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TWO-COUPLED-TANKS LEVEL REGULATION WITH CONSTRAINTS ON THE CONTROL AND THE OUTPUT*

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Abstract: This paper presents the application of the positive invariance concept to real-time control of a two-tanks system in the presence of hard constraints on the output and control signals. Results show that it is possible to regulate correctly the laboratory plant without violating the output and control constraints, when controlling the system with the output-feedback controller designed using the positive invariance concept. *Copyright © 2002 IFAC*

Keywords: Positive invariance; Constraints.

1. INTRODUCTION

Generally, real systems are subjected to limitations of physical or technological order. These limitations can usually be described as inequality constraints on the process variables. The positive invariance concept is largely used to take into account such constraints in the design of the control laws, [Benzaouia et al (1988), Benzaouia (1991, 1994), Blanchini (1999) and the references therein]. This enables the designer to avoid the saturation phenomenon, which could induce bad performance, or even unstability, when the controller is implemented in the real system.

However, from the authors knowledge, very few applications using these techniques to solve a real problem have been presented in the literature, even at laboratory scale [Pittet (1998)]. In this work, this concept is used to achieve the regulation of the water level in a two-coupled-tanks process. An output-feedback controller is designed to solve the regulation problem in the presence of constraints on the input and the output vectors.

The paper is organized as follows: Section II is reserved to the preliminaries while the process is described in section III. Simulation and implementation are done in section IV. Finally, some conclusions are presented.

1.1 *Notations:*

For two vectors x and y of $\mathbb{R}^n$ and a matrix $A \in \mathbb{R}^{nxn}$, the following notation will be used:

- $x < y$ (respectively $x \leq y$) if $x_i < y_i$ (respectively $x_i \leq y_i$), $i = 1, ..., n$.
- $\mathbb{I}_n$ is the identity matrix of $\mathbb{R}^n$
- Matrix $\widetilde{A} \in \mathbb{R}^{2nx2n}$ is defined by,

$$\widetilde{A} = \begin{bmatrix} A^+ & A^- \\ A^- & A^+ \end{bmatrix}$$

where

$$\left.\begin{array}{l} A^+(i,j) = \sup(A(i,j), 0) \\ A^-(i,j) = \sup(-A(i,j), 0) \end{array}\right\} i,j = 1, ..., n.$$

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2. PRELIMINARY RESULTS

This note is devoted to study the system described by,

$$x_{k+1} = Ax_k + Bu_k \quad (1)$$
$$y_k = x_k + z_k$$

where $x_k \in \mathbb{R}^n$ is the state of the system, $u_k \in \mathbb{R}^m$ and $y_k \in \mathbb{R}^p$ are the control and output signals, respectively. Vector z_k is the output of a noise bloc. Matrices A, B and C are constants with appropriate dimensions. The control vector is constrained to evolve in the set Ω defined by,

$$\Omega = \{u \in \mathbb{R}^m / -u_{\min} \leq u_k \leq u_{\max}\} \quad (2)$$

Vectors $u_{\min}$ and $u_{\max}$ are constant. Furthermore, the output is also constrained to evolve in the set D defined by,

$$D(\mathbb{I}_n, y_{\min}, y_{\max}) = \{y_k \in \mathbb{R}^p / -y_{\min} \leq y_k \leq y_{\max}\} \quad (3)$$

The noise affecting the plant is modeled by the following system,

$$s_{k+1} = Ps_k + Qe_k \quad (4)$$
$$z_k = s_k$$

where the scalar e_k is a white noise such that,

$$|e_k| < 1 \quad (5)$$

Definition *A set $D \subset \mathbb{R}^n$ is said to be positively invariant with respect to system*

$$x_{k+1} = f(x_k, u_k)$$

if for any $x_o \in D$, the trajectory $x(x_o, k, u_k)$ of the system does not leave the set D for any $k \geq 0$.

Now, consider the first equation of system (1), i.,e.,

$$x_{k+1} = Ax_k + Bu_k \quad (6)$$

The following result recalls the necessary and sufficient condition allowing the positive invariance property of the set,

$$D(\mathbb{I}_n, x_{\max}, x_{\min}) = \{x \in \mathbb{R}^n / -x_{\min} \leq x \leq x_{\max}\}$$

Theorem 1 *[Benzaouia et al., 1988b] Domain $D(\mathbb{I}_n, x_{\max}, x_{\min})$ is positively invariant with respect to system (6) if and only if,*

$$(\mathbb{I}_{2n} - \widetilde{A})X \geq \widetilde{B}U$$

Where

$$X = \left[\, x_{\max}^T \;\; x_{\min}^T \,\right]^T$$

$$U = \left[\, u_{\max}^T \;\; u_{\min}^T \,\right]^T$$

3. DESCRIPTION OF THE PLANT

The plant is in the Department of Engineering and Automatic Systems of the University of Valladolid, Spain. As depicted in Figure 1, the process is composed of two tanks: the water is pumped independently to both tanks. The control signals are the pumps flowrates. The liquid leaves the tanks by gravity from an outlet near the bottom of the tanks. There is an additional outlet that connect the tanks via a short pipe. The liquid level in the tank is measured by a capacitive sensor

