# POSITION SYNCHRONIZATION OF MULTIPLE MOTION AXES WITH ADAPTIVE COUPLING CONTROL

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Abstract: A new control approach to position synchronization of multiple motion axes is developed, by incorporating cross-coupling technology into adaptive control design. The strategy is to stabilize position tracking of each axis while causing differential position errors between each axis and the other two axes converge to zero. If motions of every pair of axes are synchronized, motions of all axes are synchronized accordingly. The proposed adaptive controller and parameter adaptor employ coupling control by feeding back the position error of each axis and differential position errors between this axis and the other two axes. The proposed algorithm guarantees asymptotic convergence to zero of both position errors and synchronization errors. Simulation results demonstrate the effectiveness of the method, where the system is subject to different dynamic model parameters of motion axes, plant parameter variation, and external disturbances.

Keywords: synchronization, adaptive control, coupling, multiple axes

## 1. INTRODUCTION

Interest in synchronization of multiple number of motion axes or motors has grown recently in modern manufacturing. In the control of surface mounting technology (SMT) machineries or machine tools, for example, there exists increasing demand to drive multiple motion axes simultaneously for rapid development. Poor synchronization of relevant motion control axes results in diminished dimensional accuracy of the work-piece or even in unusable products (Tomizuka et al., 1992). In 1980s, Koren (1980) proposed the cross-coupling concept to solve the synchronization problem. The effort of using the cross-coupling technology to improve synchronization performance of two-axis motions includes Kulkarni and Srinivasan (1990), Tomizuka et al. (1992), and Koren and Lo (1992). Other approaches include fuzzy logic coupling control (Moore and Chen, 1995) and neuro-controller for synchronization (Lee and Jeon, 1998).

Adaptive control is an effective strategy used to address the synchronization problem. Tomizuka et al. (1992) designed and implemented an adaptive feedforward controller for speed synchronization of two motion axes, which was followed by Kamano *et al.* (1993) and Yang and Chang (1996). However, the feedforward adaptive synchronization deals with velocity synchronization only, and fails to address position synchronization problem. On the other hand, most of approaches mentioned above focused on two-axis synchronization. There exists increasing demand for developing a new control algorithm that can address *position* synchronization of *multiple* axes. This is re-

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quired by considerable industrial applications such as operations of machine tools or SMT machines.

In this study, a new adaptive coupling control algorithm is developed for position synchronization of multiple axes. The cross coupling technology is incorporated into adaptive control design through feedback of position errors and synchronization errors in the controller and the parameter adaptor. Synchronization errors are defined to be differential position errors of every pair of axes. Including all synchronization errors in the controller formulation for each axis results in intensive on-line computation work, especially when the number of axes is large. Therefore, the synchronization strategy proposed in this study is to stabilize synchronization errors between each axis and the other two axes (but not all other axes) to zero. This is based on an assumption that motion of all axes are synchronized if every pair of axes are synchronized. It is proved in theory that the proposed algorithm guarantees asymptotic convergence to zero of both position errors and synchronization errors, which has yet to be reported in the literature. Simulation results demonstrate the effectiveness of the proposed method, where the system is subject to different dynamic model parameters of motion axes, plant parameter variation, and external disturbances.

### 2. STRATEGY OF MULTI-AXIS SYNCHRONIZATION

Consider a motion control system with n axes. The dynamics of the *i*th axis is described by

$$H_i(x_i)\ddot{x}_i(t) + C_i(x_i, \dot{x}_i)\dot{x}_i(t) + F_i(x_i, \dot{x}_i) = \tau_i(1)$$

where  $x_i(t)$  denotes the motion coordinate of the *i*th axis,  $H_i(x_i)$  and  $C_i(x_i, \dot{x}_i)$  are terms representing inertia and nonlinear effect of the *i*th axis. (Note that  $\frac{1}{2}\dot{H}_i(x_i) - C_i(x_i, \dot{x}_i) = 0$ .)  $F_i(x_i, \dot{x}_i)$  denotes the external disturbance.  $\tau_i$  is the input torque.

