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ITERATIVE OPTIMIZING SET-POINT CONTROL – THE BASIC PRINCIPLE REDESIGNED

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Abstract: The paper is concerned with on-line process steady-state optimization under uncertainty. In such cases a single process model optimization can yield a set-point far away from the one optimal for the true process. The way to improve the set-point is to apply steady-state feedback, i.e., an iterative optimizing algorithm utilizing new measurements available after every subsequent set-point application. Integrated System Optimization and Parameter Estimation (ISOPE) method yields subsequent set-points converging to the true process optimum, despite uncertainty. It requires, at every iteration, model parameters to be updated under certain additional equality constraint. The aim of the paper is to present how the ISOPE can be redesigned resulting in a new structure without this constraint. Moreover, the parameter estimation itself is then not necessary at every iteration, although possible when reasonable. *Copyright © 2002 IFAC*

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1. INTRODUCTION

Optimizing set-point control of industrial plants in a multilayer structure is the subject of the paper, i.e. on-line set-point optimization under uncertainty in available plant models and disturbance estimates (Findeisen *et al.*, 1980; Morari *et al.*, 1980; Tatjewski, 1988). The steady-state case is considered, i.e. when uncontrolled inputs (disturbances) vary slowly or abrupt but rare, as compared to the controlled plant dynamics. Practically important cases with significant uncertainty are under consideration, when the set-points optimal for the available model can differ significantly from the set-points optimal for the true plant. Therefore, the evaluated model-optimal set-points can lead to substantial deterioration in productivity when applied to the true plant. To cope with the uncertainty, on-line measurement information from the plant must be then combined with model optimizations to improve the plant productivity. This leads to iterative optimizing set-point control algorithms (iterative set-point improvement algorithms). The classical approach, the iterative two-step procedure of subsequently repeated process model optimizations and process model parameter estimations, yields suboptimal results and does even not guarantee improvement. However, the modified iterative two-step method, more commonly called the Integrated System Optimization and Parameter Estimation (ISOPE), generates set-points converging towards the true plant optimum. The method was originally proposed by Roberts (1979). Theoretical optimality and convergence analysis for the improved, augmented algorithm (with certain convexification) was given in (Brdyś *et al.*, 1987*a*). Numerous papers were

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then published: to cope with output constraints (Brdyś *et al.*, 1986; Lin *et al.*, 1988; Tatjewski *et al.*, 1995), to improve effectiveness of the updating strategy (Tatjewski and Roberts, 1987), to extend the algorithm for complex, interconnected systems, e. g. (Brdyś *et al.*, 1987*b*; Tatjewski *et al.*, 1990). Many efforts were devoted to improve the crucial, from a practical viewpoint, part of the ISOPE algorithms – the technique to estimate effectively derivatives of the plant input-output mapping, utilizing on-line output measurements. The development of the dual ISOPE algorithm proposed by Brdyś and Tatjewski (1994) was here a breakthrough, a recent result by Roberts (2000) also addresses this point.

In all cited papers the basic ISOPE structure is preserved, consisting of two main steps: the model parameter estimation problem (MPE) and the modified model optimization problem (MMOP). The task of the MPE is to adjust (estimate, adapt) the model parameters utlizing new available measurements and under certain additional equality constraint. The aim of this paper is to present how the ISOPE can be redesigned resulting in a new algorithm structure not requiring this constraint. Moreover, the MPE itself occurs to be not necessary at every iteration, although possible when reasonable. The obtained structure shows clearly what is really necessary to preserve the appealing advantage of the ISOPE approach, i.e. convergence of the subsequent set-points to the true plant optimum, despite uncertainty.

Organization of the paper is as follows. First, the task of the optimizing set-point control in the multilayer structure and the algorithms for iterative set-point optimization under uncertainty will be briefly reminded. Then the ISOPE approach will be developed and discussed, in a simple and original way not published previously. Finally, basing on this development, the proposed new modified structure will be presented and commented.