The plant can be modeled by the following discrete-time state-space model:

$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = x_k + z_k$$

where

$$A = \begin{bmatrix} 0.4611 & -0.0002 \\ 0.0000 & 0.4584 \end{bmatrix}, B = \begin{bmatrix} 0.6708 & 0.0518 \\ 0.0102 & 0.7780 \end{bmatrix}$$

The noise bloc is modeled by system (4) with,

$$P = \begin{bmatrix} 0.4611 & 0 \\ 0 & 0.4584 \end{bmatrix}, Q = \begin{bmatrix} 16.7075 \\ 22.2192 \end{bmatrix}$$

Using theorem 1, given e_k such that

$$-1 \leq e_k \leq 1$$

It is possible to find $z_{\max}$ and $z_{\min}$ such that,

$$(\mathbb{I}_{2n} - \widetilde{P})Z \geq \widetilde{Q}E$$

where

$$Z = \left[\, z_{\max}^T \;\; z_{\min}^T \,\right]^T, E = \left[\, 1 \;\; 1 \,\right]^T$$

indeed, $(\mathbb{I}_{2n} - \widetilde{P})$ is a positive diagonal matrix, that is $(\mathbb{I}_{2n} - \widetilde{P})^{-1}$ is a positive matrix, consequently, vector Z can be chosen as,

$$Z = (\mathbb{I}_{2n} - \widetilde{P})^{-1}\widetilde{Q}E$$
$$= \left[\, 31.0045 \; 41.0256 \; 31.0045 \; 41.0256 \,\right]^T$$

In the sequel, the system will be closed using an output feedback control law,

$$u_k = Fy_k = Fx_k + Fz_k$$

using (1) and (4), one can write,

$$u_{k+1} = F(A + BF)x_k + F(P + BF)z_k + FQe_k$$

If there exist a matrix H_o such that,

$$FA + FBF = H_oF$$

then,

$$u_{k+1} = H_ou_k + (FP + FBF - H_oF)z_k + FQe_k$$

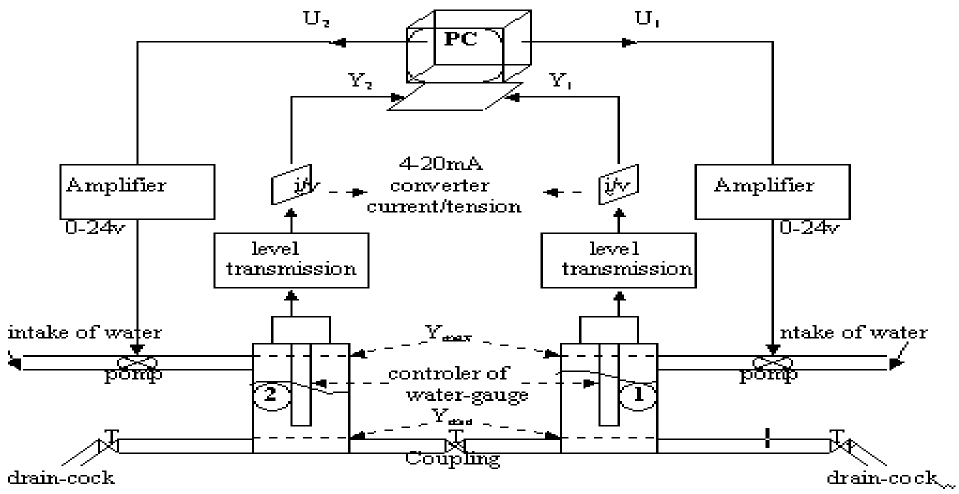

Fig. 1. The two-coupled-tanks process.

which can be written as,

$$u_{k+1} = H_o u_k + H_1 z_k + H_2 e_k \tag{7}$$

with

$$H_1 = FP + FBF - H_o F \text{ and } H_2 = FQ$$

using Theorem 1, Ω is positively invariant with respect to system (7) if,

$$(\mathbb{I}_{2n} - \widetilde{H_o})U \geq \widetilde{H_1}Z + \widetilde{H_2}E \tag{8}$$

The constraints on the output must also be considered. For this recall that the output signal can be expressed as:

$$\begin{aligned} y_{k+1} &= x_{k+1} + z_{k+1} \\ &= (A + BF)y_k + (P - A)z_k + Qe_k \end{aligned}$$

using Theorem 1, constraints on the output are respected if,

$$(\mathbb{I}_{2n} - (\widetilde{A + BF}))Y \geq (\widetilde{P - A})Z + \widetilde{Q}E \tag{9}$$

with

$$Y = \left[\, y_{\max}^T \; y_{\min}^T \,\right]^T$$

To design the control law, the following steps must be fulfilled:

(1) Given a stable matrix H_o such that,

$$\widetilde{H_o}U < U$$

(2) Solve equation

$$XA + XBX = H_o X \tag{10}$$

to find the feedback gain F. Compute matrices H_1 and H_2.