Define  $\theta_i$  as the vector containing unknown model parameters in (1). Then, the dynamics (1) can be rearranged in terms of  $\theta_i$ , i.e.,

$$H_{i}(x_{i})\ddot{x}_{i}(t) + C_{i}(x_{i}, \dot{x}_{i})\dot{x}_{i}(t) + F_{i}(x_{i}, \dot{x}_{i})$$
  
=  $Y_{i}(x_{i}, \dot{x}_{i}, \ddot{x}_{i})\theta_{i} = \tau_{i}$  (2)

where  $Y_i(x_i, \dot{x}_i, \ddot{x}_i)$  denotes a regression matrix. Since the value of the dynamic parameter  $\theta_i$  is hard to be known exactly in practice, one defines  $\hat{\theta}_i(t)$  as the estimate of  $\theta_i$ .

Define the position tracking error of the *i*th axis as

$$e_i(t) = x_i^d(t) - x_i(t)$$
 (3)

where  $x_i^d(t)$  denotes the desired position of the *i*th axis. In addition to causing  $e_i(t) \rightarrow 0$  in position

tracking, it is aimed to achieve the following goal for motion synchronization:

$$e_1(t) = e_2(t) = \dots = e_n(t)$$
 (4)

Define synchronization errors of every two axes with adjacent sequence numbers as

$$\epsilon_1(t) = e_1(t) - e_2(t)$$

$$\epsilon_2(t) = e_2(t) - e_3(t)$$

$$\vdots$$

$$\epsilon_n(t) = e_n(t) - e_1(t)$$
(5)

Obviously, if  $\epsilon_i(t) = 0$  for all  $i = 1 \sim n$ , the goal of multi-axis synchronization of (4) can be achieved.

Now the control objective is defined to design control torques  $\tau_i$  to cause position errors  $e_i(t)$  and synchronization errors  $\epsilon_i(t)$  converge to zero. Unlike traditional non-synchronized control that only concerns the convergence of position tracking errors  $e_i(t)$ , here one additionally concerns how these position errors converge to zero so that (4) holds. To meet this requirement, the control design of each axis needs to consider motion responses of the other axes. Note that incorporating motion responses of all axes into the controller formulation of each axis may result in intensive on-line computation work, especially when the number n is large. It is necessary to propose a control strategy that is able to synchronize multiple axes with the least number of axes involved in each axis control, for feasible implementation. For this purpose, the following strategy is proposed in this study:

**Strategy**: The control torque of each axis is designed to stabilize the position tracking of this axis while synchronizing motions between this axis and two other axes. Specifically, the control torque  $\tau_i$  for the *i*th axis is to control  $e_i(t) \rightarrow 0$  and at the same time, to synchronize motions of the (i-1)th axis, the *i*th axis, and the (i + 1)th axis, so that synchronization errors  $\epsilon_{i-1}(t)$  and  $\epsilon_i(t)$  converge to zero.

Under the above strategy, motions of all axes are synchronized. The control of each axis only additionally considers motion responses of two other axes, but not all other axes, for synchronization. This significantly simplifies the implementation especially when the number of axes is large.

### 3. CONTROL DESIGN

Under the proposed synchronization strategy, the *i*th axis controller not only controls the position tracking of the *i*th axis but also synchronizes motions of the *i*th axis, the (i-1)th, and the (i+1)th axes. Accordingly, a new concept named *coupled position error*, denoted

by  $e_i^*(t)$ , is introduced here.  $e_i^*(t)$  contains the position error  $e_i(t)$  and synchronization errors  $\epsilon_i(t)$  and  $\epsilon_{i-1}(t)$ , i.e.,

$$e_{1}^{*}(t) = e_{1}(t) + \beta \int_{0}^{t} (\epsilon_{1}(w) - \epsilon_{n}(w))dw$$

