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SYNCHRONIZATION OF DECENTRALIZED MULTIPLE-MODEL AND FUZZY SYSTEMS BY MARKET-BASED OPTIMIZATION

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Abstract: Market-based optimization is a new optimization method for large decentralized systems where the distributed resource allocation of an economic system is adopted. Market-based algorithms can be interpreted as multi-agent scenarios where producer and consumer agents both compete and cooperate on a market of specified commodities. The market-based approach is applied to the synchronization of a set of local multiple-model systems. The method is extended to the case where each of the subsystems is represented by a Takagi-Sugeno (TS) fuzzy system. Although all local systems are provided with the same control input, the behaviors of the local systems are, in general, different because of different parameters in the subsystems. The task of the market-based optimization is to find an appropriate composition of subsystems so that all local systems exhibit a similar dynamical behavior. Examples show that even systems with unstable subsystems can be synchronized if there exists a stable combination of weighted subsystems. *Copyright © 2002 IFAC*

Keywords: Market-based optimization, agents, decentralized systems, synchronization, multiple-model systems, fuzzy control.

1. INTRODUCTION

Centralized optimization and control of systems becomes difficult if they are composed of a large number of complex local systems. Therefore, decentralized methods like multi-agent control are able to handle optimization tasks better. An important application for agent based control is the optimization of manufacturing processes and the synchronization of production lines, respectively. Dimensioning and planning of manufacturing systems by multi agent methods is described in (Fleury, et al.1996). In (Zaremba et al.,1999) a distributed resource allocation method for a repetitive manufacturing process based on the concept of critical resources (i.e. system bottlenecks) is described. (Wallace, 1998) deals with the flow control of mobile platforms in a manufacturing plant using intelligent agents. Another important field of application for multi agent optimization and control is congestion control in traffic networks (Lei and Özgüner, 1999, Altmann et al., 1999). Other decentral control strategies are the so-called “utility approach“ (Gold, 2000) and the “behavioral approach“ (Large, et al., 1999) used for mobile robot navigation. In (Barret and Lafortune, 1999) a behavioral control strategy dealing with communicating decentralized controllers is presented. One of the most interesting and, in our opinion, promising approaches to large decentralized systems is the market-based optimization. Market-based algorithms imitate the behavior of economic systems in which producer and consumer agents both compete and cooperate on a market of commodities. This simultaneous cooperation and competition of agents is also called “coopetition“ (Teredesai and Ramesh, 1998). General ideas and some results of market-based control strategies are presented in (Clearwater, 1996, Guenther, 1997). In (Voos and Litz, 1999) a more detailed description of the optimization algorithm is presented. The authors show how to optimize distributed systems by so-called producer and consumer agents using local cost functions. Given desired setpoints and, with this, a cost function for each local system a set of controls is to find that leads the whole set of local systems to a so-called Pareto-optimum (http, 2001). The present paper adopts many ideas from (Guenther, 1997) and (Voos and Litz, 1999), but it deals with a different goal of optimization namely with the synchronization of the behavior of local systems. The market-based approach is applied to a set of decoupled local systems each of them being composed of a weighted sum of linear multiple-model subsystems, or Takagi-Sugeno (TS) fuzzy subsystems, respectively. In general, TS fuzzy systems are nonlinear subsystems that are composed of local dynamical systems each of which being weighted by a certain membership degree depending on the state and control input of the system (Palm, 1997). While each local system is provided with the same control input, the behaviors of the individual local systems are, in general, different because of different parameters in the subsystems. It is assumed to be feasible to change the behaviors of the local systems by tuning the weights for the regarding subsystems. The task of the market-based optimization is, by tuning those weights, to find a composition of subsystems so that the behavior of all local systems is synchronized. The paper is organized as follows. Section 2 gives an introduction into the market-based optimization method used in this paper. Section 3 deals with a restricted communication between agents. In Section 4 the market-based algorithm is applied to the synchronization of a set of local systems each of them being composed of linear subsystems, and nonlinear TS fuzzy subsystems, respectively. The subsystems are composed in such a way that the dynamical behavior of the local systems are synchronized. Section 5 gives corresponding simulation results. Section 6 concludes with a summarizing discussion about the methods presented and an outlook on future work.

