15th Triennial World Congress, Barcelona, Spain

POLYNOMIAL DESIGN OF CONTROLLERS FOR TWO – VARIABLE SYSTEMS – PRACTICAL IMPLEMENTATION

Marek Kubalčík, Vladimír Bobál, Miroslav Maca

Department of Control Theory, Institute of Information Technologies
Tomas BataUniversity
Nám. TGM 275, 762 72 Zlín, Czech Republic
Phone, Fax: +420 67 754 3103, E – mail:kubalcik@ft.utb.cz

Abstract: This paper presents the design and simulation of adaptive control for a two input – two output system together with the real – time control of a laboratory model using this designed method. The synthesis is based on a polynomial approach. Decoupling, where the compensator is placed ahead of the system, suppresses the interactions between control loops. The results of the simulation and the real-time experiments are also given. *Copyright © 2002 IFAC*

Keywords: Two input – two output system, controller synthesis, decoupling, compensators, adaptive control

1 INTRODUCTION

Many technological processes require that several variables relating to one system are controlled simultaneously. Each input may influence all system outputs. The design of a controller able to cope with such a system must be quite sophisticated. There are many different methods of controlling multivariable systems. Several of these use decentralized PID controllers (Luyben 1986), others apply single input-single-output (SISO) methods extended to cover multiple inputs (Chien *et al.* 1987). Here decoupling methods are used to transform the multivariable system into a series of independent SISO loops (Krishnawamy *et al.* 1991, Tade *et al.* 1986, Wittenmark *et al.* 1987, Skogestad and Postlethwaite 1996).

This paper is organized as follows: Section 2 presents the controlled model; Section 3 describes how feedback control without decoupling is designed; Section 4 describes two decoupling methods; Section 5 describes the system identification method; Section 6 gives the simulation results; Section 7 contains the experimental results; finally, Section 8 concludes the paper.

2 A DESCRIPTION OF A TWO INPUT – TWO OUTPUT SYSTEM

The internal structure of the the system is shown in Fig. 1

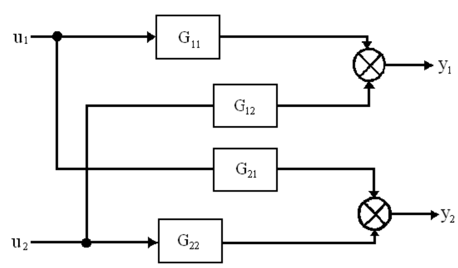

Fig. 1 A two input – two output system – the "P" structure

The transfer matrix of the system is

$$\boldsymbol{G} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \quad (1)$$

We can assume that the system is described by the matrix fraction

$$\boldsymbol{G}\left(z^{-1}\right) = \boldsymbol{A}^{-1}\left(z^{-1}\right)\boldsymbol{B}\left(z^{-1}\right) = \boldsymbol{B}_1\left(z^{-1}\right)\boldsymbol{A}_1^{-1}\left(z^{-1}\right) \quad (2)$$

Where polynomial matrices $\boldsymbol{A}\in R_{22}[z^{-1}]$, $\boldsymbol{B}\in R_{22}[z^{-1}]$ are the left indivisible decomposition of matrix $\boldsymbol{G}(z^{-1})$ and matrices $\boldsymbol{A_1}\in R_{22}[z^{-1}]$, $\boldsymbol{B_1}\in R_{22}[z^{-1}]$ are the right indivisible decomposition.

The matrices of our discrete model are

$$\boldsymbol{A}\left(z^{-1}\right)=\begin{bmatrix}1+a_1z^{-1}+a_2z^{-2} & a_3z^{-1}+a_4z^{-2}\\ a_5z^{-1}+a_6z^{-2} & 1+a_7z^{-1}+a_8z^{-2}\end{bmatrix} \quad (3)$$

$$\boldsymbol{B}\left(z^{-1}\right)=\begin{bmatrix}b_1z^{-1}+b_2z^{-2} & b_3z^{-1}+b_4z^{-2}\\ b_5z^{-1}+b_6z^{-2} & b_7z^{-1}+b_8z^{-2}\end{bmatrix}$$

and the differential equations of the model are

$$\begin{aligned}y_1(k)&=-a_1y_1(k-1)-a_2y_1(k-2)-a_3y_2(k-1)-\\&-a_4y_2(k-2)+b_1u_1(k-1)+b_2u_1(k-2)+\\&+b_3u_2(k-1)+b_4u_2(k-2)\end{aligned} \quad (4)$$

$$\begin{aligned}y_2(k)&=-a_5y_1(k-1)-a_6y_1(k-2)-a_7y_2(k-1)-\\&-a_8y_2(k-2)+b_5u_1(k-1)+b_6u_1(k-2)+\\&+b_7u_2(k-1)+b_8u_2(k-2)\end{aligned}$$

