# FAULT DIAGNOSIS IN NONLINEAR DYNAMIC SYSTEMS VIA LINEAR METHODS 

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#### Abstract

The problem of observer-based fault diagnosis of a certain class of nonlinear dynamic systems is studied. To solve this problem, the following approach is suggested: replacing the initial nonlinear system by certain linear logic-dynamic system, obtaining the bank of linear logic-dynamic observers, and transforming these observes into the nonlinear ones. The procedure of the linear logic-dynamic observers synthesis is developed. Copyright ${ }^{\oplus} 2002$ IFAC


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## 1. INTRODUCTION

There are a lot of papers dealing with the problem of fault detection and isolation (FDI) in dynamic systems (see surveys by Frank, 1990; Gertler, 1993; Patton, 1994). Most of them concern linear systems and far fewer nonlinear dynamic systems (Seliger and Prank, 1991; Shields, 1996; Zhirabok and Shumsky, 1987; Zhirabok, 1997). Last paper is based on mathematical techniques requiring rather complex analytical transformations therefore it is difficult to use them in practice.

An interesting approach to the FDI was developed by Frank and Wunnenberg (1989) for systems described by the following type of equations

$$
\begin{equation*}
\dot{x}=F x(t)+B(y(t), u(t)), \quad y(t)=H x(t) \tag{1}
\end{equation*}
$$

where $\mathrm{x}(\mathrm{t})$ is the $\mathrm{n} \times 1$ state vector, $u(\mathrm{t})$ the $\mathrm{m} \times 1$ vector of control, $\mathrm{y}(\mathrm{t})$ the $l \times 1$ vector of measured outputs, $\mathrm{F}, \mathrm{H}$
known matrices and B known vector function of appropriate dimensions. The feature of this type of systems is that the system nonlinearities can be expressed as a function of the input, $u(t)$, and output, $\mathrm{y}(\mathrm{t})$.

In this paper, we consider more general class of nonlinear systems described by the equations

$$
\begin{align*}
& \dot{x}(\mathrm{t})=\mathrm{F}(\gamma(\mathrm{t})) \mathrm{x}(\mathrm{t})+\mathrm{G}(\gamma(\mathrm{t})) \mathrm{u}(\mathrm{t})+\mathrm{B}(\mathrm{x}(\mathrm{t}), \mathrm{u}(\mathrm{t}))+\mathrm{E} \rho(\mathrm{t})  \tag{2}\\
& \mathrm{y}(\mathrm{t})=\mathrm{Hx}(\mathrm{t})
\end{align*}
$$

Here G and E are known constant matrices of appropriate dimensions, $\gamma(\mathrm{t})$ is a parameter, the term $\mathrm{E} \rho(\mathrm{t})$ models unknown inputs to the actuator and to the dynamic process and unknown parameters; the evaluation of $\mathrm{v} \times 1$ vector function $\rho(\mathrm{t})$ are considered unknown. It is supposed also that if there are no faults, then $\gamma(\mathrm{t})=\gamma_{0}$; if a fault occurs, $\gamma(\mathrm{t})$ becomes an unknown function. Denote the system (2) with $\mathrm{F}=\mathrm{F}\left(\gamma_{0}\right)$ and $\mathrm{G}=\mathrm{G}\left(\gamma_{0}\right)$ as


Figure 1. Linear logic-dynamic system
$\Sigma=(\mathrm{F}, \mathrm{B}, \mathrm{G}, \mathrm{H})$.
The suggested approach to solve the FDI problem for system (2) includes the following steps.

1. Replacing the initial nonlinear system (2) by certain linear logic-dynamic (LLD) system containing several linear subsystems and linear logical conditions.
2. Solving the FDI problem for the LLD system and obtaining the bank of the LLD observers.
3. Transforming the LLD observers into nonlinear ones.

## 2. BASIC RELATIONSHIPS

To perform this approach, consider the. simple case with single nonlinearly (Coulomb friction) of the form

$$
\begin{equation*}
\mathrm{B}(\mathrm{x}(\mathrm{t}), \mathrm{u}(\mathrm{t}))=\left(\mathrm{G}^{\prime} \mathrm{u}(\mathrm{t})\right) \operatorname{sign}(\mathrm{Ax}(\mathrm{t})) \tag{3}
\end{equation*}
$$

for some matrices $\mathrm{G}^{\prime}$ and A .
On the first step of our approach, replace the system $\Sigma=(\mathrm{F}, \mathrm{B}, \mathrm{G}, \mathrm{H})$ by the LLD system with three linear subsystems $\Sigma_{1}=\left(\mathrm{F}, 0, \mathrm{G}-\mathrm{G}^{\prime}, \mathrm{H}\right), \Sigma_{2}=(\mathrm{F}, 0, \mathrm{G}, \mathrm{H})$ and $\Sigma_{3}=\left(\mathrm{F}, 0, \mathrm{G}+\mathrm{G}^{\prime}, \mathrm{H}\right)$ and two linear logical conditions $A x(t) \geq 0$ and $A x(t)>0$ (see Figure 1). If the condition $A x(t)<0$ holds, then (in the unfaulty case) model (2) reduces to