2. OPTIMIZING SET-POINT CONTROL

The well known idea of multilayer control is to split the general control task, which is the generation of optimal trajectories of the manipulated variables, into a sequence of different and hierarchically structured sub-tasks handled by dedicated control layers. *Regulatory control* and *set-point optimization* are the two basic, well established layers. The plant together with regulatory controllers constitute the first layer which can be treated as a fast dynamic part of the system. It is assumed to follow desired steady-states prescribed by the set-points c for controllers, in spite of fast disturbances. Evaluation of economically optimal values of the set-points is the task of the second, optimizing control layer, using measurements of certain process outputs y. These are those technological variables which, together with the set-points c, are essential for the process efficiency, i.e. which enter the performance function Q or formulations of constraints. For steady-state optimization the static controlled plant model relating c and y is needed, valid for possible range of changes in the set-points c and uncontrolled outputs w. This is usually a nonlinear model and a corresponding optimization problem is often a difficult one. Moreover, the influence of uncontrollable inputs w usually has significant impact on the overall plant performance.

It is assumed that a scalar performance function $Q(c, y)$ of the plant economical performance is used and all other objectives are formulated as constraints, e.g., on product quality. The steady-state *optimizing set-point control problem* (OCP) can be formulated as finding the optimal value $\hat{c}_*(w)$ of the set-points, despite uncertainty:

$$\begin{aligned} &\text{minimize } Q(c,y) \\ &\text{subject to } \quad y = F_*(c,w) \\ &\qquad\qquad\quad c \in C \end{aligned} \tag{1}$$

where $F_* : R^n \times R^r \mapsto R^m$ represents the input–output mapping (static characteristics) of the true plant and C is a fesible set for the set-points. It is assumed, consequently, that F_* is unknown and only its *approximate model* F is available, $y = F(c, \tilde{w}, \alpha)$, where $\tilde{w}$ is a disturbance estimate and α denotes adjustable model parameters. Therefore, the following steady-state *model optimization problem* (MOP) corresponds to the OCP:

$$\begin{aligned} &\text{minimize } Q(c,y) \\ &\text{subject to } \quad y = F(c,\tilde{w},\alpha) \\ &\qquad\qquad\quad c \in C \end{aligned} \tag{2}$$

with a solution $\hat{c}_m(\tilde{w}, \alpha)$. Obviously, in the presence of uncertainty $\hat{c}_m(\tilde{w}, \alpha)$ can significantly differ from $\hat{c}_*(w)$, leading to deterioration in the performance function value.

To cope with uncertainty in modeling and in disturbance estimates, on-line measurement information from the plant must be combined with model optimizations to improve the performance function. This leads to *iterative optimizing set-point control algorithms*. A rather straightforward and classical approach, the *iterative two-step procedure*, consists of repeatedly performed:

- steady-state model optimization (i.e., solving MOP) followed by implementation of the resulting set-points in the plant controllers (*first step*), and then
- output measurements in a new steady-state followed by estimation (adaptation) of the model parameters α (*second step*) – and back to solving the MOP, etc.

It is known and rather obvious that this procedure is generally not optimal, i.e., it does not lead (if convergent) to the plant true optimal set-point $\hat{c}_*(w)$ (which a solution of the OCP (1)). Starting from c^0, it generates a sequence of set-points c^n, $c^n = \hat{c}_m(\tilde{w}, \alpha^{n-1})$, resulting from subsequent parameter adaptations and model optimizations

$$c^0 \rightarrow \alpha^0 = \alpha(c^0) \rightarrow \hat{c}_m(\tilde{w}, \alpha^0) = c^1 \rightarrow$$
$$\rightarrow \alpha^1 = \alpha(c^1) \rightarrow \hat{c}_m(\tilde{w}, \alpha^1) = c^2 \rightarrow \cdots \rightarrow c^\infty$$

where $\hat{c}_m(\tilde{w}, \alpha^i)$ denotes a solution of the model optimization problem MOP for given parameter values α^i, $\alpha(c^i)$ denotes parameter values obtained from model adaptation using steady-state measurements after application of the set-point c^i.