2. MARKET-BASED OPTIMIZATION

The imitation of market control mechanisms of an economic system and the application to technical or communication systems requires the modeling of both the system to be optimized and the optimization strategy itself. In the following, system and optimization strategy are presented as continuous models although the computational realization is usually discrete. Let a system S be composed by a set of M local systems S_i described by differential equations

$$\dot{x}_i = \sum_{j=1}^{N} w_i^j f_i^j(x_i) + Bu \qquad (1)$$

where $x_i \in \Re^n$ is the i-th state vector, $i=1,...,M$, $w_k^1 = 1; w_k^2 = 0$ is a common control vector, $B \in \Re^{n\times m}$, w_i^j is a vector of weights where

$$w_i = \{w_i^1,...,w_i^j,...,w_i^N\}^T, w_i^j \geq 0, \sum_{j=1}^{\mathbf{N}} w_i^j = 1$$

The restriction to a constant matrix B is done for the sake of simplification. Note, however, that the method presented here can also be extended to local B_i matrices. The task is to change the weights w_i^j so that all local systems exhibit a similar dynamical behavior on the condition of minimum local energies. This in turn leads, provided a common control vector u, to the synchronization of the behavior of the local systems. One possible option for tuning the w_i^j is to find a global optimum over all local systems and their subsystems. This, however, is a difficult task especially in the presence of many local systems.

Therefore a multi-agent approach has been preferred. The determination of the weights w_i^j is done by producer-consumer agent pairs in a market-based scenario that is presented in the following.

Assume that to every local system S_i belongs a set of N producer agents Pag_i^j and N consumer agents Cag_i^j. Producer and consumer agents sell and buy, respectively, the weights w_i^j on the basis of common prices p^j. Producer agents Pag_i^j supply weights $w_{i\ p}^j$ and try to maximize specific local profit functions ρ_i^j where "local" means "belonging to system S_i". On the other hand, consumer agents Cag_i^j buy weights $w_{i\ c}^j$ from the producer agents and try to maximize specific local utility functions U_i^j. The whole "economy" is in equilibrium as the sum over all supplied weights $w_{i\ p}^j$ is equal to the sum over all utilized weights $w_{i\ c}^j$.

$$\sum_{i=1}^{M} w_{i\ p}^j(p^j) = \sum_{i=1}^{M} w_{i\ c}^j(p^j) \qquad (2)$$

As we will see later, the dependence of the weights on prices p^j in (2) makes a computation of the prices p^j, and the final weights w_i^j possible.

The trade between the producer and consumer agents is based on the definition of cost functions for each type of agent. Therefore we define a local utility function for the consumer agent Cag_i^j

Utility = benefit - expenditure

$$U_i^j = \tilde{b}_i^j w_{i\ c}^j - \tilde{c}_i^j p^j w_{i\ c}^{j\ 2} \qquad (3)$$

where $\tilde{b}_i^j, \tilde{c}_i^j > 0$ and a local profit function for the producer agent Pag_i^j

profit = income - costs

$$\rho_i^j = g_i^j p^j w_{i\ p}^j - e_i^j (w_{i\ p}^j)^2 \qquad (4)$$

where $g_i^j, e_i^j > 0$ are free parameters that determine the average price level. Observe here that both cost functions (3) and (4) use the price p^j on the basis of which the weights w_i^j are calculated.

Assume further that, according to (1), we can formulate a local energy function to be minimized

$$\tilde{J}_i^j = \dot{x}_i^T \dot{x}_i = a_i^j + b_i^j w_i^j + c_i^j (w_i^j)^2 \to \min \qquad (5)$$

where $\tilde{J}_i^j \geq 0,\ a_i^j, c_i^j > 0$.

