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DECENTRALIZED ADAPTIVE ROBUST CONTROLLERS GUARANTEEING UNIFORM ULTIMATE BOUNDEDNESS FOR UNCERTAIN LARGE SCALE SYSTEMS

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Abstract: The problem of decentralized control is considered for a class of linear time–varying large scale systems with uncertainties and external disturbances in the interconnections. In the paper, the upper bounds of the uncertainties and external disturbances are assumed to be unknown, and control inputs are represented by the nonlinear functions satisfying the condition of the series nonlinearity. The adaptation laws are proposed to estimate such unknown bounds, and by making use of the updated values of these unknown bounds, a class of decentralized state feedback controllers are constructed. It is shown that by employing the proposed decentralized state feedback controllers, the solutions of the resulting adaptive closed–loop large scale system can be guaranteed to be uniformly ultimately bounded.

Keywords: Large scale systems, decentralized control, adaptive control, robust control, uncertainties, ultimate boundedness.

1. INTRODUCTION

It is well known that large scale dynamical systems are essential features of our modern society. For instance, transportation systems, power systems, communication systems, economic systems, and so on, can be considered as such a class of systems. Generally, a large scale system can be characterized by a large number of variables representing system, a strong interaction between the system variables, and a complex structure (see, e.g. (Siljak, 1978), (Lunze, 1992)). In particular, a large scale system is often considered as a set of interconnected subsystems, and referred to as large scale interconnected systems. The advantage of this aspect in controller design is to reduce complexity and this therefore allows the control implementation to be feasible. Thus, the problem of decentralized control of large scale interconnected systems has been widely studied (see, e.g. (Siljak, 1991), (Anderson, 1982), (Ikeda and Siljak, 1980), and the references therein).

On the other hand, it is not avoidable to include uncertain parameters and external disturbances in the practical control systems due to modeling errors, measurement errors, linearization approximations, and so on. In particular, for large scale interconnected systems, the essential uncertainties and disturbances are represented by the interconnections among the subsystems. Therefore, the problem of decentralized robust control of large scale interconnected systems with significant uncertainties has been also received considerable attention of many researchers, and many approaches to designing decentralized robust state (or output) feedback controllers have been developed (see, e.g. (Wang *et al.*, 1989), (Chen, 1987), (Chen *et al.*, 1991), (Wu, 1996), (Wu, 1999*b*), and the references therein).

It is worth pointing out that a salient feature of those schemes is that the decentralized state (or output) feedback controllers explicitly depend on the upper bounds of the uncertainties and external disturbances. Therefore, for the decentralized robust controller design problem, one has to assume that the upper bounds of the uncertainties and external disturbances are known. However, in a number of practical control problems, such bounds may be unknown, or be partially known. In some cases, it may also be difficult to evaluate the upper bounds of the uncertainties and external disturbances. Thus, one must develop some new controller design methods to relax this assumption. For composite systems, some updating laws to such unknown (or partially known) bounds have been introduced to construct some types of adaptive robust feedback controllers (see, e.g. (Chen, 1992), (Choi and Kim, 1993), (Wu, 1999*a*), (Wu, 2000), (Wu and Shigemaru, 1999), and the references therein). However, few efforts are made to consider the problem of decentralized feedback control for large scale interconnected systems with the unknown upper bounds of uncertainties and external disturbances because of its complexity. In (Chen, 1991), for example, a class of saturation–type decentralized adaptive robust state feedback controllers is proposed for a class of uncertain large scale interconnected systems with partially known bounds of uncertainties and external disturbances to guarantee the uniform ultimate boundedness of closed–loop large scale systems. In (Tang *et al.*, 2000), the decentralized (adaptive) robust feedback controllers are proposed for a class of mechanical systems described by Euler–Lagrange equations and involving high–order interconnections, which can guarantee the uniform ultimate boundedness when the control objective is to tracking a smooth desired trajectory.