At every model-optimal point $\hat{c}_m(\tilde{w}, \alpha^i)$ optimality conditions for the MOP are satisfied. On the other hand, at true plant optimal set-point $\hat{c}_*(w)$ optimality conditions for the optimizing control problem OCP are satisfied. Looking at differences between MOP and OCP it can easily be seen that in order to achieve the same necessary optimality conditions for the points c^∞ and $\hat{c}_*(w)$, *derivatives of the plant mapping and its model must be equal at these points*, i.e.,

$$F_c^{'}(\hat{c}_m(\tilde{w}, \alpha^\infty), \tilde{w}, \alpha^\infty) = (F_*)_c^{'}(\hat{c}_*(w), w) \quad (3)$$

where $\alpha^\infty = \alpha(c^\infty)$ and $F_c^{'}(c, w, \alpha)$ denotes partial derivative with respect to c, $F_c^{'}(c, w, \alpha) = \frac{\partial}{\partial c} F(c, w, \alpha)$, etc. It is difficult, if at all possible, to satisfy the condition (3) – from obvious reasons: the model is only approximate and location of the point $\hat{c}_*(w)$ is not known in advance. Therefore, matching true plant mapping derivatives by its model derivatives in the whole domain of possible set-point changes would only guarantee the required equality at the limit point of iterations. But this is a contradiction – we assumed significant uncertainty in the plant model, and a model with accurate derivatives would mean a highly precise model !

It follows from the presented argument that special attention must be paid to the accuracy of the plant model and, especially, its derivatives when applying the single model optimization or the two-step procedure of interchanging parameter estimations and model optimizations. However, it is usually not possible to construct very accurate plant models, especially when the range of admissible input signal variations is broad. Moreover, the iterative approach is not needed when having a good model, a single model optimization would then suffice.

In practice, models often occur to be quite crude. An alternative approach would then be to try to acquire precise knowledge of the plant mapping derivatives only *locally*, utilizing in a suitable way the plant output measurements at current points. It turns out that this information can be used in the model-based optimizations in a way that forces convergence of the model-based solutions towards the true plant optimum. This is precisely the essence of the *modified iterative two-step procedure*, more commonly known as the *Integrated System Optimization and Parameter Estimation (ISOPE) method* (Roberts, 1979; Brdyś *et al.*, 1987*a*).

3. THE ISOPE METHOD

Precise derivation and formal analysis of the ISOPE method can be found in e.g. (Brdyś *et al.*, 1987*a*). Briefly speaking, the idea of the ISOPE approach is as follows:

- to use, instead of the MOP, the following *modified model-based optimization problem* (MMOP):

$$\begin{aligned} &\text{minimize}_c \{ q_{\text{mod}}(c, c^i, \tilde{w}, \alpha^i) = \\ &\qquad = Q(c, F(c, \tilde{w}, \alpha^i)) - \lambda(c^i, \alpha^i)^T c \} \\ &\text{subject to} \quad c \in C \end{aligned} \quad (4)$$

  where

$$\lambda(c^i, \alpha^i)^T = Q_y^{'}(c^i, F(c^i, \tilde{w}, \alpha^i)) \cdot \cdot [F_c^{'}(c^i, \tilde{w}, \alpha^i) - (F_*)_c^{'}(c^i, w)] \quad (5)$$

  and the subscript i indexes iterations,
- to perform *the model parameter estimation* MPE (yielding adapted parameter values α^i) at the point c^i under the additional constraint

$$F(c^i, \tilde{w}, \alpha^i) = F_*(c^i, w) \quad (6)$$

  This constraint requires that, at the current set-point c^i, the output from the model after the parameter update should be equal to the measured process output.

Now, it follows directly from the construction of the MMOP that

the performance function of the MMOP (4) has at each point c^i the derivative equal to the derivative of the performance function of the original optimizing control problem (1), provided the condition (6) is satisfied.