$$
: \begin{aligned}
& \dot{x}(\mathrm{t})=\mathrm{Fx}(\mathrm{t})+\left(\mathrm{G}-\mathrm{G}^{\prime}\right) \mathrm{u}(\mathrm{t})+\mathrm{B}(\mathrm{x}(\mathrm{t}), \mathrm{u}(\mathrm{t}))+\mathrm{E} \rho(\mathrm{t}), \\
& \mathrm{y}(\mathrm{t})=\mathrm{Hx}(\mathrm{t})
\end{aligned}
$$

if $\operatorname{Ax}(\mathrm{t})=0$, then

$$
\Sigma_{2}: \begin{aligned}
& \dot{x}(\mathrm{t})=\mathrm{Fx}(\mathrm{t})+\mathrm{Gu}(\mathrm{t})+\mathrm{B}(\mathrm{x}(\mathrm{t}), \mathrm{u}(\mathrm{t}))+\mathrm{E} \rho(\mathrm{t}) \\
& \mathrm{y}(\mathrm{t})=\mathrm{Hx}(\mathrm{t})
\end{aligned}
$$

if $\operatorname{Ax}(\mathrm{t})>0$, then
$\Sigma_{3}: \begin{aligned} & \dot{x}(\mathrm{t})=\mathrm{Fx}(\mathrm{t})+\left(\mathrm{G}+\mathrm{G}^{\prime}\right) \mathrm{u}(\mathrm{t})+\mathrm{B}(\mathrm{x}(\mathrm{t}), \mathrm{u}(\mathrm{t}))+\mathrm{E} \rho(\mathrm{t}), \\ & \mathrm{y}(\mathrm{t})=\mathrm{Hx}(\mathrm{t}) .\end{aligned}$
It is very important, that these models have the same matrices F and H .

On the second step, a bank of the LLD observers has to be obtained. It is well-known from the linear FDI theory (Frank, 1990; Gertler, 1993; Patton, 1994) that for the observer synthesis, the matrix $\Phi$ such that $\Phi x(t)=x_{*}(t)$ in the unfaulty case plays the main role. Here $x_{*}(t)$ is the state vector of the observer described by the equations

$$
\begin{align*}
& \dot{x_{*}}(\mathrm{t})=\mathrm{F}_{*} \mathrm{x}_{*}(\mathrm{t})+\mathrm{G}_{*} \mathrm{u}(\mathrm{t})+\mathrm{Jy}(\mathrm{t}), \\
& \mathrm{y}_{*}(\mathrm{t})=\mathrm{H}_{*} \mathrm{x}_{*}(\mathrm{t}) \tag{4}
\end{align*}
$$

with the output vector $y_{*}(\mathrm{t})$ and some matrices $\mathrm{F}_{*}, \mathrm{G}_{*}, \mathrm{~J}$ and $\mathrm{H}_{*}$. The observer generates the residual

$$
\mathrm{r}(\mathrm{t})=\mathrm{Cy}(\mathrm{t})-\mathrm{y}_{*}(\mathrm{t})
$$

for certain matrix $C$.
Write down the approximate equalities

$$
\begin{aligned}
& \mathrm{F}(\gamma)=\mathrm{F}\left(\gamma_{0}\right)+\left.\frac{\mathrm{dF}}{\mathrm{~d} \gamma}\right|_{\gamma=\gamma_{0}}\left(\gamma-\gamma_{0}\right)=\mathrm{F}+\mathrm{K}\left(\gamma-\gamma_{0}\right), \\
& \mathrm{G}(\gamma)=\mathrm{G}\left(\gamma_{0}\right)+\left.\frac{\mathrm{dG}}{\mathrm{~d} \gamma}\right|_{\gamma=\gamma_{0}}\left(\gamma-\gamma_{0}\right)=\mathrm{G}+\Gamma\left(\gamma-\gamma_{0}\right),
\end{aligned}
$$

and use the last expressions instead of the matrices $\mathrm{F}(\gamma(\mathrm{t}))$ and $\mathrm{G}(\gamma(\mathrm{t}))$ in (2) respectively.

In the absence of faults, the following well-known set of equations is fulfilled (Frank, 1990; Patton, 1994):

$$
\begin{equation*}
\mathrm{F}_{*} \Phi+\mathrm{JH}=\Phi \mathrm{F}, \quad \Phi \mathrm{G}=\mathrm{G}_{*}, \quad \mathrm{H}_{*} \Phi=\mathrm{CH} \tag{5}
\end{equation*}
$$

Consider the case when the residual $r(t)$ has to be sensitive to the faults and invariant under the unknown inputs $\rho(\mathrm{t})$ that is

$$
\begin{equation*}
\Phi[\mathrm{K} ; \Gamma] \neq 0, \quad \Phi \mathrm{E}=0 \tag{6}
\end{equation*}
$$