How to combine the local energy function (5) and the utility function (3) , and how are the parameters $\tilde{b}_i^{\,j}, \tilde{c}_i^{\,j}$ in (3) to be chosen? An intuitive choice

$$\tilde{b}_i^{\,j} = | b_i^j |, \qquad \tilde{c}_i^{\,j} = c_i^j \qquad (6)$$

guarantees $w_i^j \geq 0$. It can also be shown that near the equilibrium $\dot{x}_i = 0$, and for $p^j = 1$, the energy function (5) reaches its minimum, and the utility function (3) its maximum, respectively. With (6) the utility function (3) becomes

$$U_i^j = | b_i^j | \, w_{i\,c}^j - c_i^j p^j w_{i\,c}^{j\;2} \qquad (7)$$

Maximization of (7) yields

$$\frac{\partial U_i^j}{\partial w_{i\,c}^j} = | b_i^j | - 2 c_i^j p^j w_{i\,c}^j = 0 \qquad (8)$$

from which a local $w_{i\,c}^j$ is obtained

$$w_{i\,c}^j = \frac{| b_i^j |}{2 c_i^j} \cdot \frac{1}{p^j} \qquad (9)$$

Maximization of the local profit function (4) yields

$$\frac{\partial \rho_j^k}{\partial w_{i\,p}^j} = g_i^j p^j - 2 e_i^i w_{i\,p}^j = 0 \qquad (10)$$

from which a local $w_{i\,p}^j$ is obtained

$$w_{i\,p}^j = \frac{p^j}{2\eta_i^j}, \; where \;\; \eta_i^j = \frac{e_i^j}{g_i^j} \qquad (11)$$

The requirement for an equilibrium between the sums of the "produced" $w_{i\,p}^j$ and the "demanded" $w_{i\,c}^j$ leads to the balance equation

$$\sum_{i=1}^{M} w_{i\,c}^j = \sum_{i=1}^{M} w_{i\,p}^j \qquad (12)$$

Substituting (9) and (11) into (12) gives the prices p^j for $w_{i\,p}^j$'s

$$p^j = \sqrt{\left\{ \sum_{i=1}^{M} | b_i^j | / c_i^j \right\} \Big/ \left\{ \sum_{i=1}^{M} 1/\eta_i^j \right\}} \qquad (13)$$

Substituting (13) into (9) yields the final weights w_i^j to be implemented in each local system. Once the new weights w_i^j are calculated, each of them has to be normalized with respect to $\sum_{j=1}^{N} w_i^j$ which gurantees the above requirement $\sum_{j=1}^{\mathrm{N}} w_i^j = 1$.

3. MARKET-BASED ALGORITHMS WITH A RESTRICTED NUMBER OF CONNECTIONS BETWEEN AGENTS

In the previous sections the basis for the interaction of the individual agents was a common (global) price built by all agents at the same time. Every system was connected with the other (see Fig. 1).

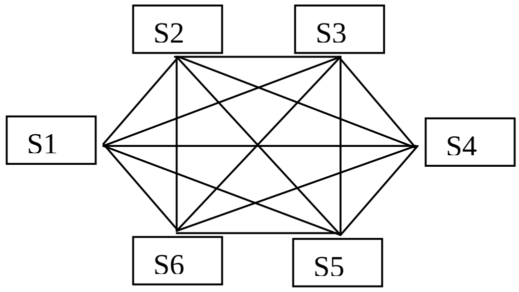

Fig. 1. Connections between all systems (agents) via a common (global) price

In real systems this may be not feasible. Therefore a restricted communication between agents can only be realized. Fig. 2 shows a graph for a restricted chain structure.

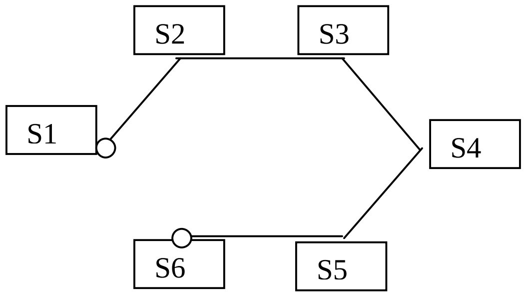

Fig. 2. Connections between neighboring systems (agents) via local prices

Then equation (13) changes into

$$p_i^j = \sqrt{\left\{ \sum_{l=i-1}^{i+1} | b_l^j | / c_l^j \right\} \Big/ \left\{ \sum_{l=i-1}^{i+1} 1/\eta_l^j \right\}} \qquad (13a)$$

Simulations showed that the results with the local price approach differ only marginally from those with a common (global) price.

