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ON SOME STABILITY REGIONS OF A MULTIPLE DELAYS LINEAR SYSTEM WITH APPLICATIONS

S.-I. Niculescu *

* *HEUDIASYC (UMR CNRS 6599), Université de Technologie de Compiègne, Centre de Recherche de Royallieu, BP. 20529, 60205, Compiègne, France.*
E-mail: silviu@hds.utc.fr

Abstract: This paper focuses on the asymptotic stability of a class of linear systems including multiple (commensurate/uncommensurate) pointwise delays. More precisely, we shall characterize some stability/instability regions in the delays parameter-space using a frequency-domain approach.
As potential applications, we shall discuss the robustness analysis in terms of delays of a fluid network approximation (see, for instance, the models proposed by Izmailov, 1996). *©2001 IFAC*

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1. INTRODUCTION

The complete characterization of stability regions for linear delay systems in the parameters space is still *open* in the general case (see, e.g. Diekmann *et al*, 1995). Furthermore, it was proved that the discrete or pointwise delays case involving uncommensurate delays (rationally independent) is $\mathcal{NP}$-*hard* (Toker and Ozbay, 1996). A different argument, but with the same conclusion was proposed by Meinsma *et al.* (1996) for the stability analysis of some feedback schemes in the case of *small delays*. However, the commensurate delays case can be handled using, for example, matrix pencils methods (Chen *et al.*, 1994; Niculescu, 1997). A complete discussion and further comments on the methodology and algorithms simplification can be found in Niculescu (2001a).

In the sequel, we shall focus on the asymptotic stability analysis of a class of linear delay systems involving a 'combination' of commensurate/uncommensurate delays suggested by one of the fluid approximation models proposed in Izmailov (1995, 1996). More precisely, we shall consider the following second order delay system:

$$\ddot{y}(t) + \sum_{k=0}^{n} a_k y\left(t - \frac{k}{n} r - \tau\right) = 0, \qquad (1)$$

with the initial condition defined by:

$$y(\theta) = \phi(\theta), \quad \forall \theta \in [-r-\tau, 0], \qquad \phi \in \mathcal{C}([-\tau - r, 0], \mathbf{R}). \qquad (2)$$

The delays τ and r are rationally independent, but the model includes commensurate delays in terms of r. The cases $n = 1, 2$ were considered by Izmailov (1996) for handling congestion in high-speed networks: τ representing the round-trip time and r some control time-interval. Some issues on robustness analysis in the delays parameter-space if $n = 1$ were proposed in Niculescu (2001b).

The aim of this paper is to analyze the stability of (1)-(2) in the delays parameter-space $Or\tau$. To the best of the author's knowledge, if the scalar case including two uncommensurate delays was treated in the 1990s (see the paper of Hale and

Huang, 1993), the second-order system case was *not completely performed*, and its characterization is still *open*. Some analysis cases can be found in Freedman and Rao (1986), Stépàn (1989), Boese (1995), to cite only a few.

Note that if there are no delays $r = \tau = 0$, and if $\sum_{k=0}^{n} a_k > 0$, the system (1) is a second-order *oscillator*. Furthermore, if one assumes $r \neq 0$, but $\tau = 0$, we have the closed-loop system of an oscillator with an appropriate output delayed feedback including commensurate delays and various gains a_k $(k = \overline{1, n})$. The case of a single delay in the output was already considered in the literature, and we have seen that the delay may have a *stabilizing* effect (see, for instance, Abdallah *et al.*, 1993, Niculescu and Abdallah, 2000).

Note that the method of Chen *et al.* (1994) based on matrix pencils can not be applied directly for the first delay-interval guaranteeing stability in the commensurate delays case $(r \neq 0, \tau = 0)$ since the system free of delays $(r = \tau = 0)$ is *not stable*.

In the sequel, we shall use some of the ideas proposed in Niculescu (1997) (see also Niculescu, 2001a) in the commensurate delays case to characterize the first stability region in the delays parameter-space if one assumes r and τ rationally independent. Furthermore, the methodology proposed here can be extended to more general systems including commensurate/uncommensurate delays.

The paper is organized as follows: Section 2 includes some preliminary results on matrix pencils. Section 3 is devoted to the main results, and Section 4 to some interpretations in networks congestion control. Some concluding remarks end the paper. The notations are standard.