3 DESIGNING FEEDBACK CONTROL

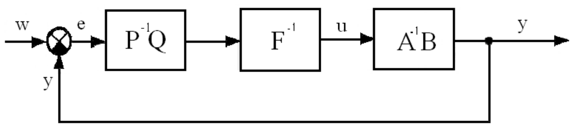

Fig. 2 Block diagram of the closed loop system

In the same way as the controlled system, the transfer matrix of the controller takes the form of matrix fraction

$$\boldsymbol{G}_R\left(z^{-1}\right)=\boldsymbol{P}^{-1}\left(z^{-1}\right)\boldsymbol{Q}\left(z^{-1}\right)=\boldsymbol{Q}_1\left(z^{-1}\right)\boldsymbol{P}_1^{-1}\left(z^{-1}\right) \quad (5)$$

The matrix of an integrator for permanent zero control error is

$$\boldsymbol{F}\left(z^{-1}\right)=\begin{bmatrix}1-z^{-1} & 0\\ 0 & 1-z^{-1}\end{bmatrix} \quad (6)$$

The control law apparent in the block diagram (operator z^{-1} will be omitted from some operations for the sake of simplification) has the form

$$\boldsymbol{U}=\boldsymbol{F}^{-1}\boldsymbol{Q}_1\boldsymbol{P}_1^{-1}\boldsymbol{E} \quad (7)$$

We can derive the following equation for the system output

$$\boldsymbol{Y}=\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\boldsymbol{Q}\boldsymbol{E}=\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\boldsymbol{Q}(\boldsymbol{W}-\boldsymbol{Y}) \quad (8)$$

which can be modified to give

$$\boldsymbol{Y}=\boldsymbol{P_1}(\boldsymbol{AFP_1}+\boldsymbol{BQ_1})^{-1}\boldsymbol{BQ_1P_1^{-1}W} \quad (9)$$

The closed loop system is stable when the following diophantine equation is satisfied

$$\boldsymbol{AF\,P}_1+\boldsymbol{BQ}_1=\boldsymbol{M} \quad (10)$$

where $\boldsymbol{M}(z^{-1})\in R_{22}[z^{-1}]$ is a stable diagonal polynomial matrix.

$$\boldsymbol{M}\left(z^{-1}\right)=\begin{bmatrix}\begin{matrix}1+m_1z^{-1}+m_2z^{-2}+\\+m_3z^{-3}+m_4z^{-4}\end{matrix} & 0\\ 0 & \begin{matrix}1+m_5z^{-1}+m_6z^{-2}+\\+m_7z^{-3}+m_8z^{-4}\end{matrix}\end{bmatrix} \quad (11)$$

The roots of this polynomial matrix are the ruling factor in the behaviour of the closed loop system. They must be inside the unit circle if the system is to be stable.

The product $\boldsymbol{AFP}_1$ is dimensionally correct if the number of inputs is equal to the number of outputs.

The degree of the controller matrices polynomials depends on the internal properness of the closed loop. The structure of matrices $\boldsymbol{P}_1$ and $\boldsymbol{Q}_1$ was chosen so that the number of unknown controller parameters equals the number of algebraic equations resulting from the solution of the diophantine equation using the uncertain coefficients method.

$$\boldsymbol{P}_1\left(z^{-1}\right)=\begin{bmatrix}1+p_1z^{-1} & p_2z^{-1}\\ p_3z^{-1} & 1+p_4z^{-1}\end{bmatrix} \quad (12)$$

$$\boldsymbol{Q}_1\left(z^{-1}\right)=\begin{bmatrix}q_1+q_2z^{-1}+q_3z^{-2} & q_4+q_5z^{-1}+q_6z^{-2}\\ q_7+q_8z^{-1}+q_9z^{-2} & q_{10}+q_{11}z^{-1}+q_{12}z^{-2}\end{bmatrix}$$

The solution to the diophantine equation results in a set of sixteen algebraic equations with unknown controller parameters. We obtain the controller parameters by solving these equations.