$$e_{2}^{*}(t) = e_{2}(t) + \beta \int_{0}^{t} (\epsilon_{2}(w) - \epsilon_{1}(w))dw \qquad (6)$$

$$\vdots$$

$$e_n^*(t) = e_n(t) + \beta \int_0 (\epsilon_n(w) - \epsilon_{n-1}(w)) dw$$

where  $\beta$  is a positive coupling parameter. Note that the synchronization error  $\epsilon_i(t)$  appears in  $e_i^*(t)$  and  $e_{i+1}^*(t)$  with opposite sign. As a result,  $e_i^*(t)$  and  $e_{i+1}^*(t)$  are driven in opposite directions regarding  $\epsilon_i(t)$ , which is helpful to removal of the synchronization error  $\epsilon_i(t)$ .

Define command vectors  $u_i(t)$  as

$$u_{1}(t) = \dot{x}_{1}^{d}(t) + \beta(\epsilon_{1}(t) - \epsilon_{n}(t)) + \Lambda e_{1}^{*}(t)$$

$$u_{2}(t) = \dot{x}_{2}^{d}(t) + \beta(\epsilon_{2}(t) - \epsilon_{1}(t)) + \Lambda e_{2}^{*}(t)$$

$$\vdots$$

$$u_{n}(t) = \dot{x}_{n}^{d}(t) + \beta(\epsilon_{n}(t) - \epsilon_{n-1}(t)) + \Lambda e_{n}^{*}(t)$$

where  $\dot{x}_i^d(t)$  denotes the desired velocity of the *i*th axis, and  $\Lambda$  is a positive number. Definition of  $u_i(t)$  in (7) leads to following vectors regarding coupled position/velocity errors

$$r_i(t) = u_i(t) - \dot{x}_i(t) = \dot{e}_i^*(t) + \Lambda e_i^*(t)$$
 (8)

The objective is to design control torques  $\tau_i(t)$  to restrict  $r_i(t)$  to lie on the sliding surface, so that the coupled errors  $e_i^*(t)$  and  $\dot{e}_i^*(t)$  tend to zero.

Design torque inputs as

$$\begin{aligned} \tau_{1} &= \hat{H}_{1}(x_{1})\dot{u}_{1}(t) + \hat{C}_{1}(x_{1},\dot{x}_{1})u_{1}(t) + \hat{F}_{1}(x_{1},\dot{x}_{1}) \\ &+ k_{r}r_{1}(t) + k_{\epsilon}(\epsilon_{1}(t) - \epsilon_{n}(t)) \\ &= Y_{1}(x_{1},\dot{x}_{1},u_{1},\dot{u}_{1})\hat{\theta}_{1}(t) + k_{r}r_{1}(t) \\ &+ k_{\epsilon}(\epsilon_{1}(t) - \epsilon_{n}(t)) \\ \tau_{2} &= \hat{H}_{2}(x_{2})\dot{u}_{2}(t) + \hat{C}_{2}(x_{2},\dot{x}_{2})u_{2}(t) + \hat{F}_{2}(x_{2},\dot{x}_{2}) \\ &+ k_{r}r_{2}(t) + k_{\epsilon}(\epsilon_{2}(t) - \epsilon_{1}(t)) \\ &= Y_{2}(x_{2},\dot{x}_{2},u_{2},\dot{u}_{2})\hat{\theta}_{2}(t) + k_{r}r_{2}(t) \\ &+ k_{\epsilon}(\epsilon_{2}(t) - \epsilon_{1}(t)) \end{aligned}$$
(9)  

$$\vdots$$

$$\begin{aligned} \tau_n &= \hat{H}_n(x_n) \dot{u}_n(t) + \hat{C}_n(x_n, \dot{x}_n) u_n(t) + \hat{F}_n(x_n, \dot{x}_n) \\ &+ k_r r_n(t) + k_\epsilon (\epsilon_n(t) - \epsilon_{n-1}(t)) \\ &= Y_n(x_n, \dot{x}_n, u_n, \dot{u}_n) \hat{\theta}_n(t) + k_r r_n(t) \end{aligned}$$