4. MARKET-BASED OPTIMIZATION OF LINEAR SYSTEMS AND TS FUZZY SYSTEMS

4.1. Linear subsystems

In the following the local system (1) is specified as a convex combination of *N* linear local systems

$$\dot{x}_i = \sum_{j=1}^{N} w_i^j A_i^j x_i + Bu \qquad (14)$$

where $i=1,...,M$,

$$A_i^j \in \Re^{n\times n},\ B\in \Re^{n\times m},\ w_i^j \ge 0,\ \sum_{j=1}^{\mathbf{N}} w_i^j = 1$$

According to (3) we define the energy function

$$\tilde{J}_i^j = \dot{x}_i^T \dot{x}_i$$

$$= \left(\sum_{k=1}^{N} w_i^k A_i^k x_i + Bu\right)^T \left(\sum_{l=1}^{N} w_i^l A_i^l x_i + Bu\right)$$

$$= \sum_{k=1}^{N}\sum_{l=1}^{N} w_i^k w_i^l (x_i^T A_i^{k^T} A_i^l x_i$$

$$+ 2x_i^T A_i^{k^T} Bu) + u^T B^T Bu$$

$$=(w_i^j)^2 x_i^T A_i^{j^T} A_i^j x_i$$

$$+2w_i^j \left(\sum_{\substack{k=1\\k\ne j}}^{N} w_i^k x_i^T A_i^{j^T} A_i^k x_i + x_i^T A_i^{j^T} Bu\right)$$

$$+\sum_{\substack{k=1\\k\ne j}}^{N}\sum_{\substack{l=1\\k\ne j}}^{N} w_i^k w_i^l (x_i^T A_i^{k^T} A_i^l x_i \qquad (15)$$

$$+ 2x_i^T A_i^{k^T} Bu) + u^T B^T Bu$$

$$= a_i^j + b_i^j w_i^j + c_i^j (w_i^j)^2$$

where

$$a_i^j = \sum_{\substack{k=1\\k\ne j}}^{N}\sum_{\substack{l=1\\k\ne j}}^{N} w_i^k w_i^l (x_i^T A_i^{k^T} A_i^l x_i + 2x_i^T A_i^{k^T} Bu)$$

$$+u^T B^T Bu$$

$$b_i^j = 2\left(\sum_{\substack{k=1\\k\ne j}}^{N} w_i^k x_i^T A_i^{j^T} A_i^k x_i + x_i^T A_i^{j^T} Bu\right) \qquad (16)$$

$$c_i^j = x_i^T A_i^{j^T} A_i^j x_i$$

With (16), (13), and (9) new weights w_i^j are calculated at every time step, and each local system (14) is updated. Since every local system S_i tries to optimize its own performance the dynamical behaviors of the local systems S_i become more and more adjusted.

4.2. Nonlinear TS fuzzy subsystems

In the previous section the multiple-model subsystems to be mixed within every local system S_i were linear. In the following, the subsystems are nonlinear TS fuzzy functions the system matrices of which depending on the local state x_i.

Given a set of M parallel local systems each of them being composed of N TS fuzzy subsystems

$$\dot{x}_i = \sum_{j=1}^{N} w_i^j A_i^j (x_i) x_i + Bu \qquad (17)$$

where $i=1,...,M$,

$$A_i^j = \sum_{r=1}^{R} \mu_{i\,r}(x_i) A_{i\,r}^j;\ A_i^j \in \Re^{n\times n},\quad B\in \Re^{n\times m},$$

$$w_i^j \ge 0,\ \sum_{j=1}^{\mathbf{N}} w_i^j = 1;\ A_{i\,r}^j \in \Re^{n\times n},\ \mu_{i\,r} \ge 0,\ \sum_{r=1}^{\mathbf{R}} \mu_{i\,r} = 1$$

The further development is the same as before. The only difference are the additional fuzzy parameters $\mu_{i\,r}(x_i)$ depending on x_i.