Assume that the structure of each LLD observer is analogous to the one shown in Figure 1, therefore the row matrix $A_{*}$ exists such that the following relationships hold in the unfaulty case:

$$
\begin{aligned}
& \text { if } \operatorname{Ax}(\mathrm{t})>0 \text {, then } \mathrm{A}_{*} x_{*}(\mathrm{t})>0, \\
& \text { if } \mathrm{Ax}(\mathrm{t})=0 \text {, then } \mathrm{A}_{*} \mathrm{x}_{*}(\mathrm{t})=0,
\end{aligned}
$$

$$
\text { if } \grave{A} x(t)<0 \text {, then } A_{*} X_{*}(t)<0 .
$$

Since $\mathrm{x}_{*}(\mathrm{t})=\Phi \mathrm{x}(\mathrm{t})$, then $\mathrm{A}=\mathrm{A}_{*} \Phi$ which is equivalent to the equality

$$
\begin{equation*}
\operatorname{rank}(\Phi)=\operatorname{rank}\left[\Phi^{\mathrm{T}} \vdots \mathrm{~A}^{\mathrm{T}}\right] . \tag{7}
\end{equation*}
$$

This condition imposes an additional restriction on the matrix $\Phi$.

## 3.OBSERVER DESIGN

To design an observer in the linear case, there are a number of approaches, e.g., the eigenstructure assignment (Patton, 1994), the approach based on the Kronecker canonical form developed by Frank (1990). Consider another linear procedure suggested by Zhirabok (1997) also based on the Kronecker canonical form that allows one to take into account condition (7) easily.

It is well-known (Kwakernaak and Sivan, 1972) that one can let

$$
\mathrm{F}_{*}=\left[\begin{array}{cccc}
\beta_{1} & 1 & 0 & \ldots 0  \tag{8}\\
\beta_{2} & 0 & 1 & \ldots 0 \\
& \cdot & \cdot & \cdot \\
\beta_{\mathrm{k}} & 0 & 0 & 0
\end{array}\right], \quad \mathrm{H}_{*}=\left[\begin{array}{lllll}
1 & 0 & 0 & \ldots & 0
\end{array}\right]
$$

without increasing a dimension of the considered observer. By analogy with Mironovsky (1979), transform the observer with the matrices (8) into the open-loop observer with the ones

$$
\mathrm{F}_{*}=\left[\begin{array}{cccc}
0 & 1 & 0 & \ldots 0 \\
0 & 0 & 1 & \ldots 0 \\
& \cdot & \cdot & . \\
0 & 0 & 0 & 0
\end{array}\right], \mathrm{H}_{*}=\left[\begin{array}{ccccc}
1 & 0 & 0 & \ldots & 0
\end{array}\right]
$$

This can be arranged by removing the feedback coefficients $\beta_{1}, \beta_{2}, \ldots, \beta_{\mathrm{k}}$ and transforming the matrix J . One can obtain, using equation (5) and these matrices, the following equations:

$$
\Phi_{1}=\mathrm{CH}, \mathrm{~J}_{\mathrm{i}} \mathrm{H}+\Phi_{\mathrm{i}+1}=\Phi_{\mathrm{i}} \mathrm{~F}, \mathrm{i}=1,2, \ldots, \mathrm{k}-1, \mathrm{~J}_{\mathrm{k}} \mathrm{H}=\Phi_{\mathrm{k}} \mathrm{~F}(9)
$$

where $\Phi_{i}$ and $J_{i}$ are the i-th rows of the matrices $\Phi$ and J respectively.

Equalities (9) can be transformed into the single equation

$$
\begin{equation*}
\mathrm{CHF}^{\mathrm{k}}=\mathrm{J}_{1} \mathrm{HF}^{\mathrm{k}-1}+\mathrm{J}_{2} \mathrm{HF}^{\mathrm{k}-2}+\ldots+\mathrm{J}_{\mathrm{k}} \mathrm{H} \tag{10}
\end{equation*}
$$

where the row matrix C is determined as follows. Let $\mathrm{E}_{*}$
and $\left[N_{1} ; N_{2}\right]$ be matrices of maximal rank such that $E_{*} E=0$ and $\left[N_{1} ; N_{2}\right]\left[\begin{array}{c}E_{*} \\ H\end{array}\right]=0$, then $C=-N_{2}$. Actually, $\mathrm{CH}=\Phi_{1}$ from (5) and (8); since $\Phi \mathrm{E}=0$, then $\mathrm{N}_{1} \mathrm{E}_{*}=\Phi_{1}$ for some matrix $N_{1}$ by definition of the matrix $E_{*}$.
Therefore $C H=N_{1} E_{*}$ and $C=-N_{2}$ if $\left[N_{1} ; N_{2}\right]\left[\begin{array}{c}E_{*} \\ H\end{array}\right]=0$. If $\mathrm{N}_{2}$ is not row matrix, one can use for C some row of $\mathrm{N}_{2}$ or sum of them.

## Algorithm

## Step 1. Let $\mathrm{k}=1$.

Step 2. If equation (10) is fulfilled for some row matrices $\mathrm{J}_{1}, \mathrm{~J}_{2}, \ldots, \mathrm{~J}_{\mathrm{k}}$ (this can be checked using the mathematical packages, e.g. MATLAB), go to 4.