2. PRELIMINARY RESULTS AND DEFINITIONS

Simple computations allow to rewrite the system (1)-(2) as follows:

$$\dot{x}(t) = \sum_{k=0}^{n} A_k x\left(t - \frac{k}{n} r - \tau\right), \tag{3}$$

where:

$$A_0 = \begin{bmatrix} 0 & 1 \\ -a_0 & 0 \end{bmatrix}, \ A_k = \begin{bmatrix} 0 & 0 \\ -a_k & 0 \end{bmatrix}, \quad k > 0,$$

and $x = [y \ \dot{y}]^T$.

Introduce now the following matrix pencils $\Lambda_1 \in \mathbf{C}^{2n \times 2n}$ and $\Lambda_2 \in \mathbf{C}^{n \times n}$:

$$\Lambda_1(z) = z \begin{bmatrix} 1 & & 0 & 0 \\ & \ddots & & \\ 0 & & 1 & 0 \\ 0 & \dots & 0 & a_n \end{bmatrix} + \begin{bmatrix} 0 & -1 & \dots & 0 & 0 & 0 & \dots & 0 \\ & & \ddots & & & & & \\ 0 & & & & & & & -1 \\ -a_n & -a_{n-1} & \dots & -a_1 & 0 & a_1 & \dots & a_{n-1} \end{bmatrix} \tag{4}$$

$$\Lambda_2(z) = z \begin{bmatrix} 1 & & 0 & 0 \\ & \ddots & & \\ 0 & & 1 & 0 \\ 0 & \dots & 0 & a_n \end{bmatrix} + \begin{bmatrix} 0 & -1 & & 0 \\ & & \ddots & \\ 0 & 0 & & -1 \\ a_0 & a_1 & \dots & a_{n-1} \end{bmatrix}, \tag{5}$$

where Λ_1 was defined using appropriate matrix tensor products and sums (which generalize the Kronecker products and sums, with a lower size). Using the same language as in Niculescu (2001a), the matrix pencil Λ_1 is associated to finite delays and Λ_2 to infinite delays for the system (3) with $\tau = 0$.

Recall the following result from Niculescu (1997, 2001a), valid also in the general case:

Lemma 1. The following statements are true:

1) The complex number $z \in \mathbf{C}^*$, $\mid z \mid \neq 1$ is a generalized eigenvalue of the matrix pencil Λ_1 if and only if z^{-1} is an eigenvalue of Λ_1.
2) All the generalized eigenvalues on the unit circle of the matrix pencil Λ_2 are also eigenvalues on the unit circle for Λ_1.

Consider the original system (1)-(2), and assume that $\tau = 0$. Denote $\sigma(\Lambda)$ the set of general eigenvalues of the matrix pencil Λ. The same notation will be also used for the eigenvalues of some real or complex matrix.

Let us assume that $r = \bar{r}$ be a positive value. In the sequel, one needs to introduce the following sets:

$$\begin{aligned} \Lambda_{\bar{r},+} = \Big\{ (r_{h_i}, \alpha_h) : \ & r_{h_i} = \frac{\alpha_h}{\omega_{hi}} > \bar{r} : \\ & e^{-j\alpha_h} \in \sigma(\Lambda_1) - \sigma(\Lambda_2), \\ & j\omega_{hi} \in \sigma\left(\sum_{k=0}^{n} A_k e^{-j\frac{\alpha_h k}{n}}\right) - \{0\}, \\ & 1 \leq h \leq 2n, \ 1 \leq i \leq n \Big\}, \end{aligned} \tag{6}$$

$$\begin{aligned} \Lambda_{\bar{r},-} = \Big\{ (r_{h_i}, \alpha_h) : \ & r_{h_i} = \frac{\alpha_h}{\omega_{hi}} < \bar{r} : \\ & e^{-j\alpha_h} \in \sigma(\Lambda_1) - \sigma(\Lambda_2), \\ & j\omega_{hi} \in \sigma\left(\sum_{k=0}^{n} A_k e^{-j\frac{\alpha_h k}{n}}\right) - \{0\}, \\ & 1 \leq h \leq 2n, \ 1 \leq i \leq n \Big\}. \end{aligned} \tag{7}$$

As seen in Niculescu (1997, 2001a) the set $\Lambda_{\bar{r},+}$ allows to characterize the delay values $\bar{r}_{h_i} > \bar{r}$ for which some roots of the associated characteristic equation cross the imaginary axis. The same conclusion holds for the set $\Lambda_{\bar{r},-}$, but with the difference that $\bar{r}_{h_i} < \bar{r}$.