4 DESIGNING DECOUPLING CONTROL USING COMPENSATORS

There are several ways to control multivariable systems with internal interactions. Some make use of decentralized PID controllers, whilst others are composed of a string of single input – single output methods.

One possibility is the serial insertion of a compensator ahead of the system (Krishnawamy *et al.* 1991, Peng 1990, Tade *et al.* 1986, Wittenmark *et al.* 1987). The aim here is to suppress of undesirable interactions between the input and output variables so that each input affects only one controlled variable.

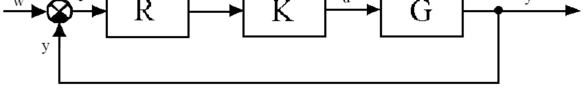

Fig. 3 A Closed loop system with compensator

The resulting transfer function $\boldsymbol{H}$ is then given by

$$\boldsymbol{H}=\boldsymbol{KG} \quad (13)$$

The decoupling conditions are fulfilled when matrix $\boldsymbol{H}$ is diagonal.

Several well – known compensators are given in (Krishnawamy *et al.* 1991, Peng 1990, Tade *et al.* 1986, Wittenmark *et al.* 1987). Control algorithms were derived for the model above with two compensators. These will be referred to as $\mathbf{C_1}$ and $\mathbf{C_2}$.

Compensator $\mathbf{C_1}$ is the inversion of the controlled system. Matrix $\boldsymbol{H}$ is,therefore, a unit matrix.

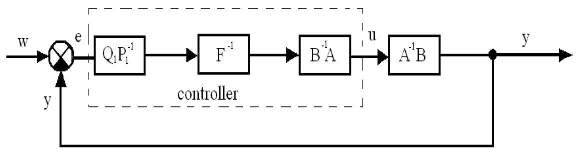

Fig. 4 The closed loop system with compensator $\mathbf{C_1}$

This block diagram leads to an equation for the system output which takes the form

$$\boldsymbol{Y} = \boldsymbol{P_1}(\boldsymbol{FP_1} + \boldsymbol{Q_1})^{-1}\boldsymbol{Q_1}\boldsymbol{P_1}^{-1}\boldsymbol{W} \tag{14}$$

The following equation must be satisfied if the closed loop system is to be stable

$$\boldsymbol{FP}_1 + \boldsymbol{Q}_1 = \boldsymbol{M} \tag{15}$$

The structure of the polynomial matrices of the controller were chosen to suit physical demands.

$$\boldsymbol{P}_1\left(z^{-1}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{16}$$

$$\boldsymbol{Q}_1\left(z^{-1}\right) = \begin{bmatrix} q_1 z^{-1} & 0 \\ 0 & q_2 z^{-1} \end{bmatrix}$$

Consequently, matrix $\boldsymbol{M}$ was chosen to be

$$\boldsymbol{M}\left(z^{-1}\right) = \begin{bmatrix} 1 + m_1 z^{-1} & 0 \\ 0 & 1 + m_2 z^{-1} \end{bmatrix} \tag{17}$$

The controller parameters are the result from the equation (15). The control law can be described by matrix equation

$$\boldsymbol{FU} = \boldsymbol{B}^{-1}\boldsymbol{AQ}_1\boldsymbol{P}_1^{-1}\boldsymbol{E} \tag{18}$$

This controller is unsuitable for non – minimum phase systems.

Compensator $\mathbf{C_2}$ is adjugated matrix $\boldsymbol{B}$. When $\mathbf{C_2}$ was included in the design of the closed loop the model was simplified by considering matrix $\boldsymbol{A}$ as diagonal. The multiplication of matrix $\boldsymbol{B}$ and adjugated matrix $\boldsymbol{B}$ results in diagonal matrix $\boldsymbol{H}$. The determinants of matrix $\boldsymbol{B}$ represent the diagonal elements. When matrix $\boldsymbol{A}$ is nondiagonal, its inverted form must be placed ahead of the system in order to obtain diagonal matrix $\boldsymbol{H}$, otherwise it may increase the order of the controller and sophistication of the closed loop system. Although designed for a diagonal matrix, compensator $\mathbf{C_2}$ also improves the control process for non – diagonal matrix $\boldsymbol{A}$ in the controlled system. This is demonstrated in the simulation results.