(3) If (8) and (9) are satisfied then stop, else return to step1 and change H_o.

4. SIMULATION AND IMPLEMENTATION

The nominal working point is selected to be 45% of each maximal input value; the corresponding outputs are 60% and 65% of the maximal outputs, respectively. To accomplish this work, the system is centered around the nominal working point. That is,

$$u_{\max} = \begin{bmatrix} 55 \\ 55 \end{bmatrix}, u_{\min} = \begin{bmatrix} 45 \\ 45 \end{bmatrix}$$

$$y_{\max} = \begin{bmatrix} 40 \\ 35 \end{bmatrix}, y_{\min} = \begin{bmatrix} 60 \\ 65 \end{bmatrix}$$

We choose

$$H_o = \begin{bmatrix} 0.3227 & 0.0273 \\ 0.0227 & 0.3273 \end{bmatrix}$$

the resolution of (10) gives,

$$F = \begin{bmatrix} -0.2072 & 0.0481 \\ 0.0372 & -0.1709 \end{bmatrix}$$

we obtain,

$$H_1 = 10^{-4} * \begin{bmatrix} -0.0142 & -0.3747 \\ 0.0697 & 0.0628 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} -2.3922 \\ -3.1763 \end{bmatrix}$$

Inequalities (8) and (9) are satisfied. Figure 2 presents the components of output and input vectors calculated using a simulation of the plant.

The initial output vector is:

$$y_o = \left[\, 7.5 \; -9.6 \,\right]^T$$

Using the same feedback gain F in the real process, and starting from the same initial condition, the trajectories of the output and the input vectors, presented in Figure 3, were experimentally obtained.

Globally, the obtained results are satisfactory despite the problem which appears in the time axis. In our

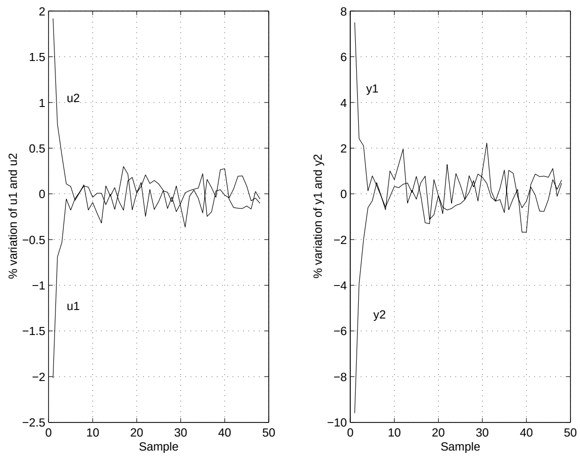

Fig. 2. Trajectories of the output and the control components.

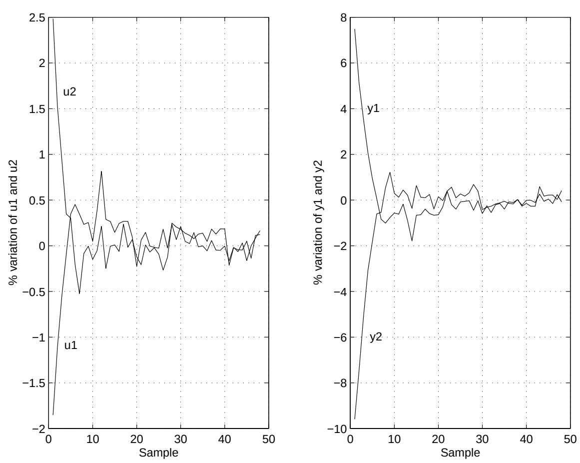

Fig. 3. Trajectories of output and control components.

opinion, this problems is generated by an incorrect modelisation of the process.

5. CONCLUSION

The positive invariance concept has been applied to regulate liquid levels in a laboratory plant that consists of two coupled tanks. An output-feedback controller is used to realize our goal. The designed controller have been tested using both a simulation of the model and an implementation of the control law on the real process. It has been shown that the control objectives has been accomplished: constraints on the output and the input vectors are respected without any saturation. The tracking problem with constraints on the input and output of the process can be an interesting question to deal with in a future work.

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