In other words, if the original system is asymptotically *stable* for the delay value $r = \bar{r}$, then $\Lambda_{\bar{r},+}$ and $\Lambda_{\bar{r},-}$ will give the upper and respectively the lower bound on r such that the corresponding delay-interval including $\bar{r}$ will guarantee (*necessary and sufficient* conditions) the asymptotic stability of the original system for all delay values inside the computed interval (Niculescu, 2001a).

If the system is *unstable* for $r = \bar{r}$ it will be unstable for all delay values inside the same interval. In such a case, we shall discuss about a *hyperbolicity* type property (see, for instance, Hale *et al.*, 1985 for the delay-independent case or the matrix pencil characterization in Niculescu, 1997).

Remark 1. The method briefly presented above can be found in Niculescu (1997) and it extends the ideas of Chen *et al* (1994) to handle the stability for delay intervals. Furtheremore, the matrix pencil Λ_1 here is $2n \times 2n$, and the corresponding matrix pencil in Chen *et al.* (1994) is $8n \times 8n$. Note that a different idea extending the approach of Kamen (1980, 1982) for delay intervals *robustness* analysis can be found in Chiasson and Abdallah (2001).

3. MAIN RESULTS

Assume that the system free of delays is an oscillator. The first step is to find conditions such that there exists sufficiently small delays guaranteeing the asymptotic stability of (1)-(2).

Proposition 1. (small delays). Assume that

$$\sum_{k=0}^{n} a_k > 0, \sum_{k=1}^{n} ka_k < 0. \tag{8}$$

Then there exists a sufficiently small positive value $\varepsilon > 0$, such that (1)-(2) with $r = \varepsilon$ and $\tau = 0$ is asymptotically stable.

Remark 2. If $n = 1$, (8) rewrites as $a_0 >| a_1 |$, $a_1 < 0$, conditions which are satisfied in the examples proposed by Abdallah *et al.* (1993), and Niculescu (2001b). Since for sufficiently small delays the system becomes stable, we have one *reversal* at $r = 0$.

The next step is to find the *upper* bound on τ guaranteeing the asymptotic stability of (1)-(2) with $\tau > 0$ and $r = 0$. We shall use the methodology proposed in Niculescu (2001a) and based on the generalized eigenvalues distribution of some appropriate matrix pencils defined using matrix tensor products and sums.

Proposition 2. (generalized eigenvalue distribution). Let $z_0 \in \mathcal{C}(0,1)$ be a simple generalized eigenvalue of the matrix pencil Λ_1, which is not eigenvalue of the pencil Λ_2, and let $(r_{i,0}, \alpha_{i,0})$ one of the corresponding (simple) generated pair in the set $\Lambda_{0,+}$ for some integer i.
Then:

i) If $\mathbf{sgn}\left(\sum_{k=1}^{n} ka_k cos\frac{k\omega_{i,0}r_{i,0}}{n}\right) = +1$, there are roots of the characteristic equation associated to (1)-(2) that cross the imaginary axis from left to right.

ii) If $\mathbf{sgn}\left(\sum_{k=1}^{n} ka_k cos\frac{k\omega_{i,0}r_{i,0}}{n}\right) = -1$, there are roots of the characteristic equation associated to (1)-(2) that cross the imaginary axis from right to left.

The proof (see Niculescu, 2001c, the full version of the paper) uses the generalized eigenvalue characterization of the corresponding associated matrix pencils Λ_1, Λ_2 with respect to the unit circle combined with some ideas developed in Cooke and van den Driessche (1986) concerning the sign of $\frac{ds}{dr}\,|_{s=j\omega_0}$. Since the generalized eigenvalue, as well as the corresponding generated pair are simple, the sign function should be either positive, either negative, *etc.*

As we have already seen that at $r = 0+$, the system becomes *stable*, the corresponding value $r = 0$ is called *delay reversal* (see, for instance, Cooke and van den Driessche (1986) and the references therein), the next step is to find the "next" delay value $r_1 > 0$ such that the system changes once again its stability behavior, from stability to instability this time. This delay value is known as a *delay switch.*