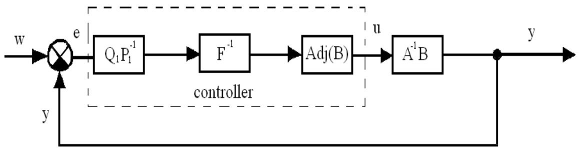

Fig. 5 The closed loop system with compensator $\mathbf{C_2}$

The equation for the system output as shown in this block diagram takes the form

$$\boldsymbol{Y} = \boldsymbol{P}_1(\boldsymbol{AFP}_1 + \boldsymbol{B}_v\boldsymbol{Q}_1)\boldsymbol{B}_v\boldsymbol{Q}_1\boldsymbol{P}_1^{-1}\boldsymbol{W} \tag{19}$$

where

$$\boldsymbol{B}_v = \boldsymbol{B}adj(\boldsymbol{B}) = \begin{bmatrix} det(\boldsymbol{B}) & 0 \\ 0 & det(\boldsymbol{B}) \end{bmatrix} \tag{20}$$

To achieve stability in the closed loop system the following diophantine equation must be fulfilled

$$\boldsymbol{AFP}_1 + \boldsymbol{B}_v\boldsymbol{Q}_1 = \boldsymbol{M} \tag{21}$$

The controller polynomial matrices are chosen as shown below

$$\boldsymbol{P}_1\left(z^{-1}\right) = \begin{bmatrix} 1 + p_1 z^{-1} & p_2 z^{-1} \\ p_3 z^{-1} & 1 + p_4 z^{-1} \end{bmatrix} \tag{22}$$

$$\boldsymbol{Q}_1\left(z^{-1}\right) = \begin{bmatrix} q_1 + q_2 z^{-1} + q_3 z^{-2} & q_4 + q_5 z^{-1} + q_6 z^{-2} \\ q_7 + q_8 z^{-1} + q_9 z^{-2} & q_{10} + q_{11} z^{-1} + q_{12} z^{-2} \end{bmatrix}$$

and matrix $\boldsymbol{M}$ is

$$\boldsymbol{M}\left(z^{-1}\right) = \begin{bmatrix} \begin{matrix} 1 + m_1 z^{-1} + \\ + m_2 z^{-2} + m_3 z^{-3} + \\ + m_4 z^{-4} + m_5 z^{-5} \end{matrix} & 0 \\ 0 & \begin{matrix} 1 + m_6 z^{-1} + \\ m_7 z^{-2} + m_8 z^{-3} + \\ + m_9 z^{-4} + m_{10} z^{-5} \end{matrix} \end{bmatrix} \tag{23}$$

Solving the diophantine equation defines a set of algebraic equations which we subsequently use to obtain the unknown controller parameters.

The control law is given by the block diagram

$$\boldsymbol{FU} = adj(\boldsymbol{B})\boldsymbol{Q}_1\boldsymbol{P}_1^{-1}\boldsymbol{E} \tag{24}$$

5 IDENTIFICATION

The algorithms designed here were incorporated into an adaptive control system with recursive identification. The recursive least squares method proved effective for self-tuning controllers (Kulhavý 1987) and was used as the basis for our algorithm. For our two-variable example we considered the disintegration of identification into two independent parts.

The parameter vectors are completed as shown below:

$$\boldsymbol{\Theta}_1^T(k) = [a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4] \qquad (25)$$

$$\boldsymbol{\Theta}_2^T(k) = [a_5, a_6, a_7, a_8, b_5, b_6, b_7, b_8]$$

The data vector is

$$\phi^T(k-1) = [-y_1(k-1), -y_1(k-2), -y_2(k-1), \qquad (26)$$
$$-y_2(k-2), u_1(k-1), u_1(k-2), u_2(k-1), u_2(k-2)]$$

The parameter estimates are actualized using the recursive least squares method plus directional forgetting.

