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FAULT DETECTION MODELS AND METHODS FOR A MULTI-TANK HYDRAULIC CONTROL PROBLEM

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Abstract: two observer-based fault detection designs are considered for detecting a fault in any inaccessible tank which forms part of a larger n-tank system. A subsystem of $n-2$ tanks is situated in a hazardous environment and only control and measurements of the outer tanks is possible. Given a set of nonlinear differential equations, several transformations are derived to aid the design of the two problems of control and fault detection. Two nonlinear observer approaches are given and illustrated by an example. Stability and detectability are examined.

Keywords: Fault detection, control, nonlinear, hydraulic.

1.INTRODUCTION

Fault detection and isolation for hydraulic systems has been investigated by using several methods including : observers (Koinig et al., 1997), parity equations with neuro-fuzzy identification (Garcia et al., 1997), residuals from physical nonlinear equations (Koscielny et al., 1994), estimation of physical parameters with use of fuzzy neural networks (Han and Frank, 1997), fuzzy model based on B-Spline networks (Benkheda and Patton, 1997) and transformations to connoical form observers (Isidori, 1995). However, new methods, both algebraic and geometric in nature, have yet to be assessed properly on real systems. Certain types of system are such that the differential equations expressing the system contain nonlinearities in the form of a repeating nonlinear term. For hydraulic systems this usually is a flow term which is not all that smooth. This paper takes a closer look at such a real system consisting of n tanks where the $n-2$ inner tanks are inaccessible. First several models are given which can be used for solving a control problem with limited (sensor) outputs and for fault diagnosis. Use of inverse models can avoid theoretical existence problems. An important result on input-output cononical forms is given from which a nonlinear observer can be designed to generate a residual for detecting a specific tank fault. The analysis here can be applied to other hydraulic systems (Shields et al., 2001a, Yu and Shields,1996).

2. APPLICATION PROBLEM

The problem considered is that of a n-tank hydraulic control system consisting of n tanks (tank 1 to tank n) connected serially with each other by cylindrical pipes. Here, the $n-2$ inner tanks and connecting pipes are situated in a hazardous environment and measurements of the tank levels are not available. Control is only possible via fluid inputs (u_1, u_2) to tank 1 and tank n. Also, only the output levels of these two tanks $(y(1), y(2))$are available for feedback control and fault detection. The flow, $Q_{i,i+1}$, between tank i and tank $i+1$ satisfies a Toricelli law,

$$Q_{i,i+1} = a_{i,i+1} g(x_i - x_{i+1}); \qquad (1)$$

where x_i is the fluid level in i,

$$g(s) = sgn(s) \cdot \sqrt{|s|}, \quad (2)$$

and where coefficient $a_{i,i+1}$ depends upon gravity, and on the flow correction term and on the cross-sectional area of the connecting pipe from tank i to tank $i+1$ $(i = 1,.,n)$. Tank n has an outflow pipe such that the flow is given by $Q_n = a_{n,n}g(x_n)$. This system is described by n differential equations (Model 1):

$$\begin{aligned} \dot{x_1} &= u_1 + \delta_1 \\ &\quad - a_{1,2}g(x_1 - x_2) & (3) \\ \dot{x_2} &= a_{1,2}g(x_1 - x_2) - a_{2,3}g(x_2 - x_3) + k_2 f & (4) \\ \dot{x_3} &= a_{2,3}g(x_2 - x_3) - a_{3,4}g(x_3 - x_4) + k_3 f & (5) \\ \dot{x_i} &= a_{i-1,i}g(x_{i-1} - x_i) - a_{i,i+1}g(x_i - x_{i+1}) \\ &\quad + k_i f \quad (i = 2,..,n-1) & (6) \\ \dot{x_n} &= a_{n-1,n}g(x_{n-1} - x_n) - a_{n,n}g(x_n) \\ &\quad + u_2 + \delta_2 & (7) \end{aligned}$$

with state vector $x = [x_1,..,x_n]'$ and output vector y,

$$\begin{aligned} y_1 &= x_1; y_2 = x_n & (8) \\ y &= Cx = [e_1, e_n]'x, & (9) \end{aligned}$$

where e_i is a unit n-vector with zero elements except for element i of value 1. Here $u_1 = \frac{Q_1}{A_1}$ and $u_2 = \frac{Q_n}{A_n}$ where A_1 and A_n are the cross-sectional areas of tank 1 and tank n and $Q_i(i = 1,n)$ is the inflow through control (actuator) pump i. A fault (a leak or a plugging) f in tank i is obtained by imposing $(k_i = 1; kj = 0; j \neq i)$. Disturbances δ_1, δ_2 are assumed only possible in the end tanks (this can be generalised). A leak model for simulation purposes could be of the form $f = a_z S_l \sqrt{2gx_i}$, for tank i. Typical parameters: $a_z = 1$, $A_1 = A_n = 0.0154m^2$, $g = 9.81m/s^2$, $S_n = 5 * 10^{-5}m^2$, $ai, i+1 =$ approximate order $= \frac{a_z S_n}{A_1}$, $(.1m < x_i < .5m)$, $S_l = 2.7 * 10^{-5}m^2$, $Q_{1max} = 1 * 10^{-4}m^3/sec$, $Q_{2max} = 1 * 10^{-4}m^3/sec$.

