is the estimate of $\xi_a(t)$ and K^* is a gain matrix that makes the matrix $(\Phi_a - K^*H_a)$ stable.

On the assumption that the matrix B_m is column full rank in addition to C1 ~ C3, the PBH test shows that the pair (H_a, Φ_a) is observable. However, this is not sufficient for the design of a target controller for the partial LTR.

The estimate of the all-pass state $x_a(t)$ provided by the observer (25) generally depends on the measured $x_m(t)$ if no constraint is imposed on the structure of the gain matrix K^* . Note that, in the output feedback case, the state $x_a(t)$ is unobservable from the output. A controller using an estimate of $x_a(t)$ based on the measured $x_m(t)$ is inappropriate as a target of the partial LTR.

To provide a target with clear design perspective, the observer gain matrix in (25) is assumed to have a constrained structure

$$K^* = \begin{bmatrix} 0 \\ K_d^* \end{bmatrix}, \tag{27}$$

where K_d^* is a gain matrix for the disturbance estimation. For this choice, the following result is obtained.

Lemma 1: Define the function

$$\phi^*(s) \triangleq \det \left[I + \frac{1}{s} B_m G_a(s) K_d^* \right]. \tag{28}$$

Then the estimator (25) with the gain matrix (27) is stable if and only if all the zeros of $\phi^*(z)$ have negative real parts.

Proof: Define the return difference matrix

$$R^*(s) \triangleq I + H_a(sI - \Phi_a)^{-1}K^*$$
. (29)

It is well known that the determinant of the return difference matrix satisfies

$$\det[R^*(s)] = \frac{\det(sI - \Phi_a + K^*H_a)}{\det(sI - \Phi_a)}.$$
 (30)

From (24) and (27), the return difference matrix can be written as

$$R^*(s) = I + \frac{1}{s} B_m G_a(s) K_d^*.$$
 (31)

Note that $\phi^*(s) = \det[R^*(s)]$. From (30), the zeros of $\phi^*(s)$ are the eigenvalues of the matrix $(\Phi_a - K^*H_a)$, which completes the proof.

The following condition is introduced:

C4: The estimator gain matrix K_d^* in (27) is chosen such that all the zeros of $\phi^*(s)$ have negative real parts.

Note that the function (28) can be rewritten as

$$\phi^*(s) = \det \left[I + \frac{1}{s} G_a(s) K_d^* B_m \right].$$
 (32)

The generalized Nyquist criterion can be applied for (28) or (32) to check the condition C4 for a given K_d^* . The problem to find a set of K_d^* satisfying C4 is not simple for the general case but is solvable possibly with a help of symbolic computation. For single input plants with a few unstable zeros, the problem is easy as shown by the following simple example.

Example: Consider a s scalar plant (m=1) with a single unstable zero at $s=\alpha>0$. To simplify discussion, define $k_m\triangleq K_d^*B_m$, which is scalar in this case. If we choose $G_a(s)=(\alpha-s)/(\alpha+s)$, the function (28) is given by $\phi^*(s)=1+k_m(\alpha-s)/[s(\alpha+s)]$. It readily follows that the set of k_m satisfying C4 is simply given by $0< k_m < \alpha$.

Remark: For minimum phase plants, a full state feedback controller can be chosen as a target. Then a large freedom exists to choose the estimator gain matrix. On the other hand, in the present case, the choice of the estimator gain matrix is restricted. If we choose K_d^* sufficiently large, the function (28) has zeros near unstable zeros of $G_a(s)$ and the condition C4 fails. This is a price paid for dealing with non-minimum phase plants.

3.2 Target feedback property

The elements of $\hat{\xi}_a(t)$ in (25) with the constrained estimator gain matrix (27) can be written as

$$\dot{\hat{x}}_{a}(t) = A_{a}\hat{x}_{a} + B_{a}[\hat{d}(t) + u(t)],
\dot{\hat{d}}(t) = K_{d}^{*}\{\dot{\hat{x}}_{m}(t) - A\hat{x}_{m}(t) - B_{m}D_{a}u(t)
-[B_{m}C_{a}\hat{x}_{a} + B_{m}D_{a}\hat{d}(t)]\}.$$
(33)

It follows from (33) that the Laplace transforms of the disturbance can be written as

$$\hat{d}(s) = [sI + K_d^* B_m G_a(s)]^{-1} K_d^*$$

$$[(sI - A)x_m(s) - B_m G_a(s)u(s)].$$
(34)

From (22) and (34), the transfer function matrix from the minimum phase state $x_m(t)$ to the control input is written as

$$C^{*}(s) \triangleq -s^{-1}\{[sI + K_{d}^{*}B_{m}G_{a}(s)]F(s) + K_{d}^{*}(sI - A)\}.$$
(35)

Straightforward matrix calculations using the above expression gives the following result for the target sensitivity matrix.