6 SIMULATION

Matlab + Simulink for Windows (The MathWork, Inc.) were used to create a program and diagrams to simulate and verify the algorithms. Verification by simulation was carried out on a range of systems with varying dynamics. The control of the model below is given here as our example.

$$\boldsymbol{A}(z^{-1}) = \begin{bmatrix} 1 + 0{,}3z^{-1} + 0{,}1z^{-2} & 0{,}1z^{-1} + 0{,}2z^{-2} \\ 0{,}1z^{-1} + 0{,}3z^{-2} & 1 + 0{,}3z^{-1} + 0{,}1z^{-2} \end{bmatrix} \qquad (27)$$

$$\boldsymbol{B}(z^{-1}) = \begin{bmatrix} 0{,}1z^{-1} + 0{,}4z^{-2} & 0{,}9z^{-1} + 0{,}4z^{-2} \\ 0{,}6z^{-1} + 0{,}2z^{-2} & 0{,}3z^{-1} + 0{,}4z^{-2} \end{bmatrix}$$

Fig. 6 shows the system‘s step response

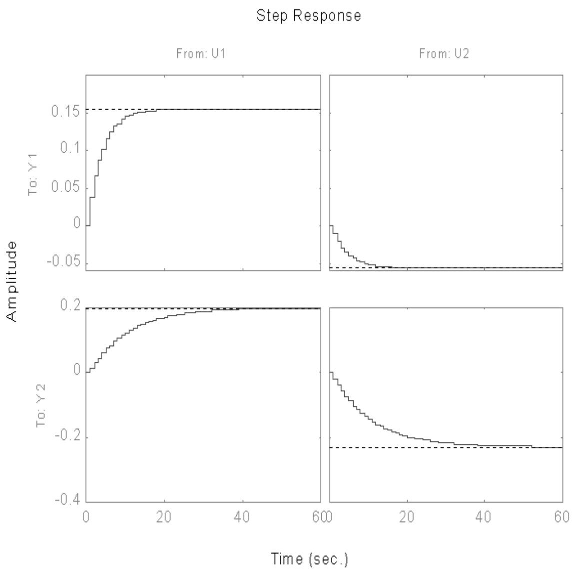

Fig. 6 The step response of the system

The right side control matrices are denoted as follows: without compensator - $\boldsymbol{M}_1$, with compensator $\mathbf{C_1}$ - $\boldsymbol{M}_2$, and with compensator $\mathbf{C_2}$ - $\boldsymbol{M}_3$.

$$\boldsymbol{M}_1(z^{-1}) = \begin{bmatrix} (1 - 0{,}1z^{-1})^4 & 0 \\ 0 & (1 - 0{,}1z^{-1})^4 \end{bmatrix} \qquad (28)$$

$$\boldsymbol{M}_2(z^{-1}) = \begin{bmatrix} 1 - 0{,}1z^{-1} & 0 \\ 0 & 1 - 0{,}1z^{-1} \end{bmatrix}$$

$$\boldsymbol{M}_3(z^{-1}) = \begin{bmatrix} (1 - 0{,}1z^{-1})^5 & 0 \\ 0 & (1 - 0{,}1z^{-1})^5 \end{bmatrix}$$

The same initial conditions for system identification were used for all the types of adaptive control we tested. The initial parameter estimates were chosen to be

$$\boldsymbol{\Theta}_1^T(0) = [0.1, 0.2, 0.3, 0.4, 0.1, 0.2, 0.3, 0.4] \qquad (29)$$

$$\boldsymbol{\Theta}_2^T(0) = [0.5, 0.6, 0.7, 0.8, 0.5, 0.6, 0.7, 0.8]$$

The results of simulation are shown in Figs 7 – 12.

We can draw several conclusions from the simulation results of our experiments on linear static systems. The basic requirement to ensure permanent zero control error was satisfied in all cases. The criteria on which we judge the quality of the control process are the overshoot on the controlled values and the speed with which zero control error is achieved. According to these criteria the controller incorporating compensator $\mathbf{C_1}$ performed the best. However, this controller appears to be unsuited to adaptive control due to the size of the overshoot and the large spread of process and controller outputs. The controller which uses compensator $\mathbf{C_2}$ seems to work best in adaptive control. With regards to decoupling, it is clear that controllers with compensators greatly reduce interaction.