The function $g(s)$ in (2) is not differentiable at $s = 0$ and not expandable as a Taylor series there. A good polynomial fit for $g(s)$ in $(-.5 < s < .5)$, where g(s) is replaced by a straight line in $(-.02 < s < .02)$, is $g_{33}(s)$, a polynomial of degree 33. Replacing g in Model 1 by $g_{33}(s)$ is then acceptable for control purposes, giving a smooth, but complex, nonlinearity in $(-.5 < s < .5)$. However, by contrast, the inverse function $h(s) = g^{-1}(s)$ is well behaved. It can be approximated in $(-.5 < s < .5)$ by a polynomial of order 7 , $h_7(s)$, an increasing function of s,

$$\begin{aligned} h_7(s) &= 2.6852s^7 - 3.3559s^5 + 2.1866s^3 + 0.1236s \\ &= 2.6852sh_1(s) & (10) \end{aligned}$$

where $h_1(s) > 0$. The derivative satisfies, $h_7'(s) > 0$. An inverse model to Model 1 can be then used. The following models are now derived, without proofs, which are used to derive results.

State space version of Model 1. Equations (3)-(7) can be written

$$\begin{aligned} \dot{x} &= -B'\Delta G(Bx) + \Omega(u + \delta) + kf, & (11) \\ &= -grad(\phi(x)) + \Omega(u + \delta) + kf, & (12) \end{aligned}$$

where $\Omega = [e_1, e_n]$,
$k = [0, k_1, k_2, .., k_{n-1}, 0]'$,
$\Delta = \text{diag}(a_{1,2}, a_{2,3}, .., a_{n-1,n}, a_{n,n})$,
$G(x) = [g(x_1), g(x_2), .., g(x_n)]'$,
and where

$$B = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 1 & -1 \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix}.$$

Here, $\phi(x) = \sum_{i=1}^{n-1} a_{i,i+1} g_s(x_i - x_{i+1}) + a_{n,n} g_s(x_n)$, and $g_s(s) = \int_0^s g(\tau)\mathrm{d}\tau$.

Model 2. The mapping, $w = Bx$, transforms Model 1 to

$$\begin{aligned} \dot{w} &= -BB'\Delta G(w) + B\Omega(u + \delta) + Bkf, & (13) \\ y &= C_w w = CB^{-1}w, & (14) \end{aligned}$$

$$\text{CB}^{-1} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}.$$

Model 3. The mapping, $z = \hat{D}Bx$, transforms Model 1 to

$$\begin{aligned} \dot{z} &= -SG(z) + \hat{D}B\Omega(u + \delta) + \hat{D}Bkf, & (15) \\ y &= C_z z = CB^{-1}\hat{D}^{-1}z, & (16) \end{aligned}$$

$$\text{CB}^{-1}\hat{D}^{-1} = \begin{bmatrix} \hat{d}_1^{-1} & \hat{d}_2^{-1} & \cdots & \hat{d}_n^{-1} \\ 0 & \cdots & 0 & \hat{d}_n^{-1} \end{bmatrix}, where \hat{D} =$$

$\text{diag}(\hat{d}_1, \hat{d}_2, ..., \hat{d}_n)$ and
$\hat{d}_i = a_{i,i+1}^{\frac{2}{3}}$ $(i = 1,..,(n-1))$, $\hat{d}_n = a_{n,n}^{\frac{2}{3}}$. Here, $S = (\hat{D}B)(\hat{D}B)'$ is positive definite , tridiagonal and symmetric.

Model 4. The mapping, $p = Wz$, transforms Model 3 to

$$\begin{aligned} \dot{p} &= -(\alpha\Sigma)p + G_1(p) + W\hat{D}B\Omega(u + \delta) \\ &\quad + W\hat{D}Bkf, & (17) \\ y &= C_p p = CB^{-1}\hat{D}^{-1}W'p, & (18) \end{aligned}$$

where W is a real orthogonal matrix such that $WSW' = \Sigma = \text{diag}(\sigma_1, .., \sigma_n)$, where $g(s)$ in (2) is approximated by $g_{33}(s) = \alpha s + g_1(s)$ and where $G_1(p)$ represents the higher-order polynomial terms in $\Sigma WG(W'p)$. Since $\sigma_i > 0$ and $\alpha > 0$ the system is locally, open-loop, asymptotically stable at $p = 0$.