Indeed,

$$\begin{aligned} (q_{\text{mod}})_c^{'}(c, c^i, \tilde{w}, \alpha^i) &= Q_{\mathbf{c}}^{'}(c, F(c, \tilde{w}, \alpha^i)) + \\ &+ Q_y^{'}(c, F(c, \tilde{w}, \alpha^i)) \cdot F_c^{'}(c, \tilde{w}, \alpha^i) + \\ -Q_y^{'}(c^i, F(c^i, \tilde{w}, \alpha^i)) \cdot &[F_c^{'}(c^i, \tilde{w}, \alpha^i) - (F_*)_c^{'}(c^i, w)] \end{aligned}$$

where $(q_{\text{mod}})_c^{'}(c, c^i, \tilde{w}, \alpha^i)$ denotes the derivative with respect to the first argument. Now, if (6) is satisfied then at $c = c^i$

$$(q_{\mathrm{mod}})'_c(c^i, c^i, \tilde{w}, \alpha^i) = Q'_c(c^i, F_*(c^i, w)) + \\ + Q'_y(c^i, F_*(c^i, w)) \cdot (F_*)'_c(c^i, w) \\ = (q_*)'_c(c^i, w) \quad (7)$$

where

$$q_*(c, w) = Q(c, F_*(c, w)) \quad (8)$$

is the performance function of the OCP (1) after its conversion to the following simplified form with the variables y eliminated:

$$\text{minimize } \{q_*(c, w) = Q(c, F_*(c, w))\} \\ \text{subject to } \; c \in C. \quad (9)$$

Imagine now the MMOP is used instead of the basic model optimization problem MOP in the iterative two-step procedure of model optimizations and parameter estimations. That is, iterations of the set-points are done in such a way that a solution $\hat{c}(c^i)$ of the MMOP problem becomes the next process set-point c^{i+1}, $c^{i+1} := \hat{c}(c^i)$, etc. Then, if the sequence $\{c^i\}$ is convergent to a point, say $\tilde{c}$, this point satisfies $\tilde{c} = \hat{c}(\tilde{c})$. So it is, in the convergence limit, both the initial and the optimal point of the MMOP. Further, the equality (7) implies then that *$\tilde{c}$ satisfies also necessary optimality conditions for the OCP.* Indeed, $\tilde{c}$ is an optimal point for the MMOP, so it satisfies its necessary optimality conditions. Provided the constraint set C is convex, these optimality conditions can be written as

$$(q_{\mathrm{mod}})'_c(\tilde{c}, \tilde{c}, \tilde{w}, \tilde{\alpha}) \cdot [c - \tilde{c}] \geq 0 \quad \text{for all } c \in C \quad (10)$$

where $\tilde{\alpha}$ is a parameter set corresponding to $c^i = \tilde{c}$. However, due to (7), (10) can be rewritten as

$$(q_*)'_c(\tilde{c}, w) \cdot [c - \tilde{c}] \geq 0 \quad \text{for all } c \in C \quad (11)$$

This means that the point $\tilde{c}$ satisfies necessary optimality conditions for the OCP (9).

Concluding, *a point satisfying necessary optimality conditions of the original optimizing set-point control problem (1) is the convergence limit of the iterations of the ISOPE* – of the the two-step procedure with the model optimization problem MOP replaced by the MMOP and the parameter estimation problem performed under the condition (6). Therefore, the set-point optimal for the true plant (precisely, satisfying necessary optimality conditions for the OCP) can be reached – not a suboptimal one as it is in the case of the classical iterative two-step procedure of (not modified) model optimizations and parameter estimations. (Certainly, the conclusion is strictly true provided all measurements are precise, in practice there is always certain suboptimality connected with measurement errors.)

However, there is a price for such an excellent result – at every subsequent set-point c^i the derivative $F'_*(c^i, w)$ of the true controlled plant input-output mapping must be estimated, to evaluate the modifiers λ, see (5). Certainly, this estimation must be based on on-line measurement information and is obviously not an easy task. In the first period of development, applicability of the ISOPE algorithms was hampered by the lack of efficient methods of this derivative estimation. The situation was changed by the development of the dual ISOPE algorithm (Brdyś and Tatjewski, 1994), further developed as the two-phase dual algorithm (Tatjewski, 1998), see also (Tatjewski *et al.*, 2001), (Roberts, 2000).