Step 3. Let $\mathrm{k}=\mathrm{k}+1$, go to 2 .
Step 4. Obtain the rows of matrix $\Phi$ : $\Phi_{1}=\mathrm{CH}$, $\Phi_{\mathrm{i}+1}=\Phi_{\mathrm{i}} \mathrm{F}-\mathrm{J}_{\mathrm{i}} \mathrm{H}, \mathrm{i}=1,2, \ldots, \mathrm{k}-1$. If the matrix $\Phi$ does not satisfy conditions (6) or (7), find another solution of equation (10) otherwise go to 3 .

Step 5. Let $\mathrm{G}_{*}^{\prime}=\Phi \mathrm{G}^{\prime}, \mathrm{G}_{*}=\Phi \mathrm{G}$ and obtain the row matrix $\mathrm{A}_{*}$ from the linear algebraic equation $\Phi^{\mathrm{T}} \mathrm{A}_{*}{ }^{\mathrm{T}}=\mathrm{A}^{\mathrm{T}}$. End.

## 4. STABILITY OF OBSERVERS

To obtain a stable matrix $\mathrm{F}_{*}$ in equation (4), it is necessary to use a feedback in the observer and to correct the matrix J correspondingly. Namely, if $\beta_{1}, \beta_{2}, \ldots, \beta_{\mathrm{k}}$ are the feedback coefficients providing the necessary stability of this matrix, then the i-th row $J_{i}$ of matrix $J$ has to be replaced by the row $J_{i}-\beta_{i} C, i=1, \ldots$, , . It is very important that the matrix $\Phi$ does not change in this case therefore the main properties of the observer (invariance under the unknown inputs and sensitivity to the faults) are not changed also. Actually, consider the i-th (i<k) row of the first matrix equality in (5) with the matrix $\mathrm{F}_{*}$ from (8) and the row matrix $J_{i}$ replaced by $J_{i}-\beta_{i} C$ :

$$
\beta_{\mathrm{i}} \Phi_{1}+\Phi_{\mathrm{i}+1}+\left(\mathrm{J}_{\mathrm{i}}-\beta_{\mathrm{i}} \mathrm{C}\right) \mathrm{H}=\Phi_{\mathrm{i}} \mathrm{~F}
$$

Since $\mathrm{CH}=\Phi_{1}$, it is easy to obtain equality (9); the same is true for $\mathrm{i}=\mathrm{k}$. Therefore, the matrix $\Phi$ obtained with Algorithm is left unchanged under that change in matrix $F_{*}$. Thus, the problems of the observer invariance under the unknown inputs and stability of matrix $\mathrm{F}_{*}$ can be solved independently of each other.

The third step of the suggested approach is formal: to transform the LLD observer into the nonlinear one, the term $\left(\mathrm{G}_{*}^{\prime} \mathrm{u}(\mathrm{t})\right) \operatorname{sign}\left(\mathrm{A}_{*} \mathrm{x}_{*}(\mathrm{t})\right)$ has to be added to equation (4):
$\mathrm{x} *(\mathrm{t})=\mathrm{F}_{*} \mathrm{X}_{*}(\mathrm{t})+\mathrm{G}_{*} \mathrm{u}(\mathrm{t})+\mathrm{Jy}(\mathrm{t})+\left(\mathrm{G}_{*}^{\prime} \mathrm{u}(\mathrm{t})\right) \operatorname{sign}\left(\mathrm{A}_{*} \mathrm{X}_{*}(\mathrm{t})\right)$.

This operation can change stability of the observer; to improve it, one has to find the appropriate feedback coefficients $\beta_{1}, \beta_{2}, \ldots, \beta_{\mathrm{k}}$ and to correct the matrix J as pointed above.

## 5. MODIFICATIONS OF SUGGESTED APPROACH

### 5.1. Some extensions

It follows from our approach that the matrix $\mathrm{G}^{\prime}$ in (3) can be replaced by the matrix function $\mathrm{G}^{\prime}(\mathrm{u}(\mathrm{t}), \gamma(\mathrm{t}))$ or $\mathrm{G}^{\prime}(\mathrm{y}(\mathrm{t})$, $\mathrm{u}(\mathrm{t}), \gamma(\mathrm{t})$ ); in this case the additional term in (4) is of form $\Phi \mathrm{G}^{\prime}\left(\mathrm{u}(\mathrm{t}), \gamma_{0}\right) \operatorname{sign}\left(\mathrm{A}_{*} \mathrm{X}_{*}(\mathrm{t})\right)$ or $\Phi \mathrm{G}^{\prime}(\mathrm{y}(\mathrm{t}), \mathrm{u}(\mathrm{t})$, $\left.\gamma_{0}\right) \operatorname{sign}\left(\mathrm{A}_{*} \mathrm{x}_{*}(\mathrm{t})\right)$ respectively.

If there exist several nonlinearities in system (2) with matrices $\mathrm{G}_{1}^{\prime}, \mathrm{G}_{2}^{\prime}, \ldots, \mathrm{G}_{\mathrm{p}}^{\prime}$ and $\mathrm{A}_{1}, \mathrm{~A}_{2}, \ldots, \mathrm{~A}_{\mathrm{p}}$, one has to form the compound matrix $\mathrm{A}=\left[\mathrm{A}_{1}^{\mathrm{T}}\left|\mathrm{A}_{2}^{\mathrm{T}}\right| \ldots \mid \mathrm{A}_{\mathrm{p}}{ }^{\mathrm{T}}\right]^{\mathrm{T}}$ and use it as pointed above.