Proposition 2: Consider the target control system consisting of the plant (1) and the controller (35). Then the sensitivity matrix at the plant input

$$\Sigma^*(s) \triangleq [I - C^*(s)G(s)]^{-1}$$
 (36)

can be factored as

$$\Sigma^*(s) = s\Sigma_F(s)[sI + K_d^* B_m G_a(s)]^{-1}, \qquad (37)$$

where $\Sigma_F(s)$ is defined in (21).

4. PARTIAL RECOVERY PROCEDURE

Due to the fact pointed out in the second remark in Section 2, it is difficult to apply the partial LTR procedure proposed by Moore and Xia (1987) directly to recover the target feedback property given in Section 3. In this section, a new partial LTR procedure is proposed to recover the target given in Section 3.

Consider a stochastic version of the model (6)

$$\dot{\xi}(t) = \Phi \xi(t) + \Gamma u(t) + \Gamma_m^* w(t)$$

$$v(t) = H \xi(t) + v(t),$$
(38)

where w(t) and v(t) are mutually independent zero-mean white noise processes. The covariance matrices are given by

$$E[w(t)w'(\tau)] = \sigma I \delta(t - \tau),$$

$$E[v(t)v'(\tau)] = V \delta(t - \tau),$$
(39)

where $\sigma > 0$ and V > 0. The matrices Φ and H are defined in (8). The key of our recovery procedure is to choose the matrix Γ_m^* as

$$\Gamma_m^* \triangleq \begin{bmatrix} B_m \\ 0 \\ K_d^* B_m \end{bmatrix}, \tag{40}$$

which includes the estimator gain matrix K_d^* used in the target.

The Kalman filter gain matrix for the above system is given by

$$K(\sigma) \triangleq \Pi H' V^{-1}$$
, (41)

where Π is a solution of the Riccati equation

$$\Pi \Phi' + \Phi \Pi$$

$$-\Pi H' V^{-1} H \Pi + \sigma \Gamma_{mer}^* (\Gamma_m^*)' = 0.$$
(42)

The existence of a stabilizing solution of the Riccati equation (42) is guaranteed by the following lemma.

Lemma 2: Assume the conditions C1 ~ C4. Then the pair (Φ, Γ_m^*) is stabilizable and (H, Φ) is detectable. In addition, the invariant zeros of the realization (Φ, Γ_m^*, H) are stable.

Proof: The stabilizability of (Φ, Γ_m^*) is shown by the PBH test. Define the matrix

$$C^*(\lambda) \triangleq \left[\lambda I - \Phi \quad \Gamma_m^* \right]. \tag{43}$$

Introduce the vectors μ , η , ζ compatible with the block matrices in Φ defined in (8). Assume that

$$\begin{bmatrix} \mu' & \eta' & \zeta' \end{bmatrix} C^*(\lambda) = 0. \tag{44}$$

From (8), (40) and (43), the following simultaneous equations are obtained.

$$\mu'(\lambda I - A) = 0 \tag{45}$$

$$\mu' B_m C_a = \eta' (\lambda I - A_a) \tag{46}$$

$$\mu' B_m D_a + \eta' B_a = \lambda \zeta' \tag{47}$$

$$\mu' B_m + \zeta' K_d^* B_m = 0 (48)$$

Assume that the real part of λ is positive. Then the matrix $(\lambda I - A_a)$ is non-singular. It follows from (46) and (47) that

$$\lambda'\zeta = \mu' B_m G_a(\lambda) . \tag{49}$$

Eliminating ζ from (48) and (49) gives

$$\mu' B_m \left[I + \frac{1}{\lambda} G_a(\lambda) K_d^* B_m \right] = 0.$$
 (50)