Summarizing, the *general structure of the ISOPE algorithms* is as follows:

Step 0. Given initial set point c^0, set iteration counter $i := 0$ and initialize algorithm internal parameters.

Step 1. Apply set-point c^i to the controlled plant, measure the corresponding steady-state outputs $y^i = F_*(c^i, w)$, calculate the derivative $F'_*(c^i, w)$ (*the technique to evaluate this derivative is a key issue, defining different ISOPE algorithms*).

Step 2. Perform the model parameter estimation (MPE) under the condition (6), yielding new parameter values $\alpha^i = \alpha^i(c^i)$.

Step 3. Calculate the modifier $\lambda(c^i, \alpha^i)$ and solve the modified model optimization problem MMOP (4), yielding the solution $\hat{c}^i_m$. Set $i := i + 1$ and go to Step 1.

Step 4. Check the termination criterion. If not satisfied then update the set-point, e. g. using the formula $c^{i+1} := \hat{c}^i_m$ – another formulae also possible and used, see (Brdyś *et al.*, 1987*a*; Tatjewski and Roberts, 1987).

4. NEW STRUCTURE OF THE METHOD

The discussion presented in the proceeding section is *original* – in previous papers on the subject formal derivation of the MMOP and the ISOPE method was performed starting from necessary optimality conditions of certain specially formulated OCP with additional variables, see e. g. (Brdyś *et al.*, 1987*a*; Tatjewski and Roberts, 1987). Our approach is simpler, more intuitive and gives deeper insight into the nature of the method. Namely, it is evident from our development that the only necessary source of the true optimality property of the ISOPE is the fundamental equality (7), obtained due to the introduction of the modifier term $\lambda(c^i, \alpha^i)^T c$ to the performance function and due to the parameter estimation condition (6). Introduction of the modifier term is unavoidable, but the method can be redesigned to the structure where the parameter estimation can be performed without the condition (6) and even not in every iteration. That will make the method more natural and easier to implement. Observe that repeating model parameter estimation problem (MPE) after adding a single new point to the data record (last steady-state measurement) is rather not very reasonable if the record consists of many points. Therefore, the MPE has to be then performed in fact mainly to satisfy the condition (6) only. A way to avoid this will now be proposed. The key idea is to introduce a model shift parameter a^i, being an internal calculation parameter of the method, not visible in the model parameter estimation problem.

Let us formulate, in place of MMOP (4), the following *modified model optimization problem*, called MMOP1 in what follows,

$$\begin{aligned} &\text{minimize}_c\{q_{\text{mod}\,a}(c, c^i, \tilde{w}, \alpha, a^i) = \\ &\qquad = Q(c, F(c, \tilde{w}, \alpha) + a^i) - \lambda_a(c^i, \alpha)^T c\} \\ &\text{subject to } \ c \in C \end{aligned} \tag{12}$$

with the following definition of the modifier (comp. (5))

$$\begin{aligned} \lambda_a(c^i, \alpha)^T = Q_y^{'}(c^i, F(c^i, \tilde{w}, \alpha) + a^i)\cdot \\ \cdot\,[F_c^{'}(c^i, \tilde{w}, \alpha) - (F_*)_c^{'}(c^i, w)] \end{aligned} \tag{13}$$

where

$$a^i \stackrel{\text{def}}{=} F_*(c^i, w) - F(c^i, \tilde{w}, \alpha) \tag{14}$$

Now, if the MMOP1 problem is used instead of the MMOP in the ISOPE algorithm structure, then the discussion of the optimality can be performed in the same way as in the previous section leading to the same main result: *a point satisfying necessary optimality conditions of the original optimizing control problem (1) is the convergence limit of the iterations.* But, the parameter estimation problem is now unconstrained. Moreover, parameter estimation is in fact not necessary at all, although possible when reasonable. The ISOPE iterations can consist of successive optimizations of MMOP1 problems and set-points updates only.

The *new, improved algorithm structure* is as follows:

Step 0. Given initial set point c^0, set iteration counter $i := 0$ and initialize algorithm internal parameters.