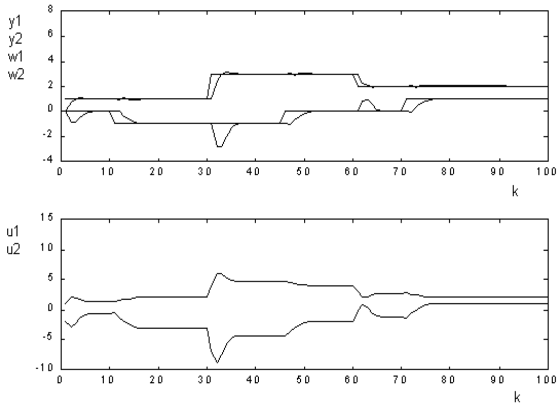

Fig. 7 Deterministic control without compensator

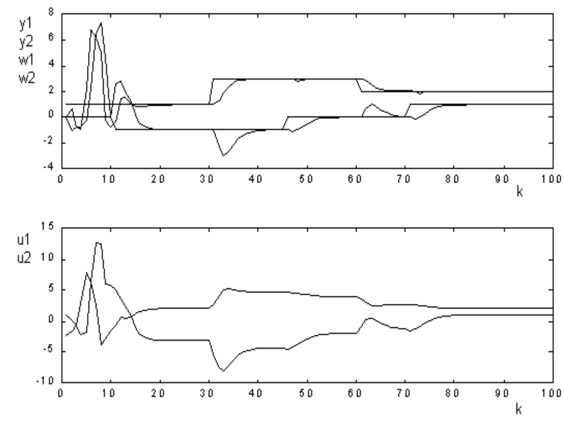

Fig. 8 Adaptive control without a compensator

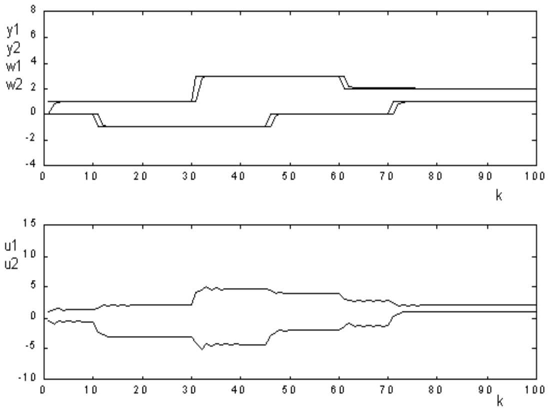

Fig. 9 Deterministic control with compensator $\mathbf{C_1}$

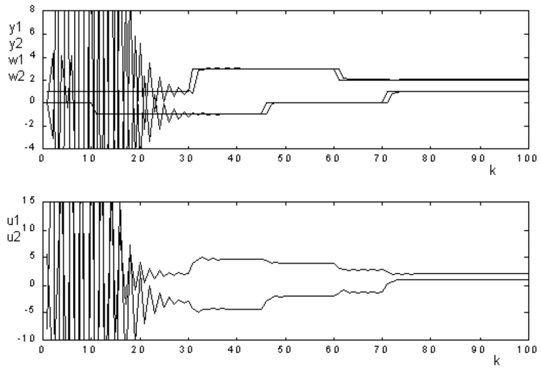

Fig. 10   Adaptive control with compensator $\mathbf{C_1}$

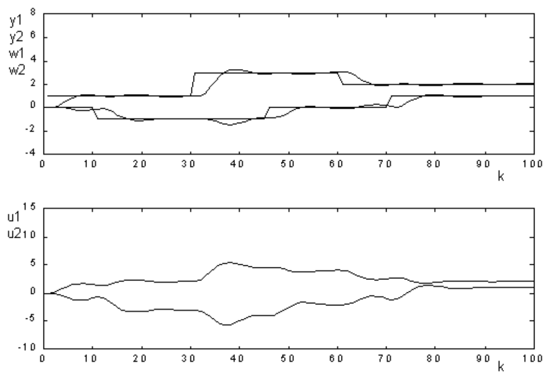

Fig. 11 Deterministic control with compensator $\mathbf{C_2}$

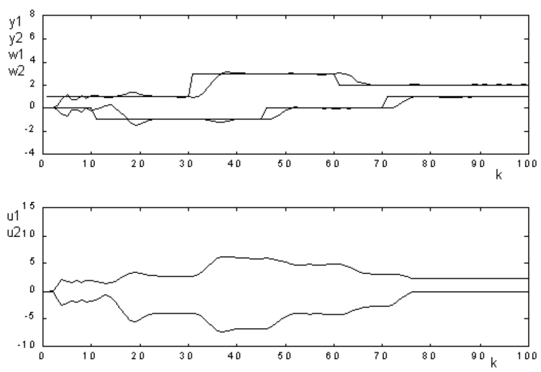

Fig. 12 Adaptive control with compensator $\mathbf{C_2}$

7 VERIFICATION – CONTROLLING A LABORATORY MODEL

Suggested controllers has been verified for the control of the laboratory through – flow air heater (Fig. 13). This laboratory equipment is two input – two output system. Manipulated variables are the heat source (electric resistance heating 2) and the air flow source (ventilator 1). Controlled variables are the air temperature, measured by resistance thermometer 4 and air flow, measured by flow speed indicator (position 7). The air flow can be changed with the throttle valve (position 5). The task was to apply the methods we designed for the adaptive control of a model representing a non-linear system with variable parameters which is, therefore, impossible to control deterministically.