The logical conditions in the LLD observers can be relaxed by the extension of the vector $\mathrm{X}_{*}(\mathrm{t})$ with the vector $y(t)$ as follows:

$$
A_{*}\left[\begin{array}{c}
x_{*}  \tag{11}\\
y
\end{array}\right]>0 \text { or } A_{*}\left[\begin{array}{c}
x_{*} \\
y
\end{array}\right]=0 \text { or } \quad A_{*}\left[\begin{array}{c}
x_{*} \\
y
\end{array}\right]<0
$$

Thus, condition (7) can be replaced by the one

$$
\begin{equation*}
\operatorname{rank}\left[\Phi^{\mathrm{T}} ; \mathrm{H}^{\mathrm{T}}\right]=\operatorname{rank}\left[\Phi^{\mathrm{T}} ; \mathrm{H}^{\mathrm{T}} ; \mathrm{A}^{\mathrm{T}}\right] . \tag{12}
\end{equation*}
$$

It is known that for the system described by the equations

$$
\begin{aligned}
& x(t+1)=F(\gamma(\mathrm{t})) \mathrm{x}(\mathrm{t})+\mathrm{G}(\gamma(\mathrm{t})) \mathrm{u}(\mathrm{t})+\mathrm{E} \rho(\mathrm{t}), \\
& \mathrm{y}(\mathrm{t})=\mathrm{Hx}(\mathrm{t}),
\end{aligned}
$$

and the proper observer relationships (5) and (6) hold. Thus, the suggested approach can be extended on the descrete-time case.

### 5.2. Another types of nonlinearities

Consider another type of nonlinearity - a backlash described by the following model:

$$
\mathrm{B}(\mathrm{x}(\mathrm{t}), \mathrm{u}(\mathrm{t}))= \begin{cases}\mathrm{G}^{\prime} \mathrm{u}(\mathrm{t}) \mathrm{k}(\mathrm{Ax}(\mathrm{t})-\sigma) & \text { if } \operatorname{Ax}(\mathrm{t})>\sigma \\ 0 & \text { if }|\operatorname{Ax}(\mathrm{t})| \leq \sigma \\ \mathrm{G}^{\prime} \mathrm{u}(\mathrm{t}) \mathrm{k}(\mathrm{Ax}(\mathrm{t})+\sigma) & \text { if } \operatorname{Ax}(\mathrm{t})<-\sigma\end{cases}
$$

where $2 \sigma$ is the backlash span and k coefficient. In this
case it is impossible to use directly the approach suggested above because it gives two different nonlinear systems $\Sigma_{1}$ and $\Sigma_{3}$. To overcome this difficult, assume that the model of the observer contains the term analogous to $\mathrm{B}(\mathrm{x}(\mathrm{t}), \mathrm{u}(\mathrm{t}))$ :

$$
\mathrm{B}_{*}\left(\mathrm{x}_{*}, \mathrm{u}\right)= \begin{cases}\mathrm{G}_{*}^{\prime} \mathrm{uk}_{*}\left(\mathrm{~A}_{*} \mathrm{x}_{*}-\sigma_{*}\right) & \text { if } \mathrm{A}_{*} \mathrm{x}_{*}>\sigma_{*} \\ 0 & \text { if }\left|\mathrm{A}_{*} \mathrm{X}_{*}\right| \leq \sigma_{*} \\ \mathrm{G}_{*}^{\prime} \mathrm{uk}_{*}\left(\mathrm{~A}_{*} \mathrm{x}_{*}+\sigma_{*}\right) & \text { if } \mathrm{A}_{*} \mathrm{x}_{*}<-\sigma_{*}\end{cases}
$$

let $\mathrm{k}_{*}=\mathrm{k}$ and $\sigma_{*}=\sigma$ without loss of generality.
Assuming that $\mathrm{A}_{*} \Phi=\mathrm{A}$ one can obtain equations (5). Therefore, this task is reduced to that considered in Section 2 with restriction (7).

Two another types of nondifferenciable nonlinearities (a saturation and a hysteresis) can be considered by analogy, and they give identical results. Moreover, the suggested approach can be used for another types of nonlinearities such as sin, cos, $\log$ and so on in spite of the fact that it is impossible to transform system (2) into any LLD system in this case. Here restriction (7) reflects not a logical condition but a condition of concordance of nonlinearities in the initial system and in the observer.