Step 1. Apply set-point c^i to the controlled plant, measure the corresponding steady-state outputs $y^i = F_*(c^i, w)$, calculate the derivative $F_*^{'}(c^i, w)$ (*the technique to evaluate this derivative is a key issue, defining different ISOPE algorithms*).

(Step 2 – optional. Perform the model parameter estimation (adaptation) yielding new parameter values α.)

Step 3. Calculate the modifier $\lambda_a(c^i, \alpha)$ (13) and solve the modified model optimization problem MMOP1 (12), yielding the solution $\hat{c}_m^i$.

Step 4. Check the termination criterion. If not satisfied then update the set-point, e.g. using the formula $c^{i+1} := \hat{c}_m^i$, another formulae also possible and used. Set $i := i + 1$, go to Step 1.

It should be noticed that now parameter estimation and model based optimization are *not integrated,* therefore the name "ISOPE – Integrated System Optimization and Parameter Estimation" is not any longer adequate for the redesigned method. The optimal properties of the method are solely due to the modification of the model based optimization, in fact introduced to obtain the performance function *gradient modification* leading to the fundamental equation (7). For the described new formulation of the method the equation (7) takes the following form

$$(q_{\text{mod}\,a})_c^{'}(c^i, c^i, \tilde{w}, \alpha, a^i) = (q_*)_c^{'}(c^i, w) \tag{15}$$

Therefore, the presented method could be better described as, e.g., the *modified gradient optimizing set-point control* to emphasize the fact that gradient modification is the clue of the approach.

The modification of the ISOPE proposed in this paper is very basic, concerning formulations of the modifier and the modified model optimization problem. The modification does not influence the way the plant input-output mapping derivative is used and estimated. Therefore, this modification can be applied to all ISOPE algorithms, as they have been referenced in many places throughout this paper. Moreover, theoretical analysis concerning optimality and convergence given in (Brdyś *et al.*, 1987*a*) remains valid for the redesigned ISOPE formulation proposed in this paper. The reason is the fundamental equation (7) is the key for the analysis, and it is certainly true.

It can easily be seen that the modified and standard ISOPE structures become identical when the performance function Q is additive with respect to c and y and linear in y, i.e.,

$$Q(c, y) = \tilde{Q}(c) + b^T y \tag{16}$$

where b is a vector of coefficients. This follows from the facts that the resulting additive term $b^T a^i$ does not influence optimization results and that the derivative of Q with respect to y is not dependent on y, therefore on a^i too.

The paper is devoted to optimizing set-point control methods. However, the ISOPE algorithm can also be applied as a *powerful optimization method (off-line)* for problems with difficult nonlinear equality constraints of the input-output type, see e.g. (Tadej and Tatjewski, 2001). The original nonlinear constraints play then the role of the "true plant", whereas their simplified versions are considered in the modified model optimization problems, solved iteratively within the ISOPE structure. In this kind of applications the parameter estimation problem was rather artificial, now it can be removed resulting in more natural and efficient optimization algorithm.

5. CONCLUSIONS

A short review of the basics of the iterative approach to on-line set-point optimization under uncertainty in plant models and disturbance estimates was first given in the paper. In particular, the idea of the known ISOPE (Integrated System Optimization and Parameter Estimation) method was explained in a simple and original way, pointing out really key issues. As a result of this analysis a different formulation of the Modified Model Optimization Problem (MMOP), being a key element of the approach, was proposed. This, in turn, led to a simplification of the general ISOPE structure – the model parameter estimation problem is not any longer constrained (by certain equality constraints connected with the plant model outputs) and is even not required to be performed at every set-point iteration. The new structure is more efficient and practical in optimizing set-point control (on-line set-point optimization). It also results in more efficient and natural applications to off-line optimization of problems with difficult nonlinear equality constraints of input-output type. The proposed modifications can be introduced in all ISOPE-type iterative algorithms, which differ generally in the way the estimation of derivatives of the plant input-output mapping is performed, at subsequent set-points. References concerning these algorithms are given and briefly commented in the paper.

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