In this paper, we consider the problem of decentralized feedback control for a class of linear time–varying large scale systems with uncertainties and external disturbances in the interconnections. Here, the upper bounds of the uncertainties and external disturbances are assumed to be unknown, and control inputs are represented by the nonlinear functions satisfying the condition of the so–called series nonlinearity. For such a class of uncertain large scale systems, we want to develop some decentralized stabilizing state feedback controllers. For this, we first propose some adaptation laws to estimate the unknown bounds of uncertainties and external disturbances. Then, by making use of their updated values, we construct a class of decentralized local state feedback controllers. We show that by employing the proposed decentralized state feedback controllers, the solutions of the resulting adaptive closed–loop large scale system can be guaranteed to be uniformly ultimately bounded.

2. PROBLEM FORMULATION

We consider an uncertain linear time–varying large scale system S composed of N interconnected subsystems $S_i, i = 1, 2, \cdots, N$, described by the following differential equations:

$$\frac{dx_i(t)}{dt} = A_i(t)x_i(t) + B_i(t)u_i(t) \tag{1}$$

where $t \in R^+$ is the time, $x_i(t) \in R^{n_i}$ is the state vector, and $u_i(t) \in R^{m_i}$ is the input vector. Each dynamical subsystem is interconnected as

$$u_i(t) = \sum_{j=1}^{N} A_{ij}(\zeta_i, t)x_j(t) + q_i(\nu_i, t) \tag{2}$$

where $i \in \{1, 2, \ldots, N\}$.

In (1) and (2), for each $i \in \{1, 2, \ldots, N\}$, $A_i(t)$, $B_i(t)$ are continuous matrices of appropriate dimensions, the matrices $A_{ij}(\cdot)$ accounts for the interconnection between the subsystems S_i and S_j, which is assumed to be continuous in all their arguments, and the vector $q_i(\cdot)$ represents the external disturbances for the ith subsystem S_i, which is also assumed to be continuous in all their arguments. Moreover, the uncertain parameters $(\zeta_i, \nu_i) \in \Psi_i \subset R^{l_i}$ are Lebesgue measurable and take values in a known compact bounding set Ω_i. In addition, $x(t) \in R^n$ denotes $\left[\, x_1^\top(t)\ x_2^\top(t)\ \cdots\ x_{_N}^\top(t) \,\right]^\top$, where $n = n_1 + n_2 + \cdots + n_{_N}$.

For this class of input–interconnected large scale dynamical systems with the uncertainties and external disturbances in the interconnections, we introduce a decentralized local state feedback controller $\bar{u}_i(t)$ given by

$$\bar{u}_i(t) = p_i(x_i,\, t) \tag{3}$$

for each subsystem which modifies (2) to

$$\begin{aligned} u_i(t) = {} & \Phi_i\left(\bar{u}_i(t)\right) + q_i(\nu_i, t) \\ & + \sum_{j=1}^{N} A_{ij}(\zeta_i, t)x_j(t) \end{aligned} \tag{4}$$

where $p_i(\cdot) : R^{n_i} \times R \to R^{m_i}$ is a continuous function, and the control input nonlinearity is represented by the continuous function $\Phi_i\left(\bar{u}_i\right)$.

Now, the main objective of this paper is to synthesize the decentralized local state feedback controller $\bar{u}_i(t)$ given in (3) such that the large scale system, described by (1) and (4), is stable in the presence of the uncertainties and external disturbances in the interconnections.

Assumption 2.1. All pairs $\{A_i(\cdot),\ B_i(\cdot)\}$ are uniformly completely controllable.

Assumption 2.2. The series nonlinearity described by $\Phi_i(\cdot):\ R^{m_i} \to R^{m_i}$ is any continuous function satisfying the following inequality:

$$\gamma_i^0 \bar{u}_i^\top \bar{u}_i \le \bar{u}_i^\top \Phi_i(\bar{u}_i) \le \gamma_i^* \bar{u}_i^\top \bar{u}_i, \forall \bar{u}_i \in R^{m_i} \quad (5)$$

where γ_i^0 and γ_i^* are two positive constants.

Remark 2.1. The series nonlinearity can well capture the inexact behaviour of an actuator. In general, γ_i^* is referred to as the gain margin and γ_i^0 as the gain reduction tolerance. It is well known (Anderson and Moore, 1971), that the optimal state feedback control law, derived from an optimal linear quadratic problem, can tolerate an infinite increase in gain and 50% gain reduction.