### 5.3. Robustness

The ideal solution of the robustness problem is the exact decoupling of the residual from the unknown inputs, that is the condition $\Phi E=0$. In many cases the exact decoupling is impossible, and we must use the approximate one. The most fundamental method of such decoupling is singular value decomposition: the matrix $E$ is expressed as

$$
\mathrm{E}=\mathrm{U} \Sigma \mathrm{~V}
$$

where U and V are orthogonal matrices,

$$
\Sigma=\left[\begin{array}{cccccc}
\sigma_{1} & & & & \vdots & \\
& \sigma_{2} & & 0 & \vdots & \\
& & \ldots & & \vdots & 0 \\
& 0 & & \sigma_{\mathrm{n}} & \vdots &
\end{array}\right],
$$

$0 \leq \sigma_{1} \leq \sigma_{2} \leq \ldots \leq \sigma_{\mathrm{n}}$ are the singular values of E , ordered by magnitude. Then the first $p, p<n$, coulombs of $U$ give the rows of matrix $\mathrm{E}^{0}$ which are the best choice for the approximate decoupling (Lou, et al., 1994).

Consider some conditions to choose the integer p from view-point of solving equation (10) (general recommendations are in (Lou, et al., 1994)). Since the condition of the exact decoupling is $\Phi E=0$, then the equality $\Phi=\mathrm{QE}^{0}$ for any matrix Q is that for the
approximate decoupling, or $\Phi_{\mathrm{i}}=\mathrm{Q}_{\mathrm{i}} \mathrm{E}^{0}, \mathrm{i}=1,2, \ldots, \mathrm{k}$, and $\mathrm{Q}_{1} \mathrm{E}^{0}=\mathrm{CH}$.

Last equality holds if

$$
\begin{equation*}
\operatorname{rank}\left[\left(\mathrm{E}^{0}\right)^{\mathrm{T}} \mathrm{H}^{\mathrm{T}}\right]<\operatorname{rank}\left(\mathrm{E}^{0}\right)+\operatorname{rank}(\mathrm{H}) . \tag{13}
\end{equation*}
$$

It follows from (9), that if one-dimensional observer exists, then $\mathrm{Q}_{1} \mathrm{E}^{0} \mathrm{~F}=\mathrm{J}_{1} \mathrm{H}$, therefore the inequality

$$
\begin{equation*}
\operatorname{rank}\left[\left(\mathrm{E}^{0} \mathrm{~F}\right)^{\mathrm{T}}, \mathrm{H}^{\mathrm{T}}\right]<\operatorname{rank}\left(\mathrm{E}^{0} \mathrm{~F}\right)+\operatorname{rank}(\mathrm{H}) \tag{14}
\end{equation*}
$$

is necessary condition to obtain such observer.
By analogy, it can be shown that the conditions for existence of two- and k -dimensional observer are (in addition to (14) because this condition corresponds to both one-dimensional observer and the last component of the vector $\mathrm{x}_{*}(\mathrm{t})$ ):

$$
\begin{gather*}
\operatorname{rank}\left[\left(\mathrm{E}^{0} \mathrm{~F}\right)^{\mathrm{T}}: \mathrm{H}^{\mathrm{T}} \mid\left(\mathrm{E}^{0}\right)^{\mathrm{T}}\right]< \\
<\operatorname{rank}\left[\left(\mathrm{E}^{0} \mathrm{~F}\right)^{\mathrm{T}}: \mathrm{H}^{\mathrm{T}}\right]+\operatorname{rank}\left(\mathrm{E}^{0}\right), \\
\operatorname{rank}\left[\left(\mathrm{E}^{0} \mathrm{~F}^{\mathrm{k}-1}\right)^{\mathrm{T}}\left|\left(\mathrm{HF}^{\mathrm{k}-2}\right)^{\mathrm{T}}\right| \ldots\left|\mathrm{H}^{\mathrm{T}}\right|\left(\mathrm{E}^{0}\right)^{\mathrm{T}}\right]< \\
<\operatorname{rank}\left[\left(\mathrm{E}^{0} \mathrm{~F}^{\mathrm{k}-1}\right)^{\mathrm{T}} \mid \ldots: \mathrm{H}^{\mathrm{T}}\right]+\operatorname{rank}\left(\mathrm{E}^{0}\right) . \tag{15}
\end{gather*}
$$

If conditions (13) or (14) or condition (15) for any $k>1$ do not hold, then the integer p must be increased.

These conditions can be used also to check if the exact decoupling is possible. In this case the matrix $\mathrm{E}^{0}$ in (13) - (15) must be replaced by $\mathrm{E}_{*}$.

It should be pointed out that if we use the approximate decoupling, then the condition $\Phi E=0$ in (6) must be replaced by $\Phi=\mathrm{QE}^{0}$ for any matrix Q which is equivalent to the equality

$$
\operatorname{rank}\left(\mathrm{E}^{0}\right)=\operatorname{rank}\left[\left(\mathrm{E}^{0}\right)^{\mathrm{T}} \mid \Phi^{\mathrm{T}}\right] .
$$

Besides, to obtain the matrix C , the matrix $\mathrm{E}_{*}$ has to be replaced by $\mathrm{E}^{0}$. This must be taken into account in algorithm.