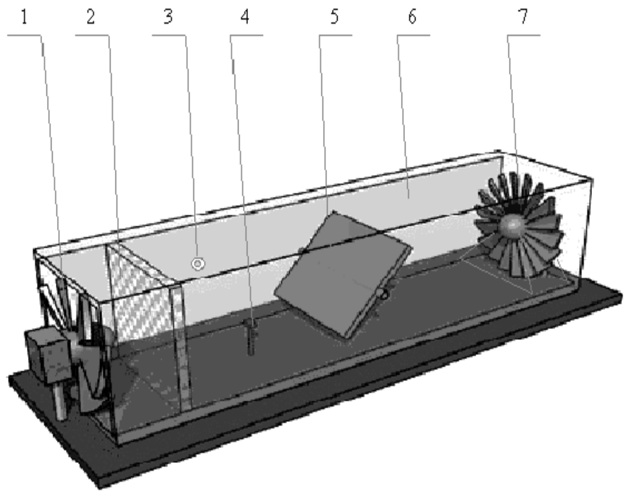

Fig. 13 Laboratory through – flow air heater
1 – ventilator, 2 – electric resistance heating , 3 – pressure sensor, 4 – resistance thermometer, 5 – throttle valve, 6 – cover of tunnel, 7 – flow indicator

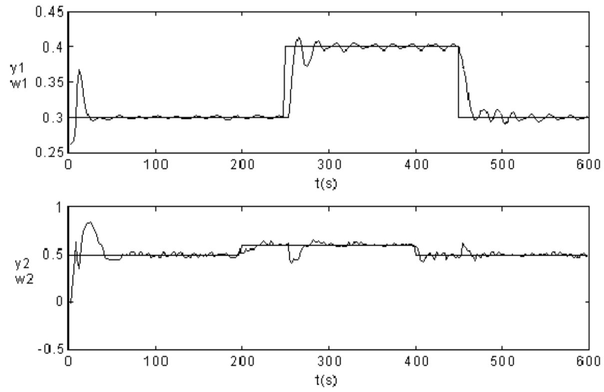

Fig. 14 Adaptive control of the real model using compensator $\mathbf{C_2}$

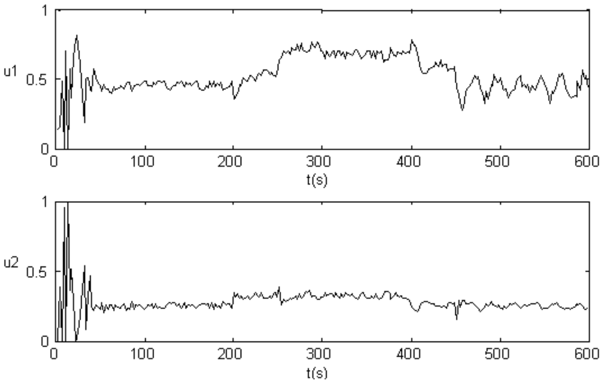

Fig. 15 The adaptive control of a real model using compensator $\mathbf{C_2}$– controller output

Adaptive control using recursive identification both with and without the use of compensators was performed. As indicated in the simulation, compensator $\mathbf{C_1}$ was shown to be unsuitable and control broke down. The other two methods gave satisfactory results. The time responses of the control for both cases are shown in Fig. 14, Fig. 15, Fig. 16 and Fig. 17. The figures demonstrate that control with a compensator reduces interaction. The controlled variable y_1 is the air temperature and the controlled variable y_2 is the air flow. The manipulated variable u_1 is the heat source and the manipulated variable u_2 is the air flow source.