Based on the results above, it follows that:

Proposition 3. (First switch). The system (1)-(2) satisfying the condition (8) is asymptotically stable for all delay values r:

$$0 < r < r_1(a_0, \ldots, a_n),$$

where

$$r_1(a_0, \ldots a_n) = \mathbf{inf}\,\{\gamma : \quad (\gamma, \alpha) \in \Lambda_{0,+}\}. \tag{9}$$

Furthermore, if $r = 0$ or $r = r_1(a_0, \ldots, a_n)$, the corresponding associated characteristic equation has at least one pair of roots on the imaginary axis.

Remark 3. Simple computations lead to the following first-delay interval if $n = 1$, and $a_0 >| a_1 |$, $a_1 < 0$:

$$0 < r < r_1(a_0, a_1) = \frac{\pi}{\sqrt{a_0 + \mid a_1 \mid}},$$

as seen in Abdallah *et al.* (1993) or Niculescu and Abdallah (2000).

Using the continuity properties of the roots of the associated characteristic equation in terms of delays (see Els'golts' and Norkin, 1973), a natural consequence of Proposition 1 is the following:

Corollary 1. (Small delays). If the system (1)-(2) satisfies the constraints (8), then there exist sufficiently small positive values $\varepsilon_1, \varepsilon_2 > 0$, such that the original system with $r = \varepsilon_1$ and $\tau = \varepsilon_2$ is asymptotically stable.

We have the following results (proof in the full version of the paper):

Proposition 4. (Delay bounds). The system (1)-(2) satisfying the constraints (8) is asymptotically stable for all delays r and τ:

$$\begin{cases} 0 < r < r_1(a_0, \ldots, a_n) \\ 0 < \tau < \tau(r) = \\ \mathbf{min}_{\omega_s} \left\{ \frac{1}{\omega_s} \cdot \mathbf{arctan} \frac{-\sum_{k=1}^{n} a_k sin \frac{\omega_s kr}{n}}{\sum_{k=0}^{n} a_k cos \frac{\omega_s kr}{n}} \right\}, \end{cases} \tag{10}$$

where ω_s belongs to the set of positive solutions of the equation:

$$\omega^4 = \sum_{k=0}^{n} a_k^2 + 2 \sum_{k=1}^{n} \sum_{h=0}^{k-1} a_k a_h cos\left(\frac{(k-h)\omega r}{n}\right). \tag{11}$$

Furthermore, assume that the chosen delay r and the solution $\tilde{\omega}_s$ defining the corresponding upper bound $\tau(r)$ in (10) satisfy the condition:

$$\tilde{\omega}_s^3 > \frac{1}{2} \sum_{k=1}^{n} \sum_{h=0}^{k-1} \frac{a_k a_h (k-h) r}{n} sin\left(\frac{(k-h)\tilde{\omega}_s r}{n}\right). \tag{12}$$

Then:

- i) if $\tau = \tau(r) + \varepsilon$, with $\varepsilon > 0$ sufficiently small, the system (1)-(2) is unstable.
- ii) if the equation (11) has only one positive solution, then there does not exist any $\tau > \tau(r)$ such that the system (1)-(2) is asymptotically stable.

Remark 4. If one takes $n = 1$, the Proposition above recovers the stability result proposed in Niculescu (2001b).

Remark 5. (Instability). It seems clear that the same delay interval $[0, \tau(\hat{r}))$ leads to the *instability* of the system (1)-(2) for a given $(n+1)$-tuple $(a_0, \ldots, a_n)$ satisfying the constraints (8) if the original system with $r = \hat{r} > r_1(a_0, \ldots, a_n)$ and $\tau = 0$ is *unstable*.
Furthermore, the instability is *delay-independent* if the equation (11) has *only* one positive solution.