## 6. EXAMPLE

Consider the following continuous-time system:

$$
\begin{aligned}
& \mathrm{x}_{1}(\mathrm{t})=\mathrm{u}_{1}(\mathrm{t})-\mathrm{x}_{2}(\mathrm{t})+\mathrm{u}_{2}(\mathrm{t}) \mathrm{x}_{3}(\mathrm{t})-\mathrm{x}_{5}(\mathrm{t}) \\
& \dot{x}_{2}(\mathrm{t})=\mathrm{x}_{1}(\mathrm{t})-2 \mathrm{x}_{2}(\mathrm{t})+\rho(\mathrm{t}) \\
& \dot{\mathrm{x}_{3}}(\mathrm{t})=\mathrm{x}_{2}(\mathrm{t})+\mathrm{u}_{1}(\mathrm{t}) \operatorname{sign}\left(\mathrm{x}_{5}(\mathrm{t})\right) \\
& \dot{x}_{4}(\mathrm{t})=\mathrm{x}_{2}(\mathrm{t})+\mathrm{x}_{3}(\mathrm{t})+\left(\mathrm{x}_{5}(\mathrm{t})+\mathrm{u}_{2}(\mathrm{t})\right) \gamma(\mathrm{t}) \\
& \dot{x_{5}}(\mathrm{t})=-\mathrm{x}_{4}(\mathrm{t}) \\
& \mathrm{y}_{1}(\mathrm{t})=\mathrm{x}_{1}(\mathrm{t}), \quad y_{2}(\mathrm{t})=\mathrm{x}_{4}(\mathrm{t}), \quad \gamma_{0}=1
\end{aligned}
$$

It contains two nonlinearities with the matrices $\mathrm{A}_{1}=$ $\left[\begin{array}{lllll}0 & 0 & 1 & 0 & 0\end{array}\right]$ and $A_{2}=\left[\begin{array}{lllll}0 & 0 & 0 & 0 & 1\end{array}\right]$, therefore $A=\left[\begin{array}{lllll}0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1\end{array}\right]$ and

$$
\begin{aligned}
& \mathrm{F}=\left[\begin{array}{ccccc}
0 & -1 & 0 & 0 & -1 \\
1 & -2 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & -1 & 0
\end{array}\right], \quad \mathrm{H}=\left[\begin{array}{lllll}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{array}\right] \\
& \mathrm{G}=\left[\begin{array}{ll}
1 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 1 \\
0 & 0
\end{array}\right], \quad \mathrm{G}_{1}^{\prime}=\left[\begin{array}{ll}
0 & 1 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{array}\right], \quad \mathrm{G}_{2}^{\prime}=\left[\begin{array}{ll}
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 0 \\
0 & 0
\end{array}\right], \\
& \mathrm{E}=\left[\begin{array}{l}
0 \\
1 \\
0 \\
0 \\
0
\end{array}\right], \quad[\mathrm{K} \vdots \Gamma]=\left[\begin{array}{lllll}
0 & \cdots & 0 & 0 & 0 \\
0 & \cdots & 0 & 0 & 0 \\
0 & \cdots & 0 & 0 & 0 \\
0 & \cdots & 1 & 0 & 1 \\
0 & \cdots & 0 & 0 & 0
\end{array}\right] .
\end{aligned}
$$

Obtain the observer invariant under $\rho(\mathrm{t})$ and sensitive to the fault $\gamma(\mathrm{t})$. In this case

$$
\mathrm{E}_{*}=\left[\begin{array}{lllll}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{array}\right],
$$

thus, $\left[\mathrm{N}_{1} \vdots \mathrm{~N}_{2}\right]=\left[\begin{array}{ccccccc}1 & 0 & 0 & 0 & \vdots & -1 & 0 \\ 0 & 0 & 1 & 0 & \vdots & 0 & -1\end{array}\right], \quad$ and $C=\left[\begin{array}{ll}1 & 1\end{array}\right]$. It can be shown that equation (10) has a solution for $\mathrm{k}=3$ :

$$
\begin{aligned}
& \mathrm{CHF}^{3}=\left[\begin{array}{llll}
1 & -2 & 0 & 0
\end{array}\right]=-\mathrm{H}_{1} \mathrm{~F}^{2}+\mathrm{H}_{2}, \\
& \mathrm{~J}=\left[\begin{array}{cc}
-1 & 0 \\
0 & 0 \\
0 & 1
\end{array}\right], \quad \Phi=\left[\begin{array}{ccccc}
1 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1
\end{array}\right] .
\end{aligned}
$$

Clearly, equality (12) is fulfilled; the matrix $\mathrm{A}_{*}$ from (11) and the matrices $\mathrm{G}_{*}, \mathrm{G}_{*_{1}}^{\prime}, \mathrm{G}_{*_{2}}^{\prime}$ are the following:

$$
A_{*}=\left[\begin{array}{ccccc}
-3 & 1 & 0 & 2 & 3 \\
0 & 0 & -1 & 0 & 0
\end{array}\right]
$$



Figure 2. The simulation results

$$
\mathrm{G}_{*}=\left[\begin{array}{ll}
1 & 1 \\
1 & 0 \\
0 & 0
\end{array}\right], \quad \mathrm{G}_{* 1}^{\prime}=\left[\begin{array}{ll}
0 & 1 \\
0 & 1 \\
0 & 0
\end{array}\right], \quad \mathrm{G}_{* 2}^{\prime}=\left[\begin{array}{ll}
0 & 0 \\
1 & 0 \\
0 & 0
\end{array}\right]
$$