For convenience, we now introduce the following notations which represent the bounds of the uncertainties.

$$\rho_{ij}(t) := \max_{\zeta_i} \left\| A_{ij}(\zeta_i, t) \right\|$$
$$\varphi_i(t) := \max_{\nu_i} \left\| q_i(\nu_i, t) \right\|$$

where $\rho_{ij}(t)$ and $\varphi_i(t)$ are assumed to be continuous and bounded for any $t \in R^+$.

Remark 2.2. It is well known that the decentralized stabilizing state feedback controllers proposed in the control literature for the large scale system described by (1) and (4) are based on the fact that the upper bounds of the uncertainties and external disturbances in the interconnections are known. That is, $\rho_{ij}(t)$ and $\varphi_i(t)$ are assumed to be the known continuous and bounded functions, and the proposed decentralized feedback control laws include such bounds $\rho_{ij}(t)$ and $\varphi_i(t)$. However, in a number of practical control problems, such bounds may be unknown, or it is difficult to evaluate them. In this paper, we will propose a class of decentralized adaptive robust state feedback controllers for such uncertain large scale interconnected systems.

3. MAIN RESULTS

Since the bounds $\rho_{ij}(t)$ and $\varphi_i(t)$ have been assumed to be continuous and bounded for any $t \in R^+$, we can suppose that there exist some positive constants ρ_{ij}^* and φ_i^*, defined as

$$\rho_{ij}^* := \max\left\{ \rho_{ij}(t)\ :\ t \in R^+ \right\} \quad (6a)$$
$$\varphi_i^* := \max\left\{ \varphi_i(t)\ :\ t \in R^+ \right\} \quad (6b)$$

Here, it is worth pointing out that the constants ρ_{ij}^* and and φ_i^*, $i, j = 1, 2, \ldots, N$, are unknown. Therefore, such unknown bounds can not be directly employed to construct decentralized local state feedback controllers.

Without loss of generality, we also introduce the following definition:

$$\psi_i^* := \sum_{j=1}^{N} \left(\rho_{ij}^*\right)^2, \quad i = 1, 2, \ldots, N$$

where ψ_i^* is still obviously unknown positive constant.

It follows from *Assumption 2.1* that for any symmetric positive definite matrix $Q_i \in R^{n_i \times n_i}$, and any positive constant γ_i^0, the matrix Riccati equation of the form

$$\frac{dP_i(t)}{dt} + A_i^\top(t)P_i(t) + P_i(t)A_i(t)$$
$$-\gamma_i^0 P_i(t)B_i(t)B_i^\top(t)P_i(t) = -Q_i \quad (7)$$

has a solution which satisfies

$$\alpha_{i1} I_{n_i} \le P_i(t) \le \alpha_{i2} I_{n_i} \quad (8)$$

for all $t \in R^+$, where α_{i1} and α_{i2} are positive numbers.

Thus, for the large scale system described by (1) and (4) we propose the following decentralized adaptive robust state feedback controllers:

$$\bar{u}_i(t) = -\frac{1}{2} k_i(t) B_i^\top(t) P_i(t) x_i(t) \quad (9a)$$

where the decentralized control gain function $k_i(t)$ is given by

$$k_i(t) = 1 + \left(\gamma_i^0\right)^{-1} \left(\delta_i^2 \hat{\psi}_i(t) + \rho_i^2 \hat{\varphi}_i^2(t)\right) \quad (9b)$$

and where for any $i \in \{1, 2, \ldots, N\}$, $P_i(t) \in R^{n_i \times n_i}$ is the solution of the Riccati equation described by (7), δ_i and ρ_i are positive constants, and δ_i is chosen such that for any $i \in \{1, 2, \ldots, N\}$,

$$\lambda_{\min}(Q_i) - \delta^{-2} N > 0 \quad (9c)$$

where $\delta := \min\{\delta_i,\ i = 1, 2, \ldots, N\}$, and $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of the matrix "·", respectively.