$$a_i^j = \sum_{\substack{k=1\\k\ne j}}^{N}\sum_{\substack{l=1\\k\ne j}}^{N}\sum_{r=1}^{R}\sum_{s=1}^{R} \mu_{i\,r}\mu_{i\,s}[w_i^k w_i^l (x_i^T A_{i\,r}^{k\ T} A_{i\,s}^l x_i$$

$$+2x_i^T A_{i\,r}^{k\ T} Bx_i) + u^T B^T Bu]$$

$$b_i^j = 2\sum_{\substack{k=1\\k\ne j}}^{N}\sum_{r=1}^{R}\sum_{s=1}^{R} \mu_{i\,r}\mu_{i\,s}[w_i^k x_i^T A_{i\,r}^{j\ T} A_{i\,s}^k x_i + x_i^T A_{i\,r}^{j\ T} Bu]$$

$$c_i^j = \sum_{r=1}^{R}\sum_{s=1}^{R} \mu_{i\,r}\mu_{i\,s} x_i^T A_{i\,r}^{j\ T} A_{i\,s}^j x_i$$

(18)

5. SIMULATION RESULTS

Example 1:

The first example deals with 3 local systems with two linear subsystems each of second order.

$$\ddot{x}_1 = -(w_1^1 + 5w_1^2)\dot{x}_1 - (8w_1^1 + 2w_1^2\cdot)x_1 + u;$$

$$\ddot{x}_2 = -(2.5w_2^1 + 4.5w_1^2)\dot{x}_2 - (1.5w_2^1 + 7.5w_1^2)x_2 + u;$$

$$\ddot{x}_3 = -(1.5w_3^1 + 5.5w_1^2)\dot{x}_3 - (8.5w_3^1 + 1.5w_1^2)x_3 + u; \qquad (19)$$

The market based parameters are $e_i^j = 1; g_i^j = 100$.

The initial weights are $w_k^1 = 1; w_k^2 = 0$ (k=1,2,3).