Let $\mathrm{x}_{*_{1}}(\mathrm{t})=\mathrm{x}_{1}(\mathrm{t})+\mathrm{x}_{4}(\mathrm{t}), \mathrm{x}_{*_{2}}(\mathrm{t})=\mathrm{x}_{1}(\mathrm{t})+\mathrm{x}_{3}(\mathrm{t}), \mathrm{x}_{*_{3}}(\mathrm{t})=-\mathrm{x}_{5}(\mathrm{t})$ and obtain the observer invariant under $\rho(\mathrm{t})$ :

$$
\begin{aligned}
\dot{x_{* 1}}(t)= & x_{* 2}(t)+u_{1}(t)+u_{2}(t)\left(-3 x_{* 1}(t)+x_{* 2}(t)+\right. \\
& \left.2 y_{1}(t)+3 y_{2}(t)+1\right)-y_{1}(t), \\
\dot{x_{* 2}}(t)= & x_{* 3}(t)+u_{1}\left(\operatorname{sign}\left(-x_{* 3}(t)\right)+1\right)+ \\
& u_{2}(t)\left(-3 x_{* 1}(t)+x_{* 2}(t)+2 y_{1}(t)+3 y_{2}(t)\right), \\
\dot{x}_{* 3}(t)= & y_{2}(t), \\
y_{*}(t)= & x_{* 1}(t), \\
r(t)= & y_{1}(t)+y_{2}(t)-y_{*}(t) .
\end{aligned}
$$

The following initial conditions were used for simulation: $\mathrm{x}_{1}(0)=0, \mathrm{x}_{2}(0)=0, \mathrm{x}_{3}(0)=1, \mathrm{x}_{4}(0)=10, \mathrm{x}_{5}(0)=5, \mathrm{x}_{* 1}(0)=10$, $x_{* 2}(0)=1, x_{* 3}(0)=-5$.

The simulation results for $\rho(\mathrm{t})=1, \mathrm{u}_{1}(\mathrm{t})=\sin (5 \mathrm{t})$ and $\mathrm{u}_{2}(\mathrm{t})=-11 \cos (7 \mathrm{t})$ are shown in Figure 2: $\gamma(\mathrm{t})$ changes from 1 to 1.1 at $t=1$ at once. Clearly, the residual is sensitive to the fault $\gamma(\mathrm{t})$ and invariant under $\rho(\mathrm{t})$.

## 7.CONCLUSION

This paper is concerned with the problem of the observer-based fault diagnosis in nonlinear dynamic systems. The suggested approach consists of the following main steps: replacing the initial nonlinear system by certain linear logic-dynamic system, obtaining the bank of the linear logic-dynamic observers and transforming them into the nonlinear ones. This approach was used for fault diagnosis in manipulation robots (Filaretov, et al., 2001).

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## REFERENCES

Filaretov V., A. Zhirabok and S. Usoltsev (2001). Fault diagnosis for nonlinear mechanic systems. Proc. 2001 IEEE/ASME Int. Conf. on Advanced Intelligent Mechatronics, Como, 2, pp. 1257-1260.
Frank, P.M. (1990). Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy - A survey and some new results. Automatica, 26, pp. 459-474.
Frank, P.M. and J. Wunnenberg (1989). Robust fault diagnosis using unknown input observer schemes. In: Fault diagnosis in dynamic systems. Theory and application (Patton R.J., P.M. Frank, R.N. Clark, Eds.), pp. 47-98. Prentice-Hall, New York.

Gertler, J. (1993). Residual generation in model-based fault diagnosis. Control Theory and Advanced Technology, 9, pp. 259-285.
Kwakernaak, H. and R. Sivan (1972). Linear optimal control systems. John Wiley \& Sons Inc., New York.
Lou, X.C., A.S. Willsky and G.C. Verghese (1996). Optimally robust redundancy relations for failure detection in uncertain systems. Automatica, 22, pp. 333-344.
Mironovsky, L. A. (1979). Functional diagnosis of linear dynamic systems. Automation and Remote Control, N 8, pp. 120-128.
Patton, R. (1994). Robust model-based fault diagnosis: the state of the art. Proc. IFAC Symp. SAFEPROCESS'94, Espoo, pp. 1-24.
Seliger, R. and P.M. Frank (1991). Fault diagnosis by disturbance decoupling nonlinear observers. Proc. 30th Conf. On Decision and Control, Brighton, pp. 2248-2253.
Shields, D.N. (1996). Qualitative approaches for fault diagnosis based on bilinear systems. Proc. 13th World Congress IFAC, San Francisco, N, pp. 151156.

Staroswiecki, M. and M.G. Comtet-Varga (1999). Fault detectability and isolability in algebraic dynamic systems. CD ROM Proc. European Control Conference ECC'99, Karlsrue.
Zhirabok, A. N. and A. Ye. Shumsky (1987). Functional diagnosis of continuous dynamic systems described by equations whose right-hand side is polynomial. Automation and Remote Control, N 8, pp. 154-164.
Zhirabok, A.N. (1997). Fault detection and isolation: linear and nonlinear systems. Prepr. IFAC Symp. SAFEPROCESS'97, Hall, pp. 903-908.