$$+k_{\epsilon}(\epsilon_n(t)-\epsilon_{n-1}(t))$$

where  $\hat{H}_i(x_i)$ ,  $\hat{C}_i(x_i, \dot{x}_i)$  and  $\hat{F}_i(x_i, \dot{x}_i)$  are estimates of  $H_i(x_i)$ ,  $C_i(x_i, \dot{x}_i)$  and  $F_i(x_i, \dot{x}_i)$ , respectively.  $k_r$ and  $k_\epsilon$  are positive control gains. Note that the regression matrix  $Y_i(x_i, \dot{x}_i, u_i, \dot{u}_i)$  is now a function of  $u_i(t)$ and  $\dot{u}_i(t)$  rather than  $\ddot{x}_i(t)$ . The estimated parameter  $\hat{\theta}_i(t)$  is subject to the adaptation law

$$\dot{\hat{\theta}}_i(t) = \Gamma_i Y_i^T(x_i, \dot{x}_i, u_i, \dot{u}_i) r_i(t)$$
(10)

where  $\Gamma_i$  is a positive diagonal control gain. Define  $\tilde{\theta}_i(t) = \theta_i - \hat{\theta}_i(t)$  as a bounded vector containing model estimation errors. The adaptation law (10) can then be rewritten by

$$\dot{\tilde{\theta}}_i(t) = -\Gamma_i Y_i^T(x_i, \dot{x}_i, u_i, \dot{u}_i) r_i(t)$$
(11)

Substituting (9) into the dynamic model (2) leads to the following closed-loop dynamics

$$\begin{aligned} H_{1}(x_{1})\dot{r}_{1}(t) + C_{1}(x_{1},\dot{x}_{1})r_{1}(t) + k_{r}r_{1}(t) \\ + k_{\epsilon}(\epsilon_{1}(t) - \epsilon_{n}(t)) &= Y_{1}(x_{1},\dot{x}_{1},u_{1},\dot{u}_{1})\tilde{\theta}_{1}(t) \\ H_{2}(x_{2})\dot{r}_{2}(t) + C_{2}(x_{2},\dot{x}_{2})r_{2}(t) + k_{r}r_{2}(t) \\ + k_{\epsilon}(\epsilon_{2}(t) - \epsilon_{1}(t)) &= Y_{2}(x_{2},\dot{x}_{2},u_{2},\dot{u}_{2})\tilde{\theta}_{2}(t) \quad (12) \\ &\vdots \\ H_{n}(x_{n})\dot{r}_{n}(t) + C_{n}(x_{n},\dot{x}_{n})r_{n}(t) + k_{r}r_{n}(t) \\ + k_{\epsilon}(\epsilon_{n}(t) - \epsilon_{n-1}(t)) &= Y_{n}(x_{n},\dot{x}_{n},u_{n},\dot{u}_{n})\tilde{\theta}_{n}(t) \end{aligned}$$

**Theorem 1** The proposed adaptive coupling controllers (6) ~ (11) guarantee asymptotic convergence to zero of both position errors and synchronization errors, i.e.,  $e_i(t) \rightarrow 0$  and  $\epsilon_i(t) \rightarrow 0$  as time  $t \rightarrow \infty$ .