The u values are with N=1000 (k – simulation step, N – maximum number of steps)

```
if k<N/4                  u=5;
if k>=N/4 & k<N/2         u=0;
if k>=N/2 & k<3*N/4       u=5;
if k>=3*N/4               u=0;
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(see Fig 3a,b)

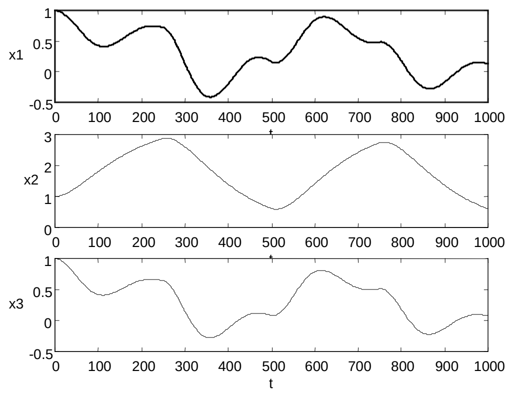

Fig 3a Stable linear subsystems, no optimization

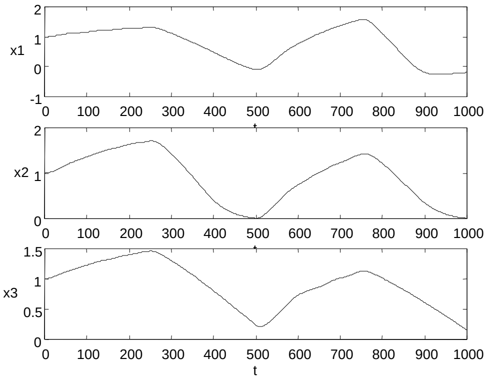

Fig 3b Stable linear subsystems, with optimization

The simulation shows clearly a synchronization of all local systems after the optimization.
The range of system parameters of each system (system 1, 2, or 3) represented by the system parameters of each subsystem (system 11 and 12, system 21 and 22, or system 31 and 32) have to have a range in common so that a common dynamic behavior may become feasible.

Example 2:
The following example deals with 3 local systems each of them composed of two subsystems of second order. Moreover, each subsystem is a nonlinear TS fuzzy system composed of two linear sub-subsystems

$$\begin{aligned}
\ddot{x}_1 &= (w_1^1 \cdot a_1^{11} + w_1^2 \cdot a_2^{11})\dot{x}_1 \\
&+ (w_1^1 \cdot a_1^{01} + w_1^2 \cdot a_2^{01})x_1 + u; \\
\ddot{x}_2 &= (w_2^1 \cdot a_1^{12} + w_1^2 \cdot a_2^{12})\dot{x}_2 \\
&+ (w_2^1 \cdot a_1^{01} + w_1^2 \cdot a_2^{02})x_2 + u; \\
\ddot{x}_3 &= (w_3^1 \cdot a_1^{13} + w_1^2 \cdot a_2^{13})\dot{x}_3 \\
&+ (w_3^1 \cdot a_1^{01} + w_1^2 \cdot a_2^{03})x_3 + u;
\end{aligned} \qquad (20)$$

$$a_i^{jk} = \mu_{k1}(x_k) a_{i\;1}^{jk} + \mu_{k2}(x_k) a_{i\;2}^{jk} \qquad (21)$$

i=1,2; j=0,1; k=1,...,3.

Figure 4 shows the the corresponding membership functions. This example deals with a combination of systems in which the 3rd system exhibits a slight oscillation tendency. Figs. 5a,b show the behavior of the systems without and with optimization, respectively, from the market-based algorithm. The initial weights are $w_k^i = 0.5$. The corresponding parameters are

$a_{1\;1}^{11} = -0.5; a_{2\;1}^{11} = -10; a_{1\;1}^{01} = -8; a_{2\;1}^{01} = -2$

$a_{1\;2}^{11} = -2.3; a_{2\;2}^{11} = -15; a_{1\;2}^{01} = -9; a_{2\;2}^{01} = -3$

$a_{1\;1}^{12} = -1.5; a_{2\;1}^{12} = -8.5; a_{1\;1}^{02} = -8; a_{2\;1}^{02} = -2$

$a_{1\;2}^{12} = -1.5; a_{2\;2}^{12} = -6.5; a_{1\;2}^{02} = -9; a_{2\;2}^{02} = -3$

$a_{1\;1}^{13} = -2.5; a_{2\;1}^{13} = -10.5; a_{1\;1}^{03} = -8; a_{2\;1}^{03} = -2$

$a_{1\;2}^{13} = -0.05; a_{2\;2}^{13} = -0.6; a_{1\;2}^{03} = -90; a_{2\;2}^{03} = -30$

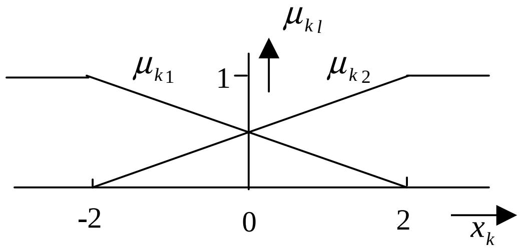

Fig. 4 Membership functions μ_{kl}, l = 0,1

In both cases a negative step u= -3 was introduced. It can be noted that the market-based optimization leads to an "alignment" of the three systems.