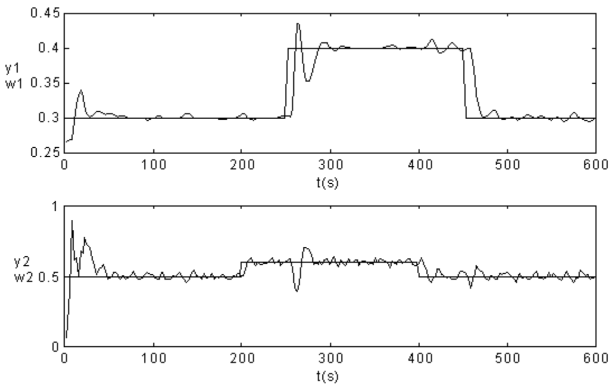

Fig. 16 Adaptive control of the real model without a compensator

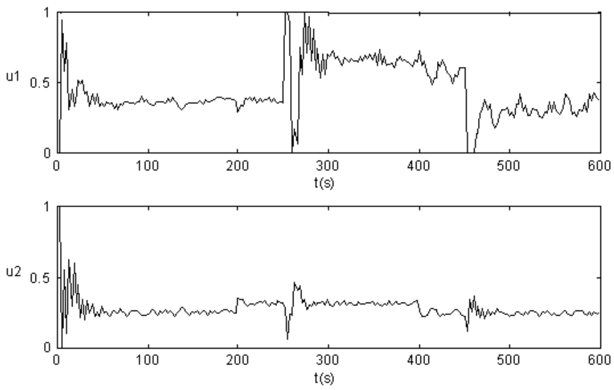

Fig. 17 Adaptive control of the real model without a compensator – controller output

8 CONCLUSIONS

The adaptive control of a two-variable system based on polynomial theory was designed. Decoupling problems were solved by the use of compensators. The designs were simulated and used to control a laboratory model. The simulation results proved that these methods are suitable for the control of linear systems. The control tests on the laboratory model gave satisfactory results despite the fact that the non-linear dynamics were described by a linear model.

ACKNOWLEDGMENTS

The work has been supported by the Grant Agency of the Czech Republic under grants No. 102/02/0204 and 102/01/P013 and by the Ministry of Education of the Czech Republic under grant MSM 281100001. This support is very gratefully acknowledged.

9 REFERENCES

Chien, I.L., Seborg, D.E., Mellichamp, D. A. (1987). Self-Tuning Control with Decoupling. *AIChE* J., 33, 7, 1079 – 1088.

Krishnawamy, P.R. et al. (1991). Reference System Decoupling for Multivariable Control. *Ind. Eng. Chem. Res*., 30, 662-670.

Kulhavý, R. (1987). Restricted exponencial forgetting in real – time identification. *Automatica*, 23, 589 – 600.

Luyben W. L. (1986). Simple Method for Tuning SISO Controllers in Multivariable Systems. *Ing. Eng. Chem. Process Des. Dev*., 25, 654 – 660.

Peng, Y. (1990). A General Decoupling Precompensator for Linear Multivariable Systems with Application to Adaptive Control. *IEEE Trans. Aut. Control*, AC-35, 3, 344-348.

Skogestad, S., Postlethwaite, J. (1996) *Multivariable Fedback Control - Analysis and Design.* New York, J. Willey.

Tade, M.O., Bayoumi, M.M., Bacon, D.W. (1986). Adaptive Decoupling of a Class of Multivariable Dynamic Systems Using Output Feedback. *IEE Proc. Pt.D*, 133, 6, 265-275.

Wittenmark, B., Middelton, R. H., Goodwin, G.C. (1987). Adaptive Decoupling of Multivariable Systems. *Int. J. Control*, 46, 6, 1993-2009.