Define a positive definite function as

$$V(t) = \sum_{i=1}^{n} \left[\frac{1}{2}H_i(x_i)r_i^2(t) + \frac{1}{2}\tilde{\theta}_i^T(t)\Gamma_i^{-1}\tilde{\theta}_i(t) + \frac{1}{2}k_\epsilon\epsilon_i^2(t)\right]$$
$$+ \frac{1}{2}k_\epsilon\Lambda\beta(\int_0^t(\epsilon_1(w) - \epsilon_n(w))dw)^2$$
$$+ \sum_{i=2}^{n}\frac{1}{2}k_\epsilon\Lambda\beta(\int_0^t(\epsilon_i(w) - \epsilon_{i-1}(w))dw)^2 \qquad (13)$$

Differentiating V(t) with respect to time yields

$$\dot{V}(t) = \sum_{i=1}^{n} [r_i^T(t)H_i(x_i)\dot{r}_i(t) + \frac{1}{2}r_i^T(t)\dot{H}_i(x_i)r_i(t) + \\ \tilde{\theta}_i^T(t)\Gamma_i^{-1}\dot{\tilde{\theta}}_i(t) + \epsilon_i(t)k_\epsilon\dot{\epsilon}_i(t)] + \\ (\epsilon_1(t) - \epsilon_n(t))k_\epsilon\Lambda\beta \int_0^t (\epsilon_1(w) - \epsilon_n(w))dw$$

$$+\sum_{i=2}^{n} (\epsilon_{i}(t) - \epsilon_{i-1}(t)) k_{\epsilon} \Lambda \beta$$
$$\int_{0}^{t} (\epsilon_{i}(w) - \epsilon_{i-1}(w)) dw$$
(14)

Multiplying both sides of the closed-loop equations in (12) by  $r_i(t)$  and then substituting the resulting equations into (14) yields

$$\dot{V}(t) = \sum_{i=1}^{n} \left[ -k_r r_i^2(t) + r_i(t) Y_i(x_i, \dot{x}_i, u_i, \dot{u}_i) \tilde{\theta}_i(t) + \tilde{\theta}_i^T(t) \Gamma_i^{-1} \dot{\tilde{\theta}}_i(t) + \epsilon_i(t) k_\epsilon \dot{\epsilon}_i(t) \right] - \sum_{i=1}^{n-1} (r_i(t) - r_{i+1}(t)) k_\epsilon \epsilon_i(t) - (r_n(t) - r_1(t)) k_\epsilon \epsilon_n(t) + (\epsilon_1(t) - \epsilon_n(t)) k_\epsilon \Lambda \beta \int_0^t (\epsilon_1(w) - \epsilon_n(w)) dw + (15)$$
$$\sum_{i=2}^n (\epsilon_i(t) - \epsilon_{i-1}(t)) k_\epsilon \Lambda \beta \int_0^t (\epsilon_i(w) - \epsilon_{i+1}(w)) dw$$

Utilizing the adaptation law (11), one obtains

$$r_i^T(t)Y_i(x_i, \dot{x}_i, u_i, \dot{u}_i)\tilde{\theta}_i(t) + \tilde{\theta}_i^T(t)\Gamma_i^{-1}\dot{\tilde{\theta}}_i(t) = 0 \quad (16)$$

In view of equations (5), (6) and (8), one obtains

$$r_{1}(t) - r_{2}(t) = \dot{\epsilon}_{1}(t) + \Lambda \epsilon_{1}(t) + \beta (2\epsilon_{1}(t) - \epsilon_{2}(t) - \epsilon_{n}(t)) + \Lambda \beta \int_{0}^{t} (2\epsilon_{1}(w) - \epsilon_{2}(w) - \epsilon_{n}(w)) dw r_{2}(t) - r_{3}(t) = \dot{\epsilon}_{2}(t) + \Lambda \epsilon_{2}(t) + \beta (2\epsilon_{2}(t) - \epsilon_{3}(t) - \epsilon_{1}(t)) + \Lambda \beta \int_{0}^{t} (2\epsilon_{2}(w) - \epsilon_{3}(w) - \epsilon_{1}(w)) dw$$
(17)

$$r_n(t) - r_1(t) = \dot{\epsilon}_n(t) + \Lambda \epsilon_n(t) + \beta (2\epsilon_n(t) - \epsilon_1(t) - \epsilon_{n-1}(t)) + \Lambda \beta \int_0^t (2\epsilon_n(w) - \epsilon_1(w) - \epsilon_{n-1}(w)) dw$$