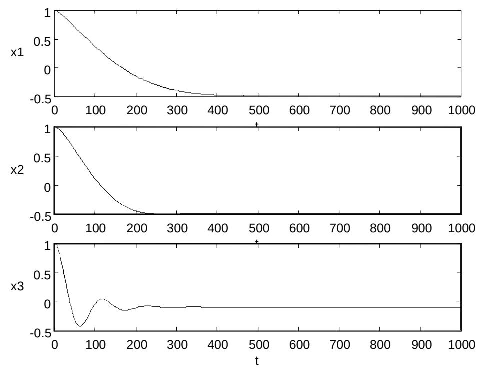

Fig. 5a Stable fuzzy subsystems, no optimization

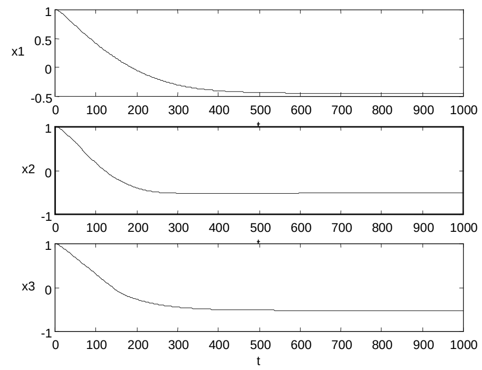

Fig. 5b Stable fuzzy subsystems, with optimization

Example 3:
This example deals with 3 systems composed of two TS fuzzy subsystems. The initial weights are $w_k^i = 0.5$. The system parameters have been chosen so that system 1 is unstable

$a_{1\;1}^{11} = -0.5; a_{2\;1}^{11} = -10; a_{1\;1}^{01} = -8; a_{2\;1}^{01} = -2$

$a_{1\;2}^{11} = -2.3;$ $\mathbf{a_{2\;2}^{11} = +15;}$ $a_{1\;2}^{01} = -9; a_{2\;2}^{01} = -3$

$a_{1\;1}^{12} = -10.5; a_{2\;1}^{12} = -4.5; a_{1\;1}^{02} = -8; a_{2\;1}^{02} = -2$

$a_{1\;2}^{12} = -7.5; a_{2\;2}^{12} = -6.5; a_{1\;2}^{02} = -9; a_{2\;2}^{02} = -3$

$a_{1\;1}^{13} = -6.5; a_{2\;1}^{13} = -1.5; a_{1\;1}^{03} = -8; a_{2\;1}^{03} = -2$

$a_{1\;2}^{13} = -1.5; a_{2\;2}^{13} = -0.5; a_{1\;2}^{03} = -9; a_{2\;2}^{03} = -3$

Figs 6a,b show the results for a negative step function u= -3 . The optimization, however, leads to a stable behavior of all three systems although slight oscillations could not be avoided.

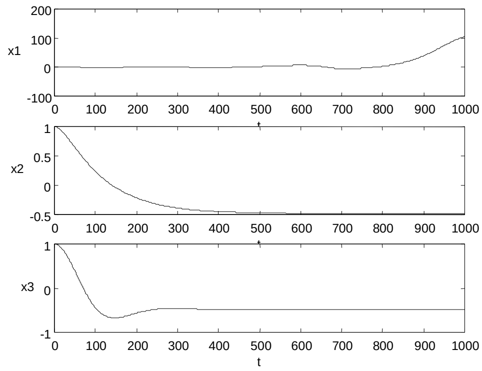

Fig. 6a Unstable fuzzy subsystems included, no optimization

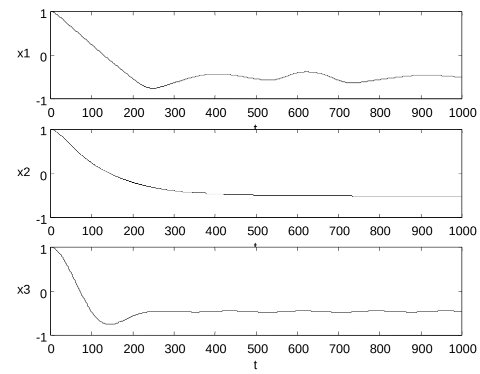

Fig. 6b Unstable fuzzy subsystems included, with optimization

6. CONCLUSIONS

Market-based optimization has been shown to be a powerful optimization strategy for decentralized systems in which producer and consumer agents both compete and cooperate on a market of commodities using a global price. A decentralized system is composed of a set of local systems each of which consisting of a convex combination of linear systems, or Takagi-Sugeno (TS) fuzzy subsystems, respectively. For given local energy functions a combination of local subsystems is to find so that a synchronization of the dynamical behavior of the local systems is provided. The simulation results show that a Pareto-optimal solution for every local system can be reached. One condition for the existence of such an optimum is that in every local system a stable combination of subsystems exists. The approach has been applied both for an unrestricted communication between agents (global price) and a restricted communi-cation (local prices). Simulation results show that this approach is successful for combinations of linear, and nonlinear TS fuzzy subsystems, respectively. Future work will be directed to the decentralized control design of TS fuzzy systems in the framework of market-based optimization and its enhancement with learning strategies.

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