In particular, for any $i \in \{1, 2, \ldots, N\}$, $\hat{\psi}_i(\cdot)$ and $\hat{\varphi}_i(\cdot)$ in (9) are respectively the estimates of the unknown ψ_i^* and φ_i^*, which are updated by the following adaptive laws:

$$\frac{d\hat{\psi}_i(t)}{dt} = -\sigma_{i1}\gamma_i\hat{\psi}_i(t)$$

$$+\delta_i^2\gamma_i \left\| B_i^\top(t)P_i(t)x_i(t) \right\|^2 \quad (10a)$$

$$\frac{d\hat{\varphi}_i(t)}{dt} = -\sigma_{i2}m_i\hat{\varphi}_i(t) + m_i \left\| B_i^\top(t)P_i(t)x_i(t) \right\| \quad (10b)$$

where σ_{i1}, σ_{i2}, γ_i, and m_i are any positive constants, and $\hat{\psi}_i(t_0)$ and $\hat{\varphi}_i(t_0)$ is finite.

Let $\hat{\psi}(t) \in R^N$ and $\hat{\varphi}(t) \in R^N$ be defined by

$$\hat{\psi}(t) := \left[\, \hat{\psi}_1(t)\ \hat{\psi}_2(t)\ \cdots\ \hat{\psi}_N(t) \,\right]^\top$$
$$\hat{\varphi}(t) := \left[\, \hat{\varphi}_1(t)\ \hat{\varphi}_2(t)\ \cdots\ \hat{\varphi}_N(t) \,\right]^\top$$

For each subsystem, applying the decentralized state feedback controller given in (9) to (1) and (4) yields the closed–loop subsystem $\hat{S}_i$, $i \in \{1, 2, \ldots, N\}$, of the form:

$$\frac{dx_i(t)}{dt} = A_i(t)x_i(t) + B_i(t)\Bigg[\Phi_i\left(\bar{u}_i(t)\right) + q_i(\nu_i, t) + \sum_{j=1}^{N} A_{ij}(\zeta_i, t)x_j(t)\Bigg] \quad (11)$$

where the decentralized control law $\bar{u}_i(t)$ is given in (9).

On the other hand, letting

$$\tilde{\psi}_i(t) = \hat{\psi}_i(t) - \psi_i^*, \quad \tilde{\varphi}_i(t) = \hat{\varphi}_i(t) - \varphi_i^*$$

we can rewrite (10) as the error system:

$$\frac{d\tilde{\psi}_i(t)}{dt} = -\sigma_{i1}\gamma_i\tilde{\psi}_i(t) - \sigma_{i1}\gamma_i\psi_i^* + \delta_i^2\gamma_i \left\| B_i^\top(t)P_i(t)x_i(t) \right\|^2 \quad (12a)$$

$$\frac{d\tilde{\varphi}_i(t)}{dt} = -\sigma_{i2}m_i\tilde{\varphi}_i(t) - \sigma_{i2}m_i\varphi_i^* + \gamma_i \left\| B_i^\top(t)P_i(t)x_i(t) \right\| \quad (12b)$$

Here, $\tilde{\psi}(t) \in R^N$ and $\tilde{\varphi}(t) \in R^N$ denote

$$\tilde{\psi}(t) := \left[\, \tilde{\psi}_1(t)\ \tilde{\psi}_2(t)\ \cdots\ \tilde{\psi}_N(t) \,\right]^\top$$
$$\tilde{\varphi}(t) := \left[\, \tilde{\varphi}_1(t)\ \tilde{\varphi}_2(t)\ \cdots\ \tilde{\varphi}_N(t) \,\right]^\top$$

In the following, by $(x, \tilde{\psi}, \tilde{\varphi})(t)$ we denote a solution of the closed–loop large scale system and the error system. Then, the following theorem can be obtained which shows the uniform ultimate boundedness of the closed–loop large scale system described by (11) and (12).

Theorem 3.1. Consider the adaptive closed–loop large scale dynamical system described by (11) and (12), which satisfies *Assumption 2.1* and *Assumption 2.2*. Then, the solutions $(x, \tilde{\psi}, \tilde{\varphi})$ $(t;\ t_0, x(t_0), \tilde{\psi}(t_0), \tilde{\varphi}(t_0))$ of the closed-loop large scale system described by (11) and the error system described by (12) are uniformly ultimately bounded in the presence of the uncertainties and external disturbances in the interconnections.