As a result,

:

$$\sum_{i=1}^{n-1} (r_i(t) - r_{i+1}(t)) k_{\epsilon} \epsilon_i(t) + (r_n(t) - r_1(t)) k_{\epsilon} \epsilon_n(t)$$

$$= \sum_{i=1}^n [\epsilon_i^T(t) k_{\epsilon} \dot{\epsilon}_i(t) + k_{\epsilon} \Lambda \epsilon_i^2(t)] + \sum_{i=1}^{n-1} k_{\epsilon} \beta((\epsilon_i(t) - \epsilon_{i+1}(t))^2 + k_{\epsilon} \beta(\epsilon_n(t) - \epsilon_1(t))^2 + (\epsilon_1(t) - \epsilon_n(t)) k_{\epsilon} \Lambda \beta \int_0^t (\epsilon_1(w) - \epsilon_n(w)) dw + \sum_{i=2}^n (\epsilon_i(t) - \epsilon_{i-1}(t)) k_{\epsilon} \Lambda \beta \int_0^t (\epsilon_i(w) - \epsilon_{i+1}(w)) dw$$
(18)

Substituting (16) and (18) into (15) yields

$$\dot{V}(t) = -\sum_{i=1}^{n} [k_r r_i^2(t) + k_\epsilon \Lambda \epsilon_i^2(t)] - \sum_{i=1}^{n-1} k_\epsilon \beta(\epsilon_i(t)) - \epsilon_{i+1}(t))^2 - k_\epsilon \beta(\epsilon_n(t) - \epsilon_1(t))^2 \le 0$$
(19)

Since  $r_i(t)$  and  $\epsilon_i(t)$  appear in (19), they are bounded in terms of  $L_2$  norm. When  $r_i(t)$  is bounded,  $\dot{e}_i^*(t)$  and  $e_i^*(t)$  are further bounded from (8), and so are  $\dot{e}_i(t)$ from differentiating (6). Thus,  $\dot{e}_i(t)$  are bounded from (5). From (12), one knows that  $\dot{r}_i(t)$  are bounded as well. Therefore,  $r_i(t)$  and  $\epsilon_i(t)$  are uniformly continuous since  $\dot{r}_i(t)$  and  $\dot{\epsilon}_i(t)$  are bounded. From Barbalat's lemma,  $r_i(t) \to 0$  and  $\epsilon_i(t) \to 0$  as time  $t \to \infty$ . It then follows from (8) that  $e_i^*(t) \to 0$  and  $\dot{e}_i^*(t) \to 0$ as  $t \to \infty$ .

When  $\epsilon_i(t) = 0$  for all  $i = 1 \sim n$ , the goal (4) is achieved. To examine the position tracking stabilization, one combines all equations in (6) to obtain

$$e_1(t) + e_2(t) + \dots + e_n(t)$$
  
=  $e_1^*(t) + e_2^*(t) + \dots + e_n^*(t) = 0$  (20)

Substituting (4) into (20) yields

$$e_1(t) = e_2(t) = \ldots = e_n(t) = 0$$

Therefore, the invariant set of the closed-loop dynamics (12) in the set  $\Omega = \{(x_i, \dot{x}_i) : \dot{V}(t) = 0\}$ contains zero position errors, namely  $e_i(t) = 0$ . Using LaSalle's theorem, one finally concludes that  $e_i(t) \rightarrow 0$  as time  $t \rightarrow \infty$ .  $\Box$ 

The advantage of the proposed adaptive coupling controller over conventional controllers without synchronization lies in the ability to regulate the relationship of coordinates during tracking process. In other words, the proposed algorithm concerns not only whether position errors  $e_i(t) \rightarrow 0$ , but also how these errors converge to zero so that  $e_1(t) = e_2(t) = \cdots = e_n(t)$ . Although the independent control without synchronization also ensures that position errors  $e_i(t) \rightarrow 0$ and hence  $\epsilon_i(t) \rightarrow 0$  eventually, they cannot guarantee satisfactory transient performance of synchronization.