Note that the function $\phi^*(s)$ defined in (28) can be written as (32). The condition C4 regarding the K_d^* implies that the $[I + \lambda^{-1}G_a(\lambda)K_d^*B_m]$ in (50) is non-singular for λ with a positive real part. It follows that $\mu' B_m = 0$ which implies that $\mu = 0$ by the controllability of (A_m, B_m) . Substitution of $\mu = 0$ into (46) and (49) gives $\eta = 0$ and $\zeta = 0$, respectively. From (44), it is concluded that the matrix (43) is row full rank for all λ with a positive real part. Therefore, the pair (Φ, Γ_{m}^{*}) is stabilizable. The detectability of the pair (H, Φ) can easily be proved by the PBH test as pointed out in Section 2. To show that the invariant zeros are stable, we note that the transfer function matrix of (Φ, Γ_m^*, H) is given by

$$H(sI - \Phi)^{-1}\Gamma_m^* = G_m(s) \left[I + \frac{1}{s} G_a(s) K_d^* B_m \right].$$
 (51)

The invariant zeros of (Φ, Γ_m^*, H) consist of the transmission zeros of $G_m(s)$ and the zeros of the

function (32). These zeros are stable under the assumptions. \blacksquare

The above result shows that the well-known result for the asymptotic behaviour of the Kalman filter gain matrix (*e.g.*, Anderson and Moore, 1990) can be applied. The result is stated as follows.

Lemma 3: Assume the conditions $C1 \sim C4$. The Kalman filter gain matrix (41) satisfies the asymptotic relation

$$\lim_{\sigma \to \infty} \sigma^{-1/2} K(\sigma) V^{1/2} = \Gamma_m^*$$
 (52)

where Γ_m^* is defined by (40).

The main result for the partial LTR is given as follows.

Proposition 3: Assume the conditions $C1 \sim C4$. Consider the output feedback controller (18) with the observer (12) \sim (14). Assume that the observer gain matrix is determined by using the Riccati equation (42). Then, as the scalar parameter σ tends to the infinity, the sensitivity matrix $\Sigma(s)$ given by (20) approaches the target sensitivity matrix $\Sigma^*(s)$ given by (37).

Proof: From Lemma 3, the Kalman filter gain matrix $K(\sigma)$ for sufficiently large σ can be written as

$$K(s) = \begin{bmatrix} K_{m}(\sigma) \\ K_{a}(\sigma) \\ K_{d}(\sigma) \end{bmatrix} = \begin{bmatrix} \sigma^{1/2} B_{m} V^{-1/2} \\ 0 \\ \sigma^{1/2} K_{d}^{*} B_{m} V^{-1/2} \end{bmatrix}.$$
 (53)

For the above gain matrix, the asymptotic expressions of the two transfer function matrices in (20) can be obtained as follows:

$$[sI - A + K_m(\sigma)C]^{-1}B_m$$

$$= (sI - A)^{-1}$$

$$\times B_m[I + \sigma^{1/2}V^{-1/2}C(sI - A)^{-1}B_m]^{-1}$$

$$\to 0 \ (\sigma \to \infty)$$
(54)

$$K_{d}(s)C[sI - A + K_{m}(\sigma)C]^{-1}B_{m}$$

$$= \sigma^{1/2}K_{d}^{*}B_{m}V^{-1/2}C(sI - A)^{-1}B_{m}$$

$$\times [I + \sigma^{1/2}V^{-1/2}C(sI - A)^{-1}B_{m}]^{-1}$$

$$= K_{d}^{*}B_{m}$$

$$\times \{I + \sigma^{-1/2}[C(sI - A)^{-1}B_{m}]^{-1}V^{-1/2}\}^{-1}$$

$$\to K_{d}^{*}B_{m} \quad (\sigma \to \infty)$$
(55)

It readily follows from (20), (54) and (55) that $\Sigma(s) \to \Sigma^*(s)$ as $\sigma \to \infty$.

5. CONCLUSIONS

A new partial LTR technique for the integral controller design based on the disturbance cancellation has been proposed. The method provides a systematic procedure for determining numerous design parameters under clear design perspective Inherent limitations of non-minimum phase plants are reflected in the design of the target controller as well as the proposed recovery procedure.

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