Proof: We first define for (11) and (12) a Lyapunov function candidate as follows.

$$V(x, \tilde{\psi}, \tilde{\varphi}) = \sum_{i=1}^{N} x_i^\top(t)P_i(t)x_i(t) + \frac{1}{2}\tilde{\psi}^\top(t)\Gamma^{-1}\tilde{\psi}(t) + \tilde{\varphi}^\top(t)M^{-1}\tilde{\varphi}(t) \quad (13)$$

where for each $i \in \{1, 2, \ldots, N\}$, $P_i(t)$ is the solution of Riccati differential equation (7), and $\Gamma^{-1} \in R^{N\times N}$ and $M^{-1} \in R^{N\times N}$ are positive definite matrices which are respectively defined by

$$\Gamma^{-1} := \mathrm{diag}\left\{\gamma_1^{-1},\ \gamma_2^{-1},\ \ldots,\ \gamma_N^{-1}\right\}$$

$$M^{-1} := \mathrm{diag}\Big\{m_1^{-1},\ m_2^{-1},\ \ldots,\ m_N^{-1}\Big\}$$

Let $(x(t),\ \tilde{\psi}(t),\ \tilde{\varphi}(t))$ be the solutions of closed–loop large scale system (11) and error system (12) for $t \geq t_0$. Then by taking the derivative of $V(\cdot)$ along the trajectories of (11) and (12) it is obtained that

$$\begin{aligned}\frac{dV(x, \tilde{\psi}, \tilde{\varphi})}{dt} = \sum_{i=1}^{N}\Bigg\{ & x_i^\top(t)\Bigg[\frac{dP_i(t)}{dt} + A_i^\top(t)P_i(t) + P_i(t)A_i(t)\Bigg]x_i(t) \\ & + 2x_i^\top(t)P_i(t)B_i(t)\Phi_i\left(\bar{u}_i(t)\right) \\ & + 2x_i^\top(t)P_i(t)B_i(t)\sum_{j=1}^{N} A_{ij}(\zeta_i, t)x_j(t) \\ & + 2x_i^\top(t)P_i(t)B_i(t)q_i(\nu_i, t)\Bigg\} \\ & + \tilde{\psi}^\top(t)\Gamma^{-1}\frac{d\tilde{\psi}(t)}{dt} + 2\tilde{\varphi}^\top(t)M^{-1}\frac{d\tilde{\varphi}(t)}{dt} \end{aligned} \quad (14)$$

From (5) and (9) we can obtain that

$$2x_i^\top(t)P_i(t)B_i(t)\Phi_i\left(\bar{u}_i(t)\right) \leq -\gamma_i^0 k_i(t)\left\| B_i^\top(t)P_i(t)x_i(t) \right\|^2 \quad (15)$$

Thus, substituting (15) into (14) yields

$$\begin{aligned}\frac{dV(x, \tilde{\psi}, \tilde{\varphi})}{dt} \leq \sum_{i=1}^{N}\Bigg\{ & -\lambda_{\min}\left(Q_i\right)\left\|x_i(t)\right\|^2 \\ & - \left(\delta_i^2\hat{\psi}_i(t) + \rho_i^2\hat{\varphi}_i^2(t)\right)\left\| B_i^\top(t)P_i(t)x_i(t) \right\|^2 \\ & + 2\sum_{j=1}^{N}\rho_{ij}^*\left\|x_j(t)\right\|\left\| B_i^\top(t)P_i(t)x_i(t) \right\| \\ & + 2\varphi_i^*\left\| B_i^\top(t)P_i(t)x_i(t) \right\|\Bigg\}\end{aligned}$$

$$+\tilde{\psi}^{\top}(t)\Gamma^{-1}\frac{d\tilde{\psi}(t)}{dt}+2\tilde{\varphi}^{\top}(t)M^{-1}\frac{d\tilde{\varphi}(t)}{dt} \quad (16)$$