#### 4. SIMULATIONS

To demonstrate the proposed approach, simulations were performed on a four-axis system in which motion synchronization is required. The desired tracking trajectory of each axes is specified by

$$x_i^d = x_i(0) + (x_i(f) - x_i(0))(1 - exp(-t))$$

where  $x_i(0)$  and  $x_i(f)$  denote the initial and the final desired coordinates of each axis, respectively. In the simulation, one selected

$$x_1(0) = x_2(0) = x_3(0) = x_4(0) = 0$$

$$x_1(f) = x_2(f) = x_3(f) = x_4(f) = 1$$

The estimated parameter vectors were set to be zero for all axes at the initial time. The control gains are chosen to be:  $\Lambda = 4$ ,  $k_r = 500$ ,  $\beta = 50$ ,  $k_{\epsilon} = 10$ , and  $\Gamma_i = diag\{0.2\}$ .

Firstly, the algorithm was tested on the system with different dynamic model parameters in four axes, i.e.,  $H_1 = C_1 = 10, H_2 = C_2 = 9, H_3 = C_3 = 8,$ and  $H_4 = C_4 = 7$ . One assumed that there was no external disturbance during the motion, namely  $F_1 = F_2 = F_3 = F_4 = 0$ . Figure 1 illustrates position tracking errors of four axes with the proposed adaptive coupling control. Besides good convergence of the position tracking in each axis, there appears satisfied performance in position synchronization of four axes. Figure 2 illustrates position tracking errors with conventional adaptive control without synchronization (namely  $\beta = 0$  and  $k_{\epsilon} = 0$ ), for comparison. Although good position tracking occurs in each axis, there appears worse transient performance of synchronization compared with that in Figure 1. Note that here synchronization errors are caused by different dynamic model parameters of axes. Figure 3 illustrates comparison of position synchronization errors of four axes (i.e.,  $\epsilon_1(t) \sim \epsilon_4(t)$ ), between the proposed adaptive coupling control (solid lines) and the independent adaptive control without synchronization (dashdot lines). It is clearly shown that the proposed coupling controller can effectively address the position synchronization problem in a multi-axis system, in which dynamic model parameters are different in each axis.

One further tested the algorithm by adding a torque disturbance to axis 1 during the motion. The disturbance was applied to axis 1 from 0.5 second to 0.7 second, which caused the change of position tracking as shown in Figures 4 and 5. It is seen that the disturbance with the proposed method has less influence to position synchronization than that with the independent control without synchronization. Figure 6 illustrates comparison of position synchronization of four axes between the proposed adaptive coupling control (solid lines) and the independent adaptive control (dashdot lines). Obviously, the proposed method exhibits significant improvement in position synchronization nization when the system is subject to the disturbance.

#### 5. CONCLUSIONS

A new adaptive coupling control algorithm has been proposed for position synchronization of multiple motion axes. Synchronization errors in this study are defined to be differential position errors of every pair of axes. The synchronization strategy is to stabilize position tracking of each axis while causing differential position errors between the axis and the other two axes converge to zero. The proposed adaptive control algorithm employs the cross-coupling technology in the design of the controller and the parameter adaptor. The proposed method guarantees asymptotic convergence to zero of both position and synchronization errors. Simulation results demonstrate the effectiveness of the approach in position synchronization, where the system is subject to different dynamic model parameters of motion axes, plant parameter variation, and external disturbances.

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Fig. 1. Position tracking errors of four axes with adaptive coupling control



Fig. 4. Position tracking errors of four axes with adaptive coupling control (with disturbance)



Fig. 2. Position tracking errors of four axes with independent adaptive control



Fig. 5. Position tracking errors of four axes with independent adaptive control (with disturbance)



Fig. 3. Synchronization errors with adaptive coupling and independent controls



Fig. 6. Synchronization errors with adaptive coupling and independent controls (with disturbance)