Remark 6. (Instability persistence). It is important to note that the equation (11) has always at least one positive solution for ω in the set

$$\cup_{i \in \mathcal{I}_n} \left[\mathbf{min} \sqrt{\sum_{k=1}^{n} (-1)^{i_k} a_k^2}, \mathbf{max} \sqrt{\sum_{k=1}^{n} (-1)^{i_k} a_k^2} \right],$$

where

$$\mathcal{I}_n = \{ i = (i_1, \ldots i_n) : \quad i_k \in \{1, 2\}, \forall k = \overline{1, n} \}$$

is an appropriate index family, *etc.*
So, in conclusion, the *upper* bound on τ will be *always finite*, that is one may expect a sequence of stability/instability delay intervals in terms of τ, with *instability persistence* for huge delay values (see also Niculescu, 2001a), that is there exists a value $\bar{\tau}$ such that for all $\tau > \bar{\tau}$ the corresponding delay system is unstable.

4. VARIOUS INTERPRETATIONS IN NETWORKS CONGESTION CONTROL ROBUSTNESS

Izmailov (1995, 1996) considered the model above with $n = 1$ and $n = 2$ as a feedback control algorithm for data transfer in high-speed networks. As it was remarked in Niculescu (2001b) in the case $n = 1$, *decreasing the duration of control-time intervals* (the delay r) is *not always the best solution to improve the performances of the algorithm in terms of delay robustness.*

Indeed, the control-time interval is an important *factor* to *induce stability* or *instability* in terms of "large" values of the round-trip time (the delay τ) for the same values of the corresponding gains (a_0, a_1 in the case $n = 1$). Note also that Izmailov (1996) has choosen a_0 and a_1 having *opposite* sign, but the argument was different.

The hypothesis here was to choose the gains such that the system free of delays is an *oscillator*. The same ideas can be applied even in the case of *instability*, the only difference beeing the *instability* of the original system for *small delays* since there are no roots crossing the imaginary axis at $r = 0+$. Furthermore, in this last case, there exists gain pairs (a_0, a_1) for which the system is *delays-independent unstable*.

The same ideas hold in the case $n = 2$, although it seems that we have more degree-of-freedom in choosing the *gains*: a_0, a_1 and a_2. In fact, improving the *performances* of the algorithm leads to *important* values of the corresponding gains. Note however some improvement of the system's behavior for *small delays* τ, r, but a *degradation* of the *performances* when the delays are increased.

Note that our intention was not to discuss the algorithm itself, but to point out some of its *limitations* in terms of delays, that is some *delay-insensitive* measures for the *uncertainty* in the *knowledge* of the round-trip times. Thus, a natural consequence of the results above is the following:

Corollary 2. (Stability robustness). The control algorithm defined by the $(n+1)$-tuple $(a_0, \ldots, a_n)$ satisfying the constraints (8) with the control-time interval $r \in (0, r_1(a_0, \ldots, a_n))$ guarantees the stability of the fluid model if the round-trip delay τ takes any positive value in the interval $[0, \tau(r))$, where $\tau(r)$ is defined in (12).

Remark 7. (Instability persistence). When approaching the *congestion phenomenon* in actual high-speed networks, the round-trip times becomes larger and larger, and it is possible to belong not to the first delay-interval guaranteeing stability, but to the next ones in terms of τ for the same control-time interval r and the same gains a_k, $k = \overline{0, n}$ till the bound $\bar{\tau}$ discussed in Remark 6 is not reached.
If $\tau > \bar{\tau}$, the instability of the continuous delay model above can be interpreted as follows: some packages will be lost in the data transfer from some source to some destination using the control algorithm above for one of the congestion nodes in the path, *etc.*

Remark 8. The maximal delay bound $\tau(r)$ guaranteeing stability can be *optimized* function of $r \in (0, r_1(a_0, \ldots, a_n))$, and we shall have a *'max-min'* problem.

Remark 9. Various numerical experiments have been performed and are included in the full version of the paper (Niculescu, 2001c). A simple case: $n = 1$, $a_0 = 2$, $a_1 = -1.75$ was largely discussed in Niculescu (2001b) including also the optimization problem cited above.

5. CONCLUDING REMARKS

In this paper, we have considered a second-order linear system involving commensurate/uncommensurate delays, system suggested by some fluid approximation model encountered in the high-speed network control congestion analysis.

The characterization of some stability regions in the delays parameter-space was performed using the generalized eigenvalues distribution of some appropriate matrix pencils. The advantage of the method lies in the possibility to handle more general systems class.

The derived results allowed various interpretations in robustness analysis of some congestion control algorithm in high-speed networks.

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