Notice that the facts that for any $i \in \{1, 2, \dots, N\}$,

$$\hat{\psi}_i(t) \;=\; \tilde{\psi}_i(t)+\psi_i^*, \;\; \hat{\varphi}_i(t) \;=\; \tilde{\varphi}_i(t)+\varphi_i^*$$

where

$$\psi_i^* \;:=\; \sum_{j=1}^{N}\left(\rho_{ij}^*\right)^2, \;\; i=1,2,\dots,N$$

it follows from (16) that

$$\begin{aligned}
\frac{dV(x,\tilde{\psi})}{dt} \;\leq\; & -\sum_{i=1}^{N}\lambda_{\min}\left(Q_i\right)\|x_i(t)\|^2 \\
& +\sum_{i=1}^{N}\sum_{j=1}^{N}\delta_i^{-2}\|x_j(t)\|^2 \\
& -\sum_{i=1}^{N}\delta_i^2\tilde{\psi}_i(t)\left\|B_i^{\top}(t)P_i(t)x_i(t)\right\|^2 \\
& -\sum_{i=1}^{N}\Big\{\rho_i^2\hat{\varphi}_i^2\left\|B_i^{\top}(t)P_i(t)x_i(t)\right\|^2 \\
& -2\varphi_i^*\left\|B_i^{\top}(t)P_i(t)x_i(t)\right\|\Big\} \\
& +\sum_{i=1}^{N}\left\{\gamma_i^{-1}\tilde{\psi}_i(t)\frac{d\tilde{\psi}_i}{dt}+2m_i^{-1}\tilde{\varphi}_i(t)\frac{d\tilde{\varphi}_i}{dt}\right\} \\
= & -\sum_{i=1}^{N}\left\{\eta_i\|x_i\|^2+\frac{1}{2}\sigma_{i1}\tilde{\psi}_i^2(t)+\sigma_{i2}\tilde{\varphi}_i^2(t)\right\} \\
& -\sum_{i=1}^{N}\Big\{\rho_i^2\hat{\varphi}_i^2(t)\left\|B_i^{\top}(t)P_i(t)x_i(t)\right\|^2 \\
& -2\hat{\varphi}_i(t)\left\|B_i^{\top}(t)P_i(t)x_i(t)\right\|\Big\} \\
& -\sum_{i=1}^{N}\Big\{\frac{1}{2}\sigma_{i1}\tilde{\psi}_i^2(t)+\sigma_{i1}\tilde{\psi}_i(t)\psi_i^*\Big\} \\
& -\sum_{i=1}^{N}\Big\{\sigma_{i2}\tilde{\varphi}_i^2(t)+2\sigma_{i2}\tilde{\varphi}_i(t)\varphi_i^*\Big\} \\
\leq & -\Big\{\sum_{i=1}^{N}\eta_i\|x_i(t)\|^2+\frac{1}{2}\tilde{\psi}^{\top}(t)\Sigma_1\tilde{\psi}(t) \\
& \qquad +\tilde{\varphi}^{\top}(t)\Sigma_2\tilde{\varphi}(t)\Big\}+\varepsilon
\end{aligned} \quad (17)$$

where

$$\eta_i := \lambda_{\min}\left(Q_i\right)-\delta^{-2}N>0, \; i=1,2,\dots,N$$

and where

$$\varepsilon := \sum_{i=1}^{N}\Big\{\rho_i^{-2}+\frac{1}{2}\sigma_{i1}\left|\psi_i^*\right|^2+\sigma_{i2}\left|\varphi_i^*\right|^2\Big\} \quad (18a)$$

$$\Sigma_1 := \mathrm{diag}\{\sigma_{11}, \;\; \sigma_{21}, \;\; \cdots, \;\; \sigma_{N1}\} \quad (18b)$$

$$\Sigma_2 := \mathrm{diag}\{\sigma_{12}, \;\; \sigma_{22}, \;\; \cdots, \;\; \sigma_{N2}\} \quad (18c)$$

On the other hand, since $P_i(t)$ is symmetric positive definite, it is obtained from the Rayleigh principle that for any $t \in R^+$,

$$\begin{aligned}
\lambda_{\min}(P_i(t))\|x_i(t)\|^2 \;\leq\; & x^{\top}(t)P(t)x(t) \\
\leq\; & \lambda_{\max}(P_i(t))\|x_i(t)\|^2
\end{aligned} \quad (19)$$

Then, in terms of (8) we can know that there exists a constant $\bar{\alpha}_i$, $i \in \{1, 2, \dots, N\}$, such that for any $t \in R^+$,

$$\lambda_{\max}(P_i(t)) \;\leq\; \bar{\alpha}_i \quad (20)$$

Thus, from (17) and (20) we obtain

$$\frac{dV(x,\tilde{\psi},\tilde{\varphi})}{dt} \;\leq\; -\tilde{\mu}V(x,\tilde{\psi},\tilde{\varphi})+\varepsilon \quad (21)$$

where

$$\bar{\eta} \;:=\; \min\{\,\bar{\eta}_i=\eta_i\bar{\alpha}_i^{-1}, \; i=1,2,\dots,N\,\}$$

and where

$$\tilde{\mu} := \min\Big\{\bar{\eta}, \; \lambda_{\min}^{-1}\left(\Gamma\right)\lambda_{\min}\left(\Sigma_1\right), \; \lambda_{\min}^{-1}\left(M\right)\lambda_{\min}\left(\Sigma_2\right)\Big\}$$

From (21), it is obvious that $V(x,\tilde{\psi},\tilde{\varphi})$ decreases monotonically along any solution of closed–loop large scale system (11) and error system (12) until the solution reaches the compact set

$$\Omega_f \;:=\; \Big\{(x,\tilde{\psi},\tilde{\varphi}) \;:\; V(x,\tilde{\psi},\tilde{\varphi}) \leq V_f\Big\} \quad (22)$$

where

$$V_f \;:=\; \tilde{\mu}^{-1}\varepsilon \quad (23)$$

Therefore, it can be concluded that the solution $(x,\tilde{\psi},\tilde{\varphi})\,(t;t_0,x(t_0)),\tilde{\psi}(t_0,\tilde{\varphi}(t_0))$ of the closed-loop large scale system described by (11) and the error system described by (12) are uniformly ultimately bounded with respect to the bound V_f given by (23).

Remark 3.1. It is worth pointing out that the parameters σ_{i1}, σ_{i2}, and ρ_i will be selected by the system designer. Therefore, by properly choosing these parameters, we can guarantee to obtain a better stability results for the adaptive large scale systems. In fact, it can be known from (18a) that by decreasing the values σ_{i1} and σ_{i2}, and by increasing the values ρ_i sufficiently, one can obtain the upper bound on the steady–state $x(t)$ and error $(\tilde{\psi}(t),\; \tilde{\varphi}(t))$ as small as desired. That is, the system designer can tune the size of the residual set by adjusting properly this parameters which are introduced in the adaptation and the control laws.

Remark 3.2. In the paper, we have constructed a class of decentralized adaptive robust state feedback controllers. However, it is not difficult to extend the results obtained in this paper to the problem of decentralized output feedback stabilization under some assumptions such as the positive realness, and minimum phase and nonsingular high–frequency gain for each isolated subsystem. In fact, on the basis of such assumptions, we can easily construct a class of decentralized adaptive robust output feedback controllers, in terms of the method proposed in this paper.

Remark 3.3. In order to illustrate the validity of the results obtained in the paper, a numerical example is also given, and the simulation is carried out. It is known from the results of the simulation that the proposed decentralized adaptive robust state feedback controllers stabilize indeed the uncertain large scale systems in the sense of uniform ultimate boundedness. (The details of the illustrative numerical example and the figures of the simulation will be displayed in the presentation.)

4. CONCLUDING REMARKS

The problem of decentralized feedback control for a class of time–varying large scale systems with uncertainties and external disturbances in the interconnections has been discussed. Here, the upper bounds of the uncertainties and external disturbances are assumed to be unknown, and control inputs have been represented by the nonlinear functions satisfying the condition of the so–called series nonlinearity. For such a class of uncertain linear time–varying large scale interconnected systems, we have proposed some adaptation laws to estimate the unknown bounds.Furthermore, by making use of the updated values of these unknown bounds we have constructed a class of decentralized state feedback controllers. We have also shown that the solutions of the adaptive closed–loop large scale system resulting from the decentralized controllers can be guaranteed to be uniformly ultimately bounded in the presence of uncertainties and external disturbances in the interconnections.

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