Model 5. The mapping, $q = DBx$, transforms Model 1 to

$$\begin{aligned} \dot{q} &= -KG(q) + DB\Omega(u + \delta) + DBkf, & (19) \\ y &= C_q q = CB^{-1}\hat{D}^{-1}q, \end{aligned}$$

$$(21)$$

$\mathrm{C}_q = \begin{bmatrix} d_1^{-1} & d_2^{-1} & \cdots & d_n^{-1} \\ 0 & \cdots & 0 & d_n^{-1} \end{bmatrix} .where \mathrm{D} = \mathrm{diag}($ $\mathrm{d}_1, d_2, ..., d_n)$ and

$d_1 = 1,\ d_i = \frac{d_{i-1}^{\frac{1}{2}}}{a_{i-1,i}}$ $(\ i = 1, .., n-1)$. Here, $K = DBB'\Delta D^{-\frac{1}{2}}$ is then of tridiagonal form where
$k_{i,i-1} = -1\ (i = 2, ..., n)$.

System (19) is now equivalent to the form

$$\dot{q_1} + k_{1,1}g(q_1) - k_{1,2}g(q_2) = \lambda_1(u_1 + \delta_1) + \gamma_1 f, \quad (22)$$
$$\dot{q_2} + k_{2,2}g(q_2) - k_{2,3}g(q_3) = g(q_1) + \gamma_2 f, \quad (23)$$
$$\dot{q_i} + k_{i,i}g(q_i) - k_{i,i+1}g(q_{i+1}) = g(q_{i-1}) + \gamma_i f, (i = 1, ..n-2), \quad (24)$$
$$\dot{q_{n-1}} + k_{n-1,n-1}g(q_{n-1}) - k_{n-1,n}g(q_n) = g(q_{n-2}) + \lambda_{n-1}(u_2 + \delta_2) + \gamma_{n-1}f, \quad (25)$$
$$\dot{q_n} + k_{n,n}g(q_n) = \lambda_n(u_2 + \delta_2). \quad (26)$$
$$(27)$$

where $\lambda_1 = d_1, \lambda_{n-1} = -d_{n-1}, \lambda_n = d_n$, $\lambda_i = 0$ otherwise, and $\gamma_i = k_i - k_{i+1}, i = 2, ., n-2$, $\gamma_1 = -k_2, \gamma_{n-1} = k_{n-1}, \gamma_n = 0$.
A Control Problem.
The control problem here is to maintain an inner tank at a given level while the end tank has a fixed outflow (x_n = fixed), using input u and mesurement y. A fault can occur in any inner tank and must be detected. Assume the specified inner tank is tank (j).
Objective 1: establish an equilibrium operating point $(\bar{x}, \bar{u})$.
Objective 2: show system is controllable and show a control stategy is feasible to maintain $x_j = \bar{x}_j$, $x_n = \bar{x}_n$, $u_i \geq 0$, with $\| x_i - \bar{x}_i \|$ small, for given $\bar{x}_i$.
Objective 3: derive nonlinear observer-based residuals for detecting a fault, f.
Equilibrium. With $f = 0$, $\delta = 0$ and $\dot{(w)} = 0$ in (13) of Model 2, there obtains

$$\Delta G(w) = (B')^{-1}\Omega u \quad (28)$$

Since $w_i = x_i - x_{i+1}$ and $w_n = x_n$ the following then hold in terms of u.

$$x_i = x_n + h(u_1)\alpha_i, \quad (i = 1, .., n-1)$$
$$a_{n,n}g(x_n) = u_1 + u_2,$$
$$\alpha_i = \sum_{j=i}^{n-1} \frac{1}{a_{j,j+1}^2}, (i \neq n); \alpha_n = \frac{1}{a_{n,n}^2}$$

Given $x_j = \bar{x}_j$, $x_n = \bar{x}_n$, the equilibrium values of $\bar{u}_i$ are obtained by solving

$$\bar{x}_j - \bar{x}_n = h(\bar{u}_1)\alpha_j, \quad (29)$$
$$a_{n,n}g(x_n) - \bar{u}_1 = \bar{u}_2, \quad (30)$$

assuming $\bar{x}_j - \bar{x}_n > 0$ and $a_{n,n}g(x_n) - u_1 > 0$. The other equilibrium tank levels are then specified as

$$\bar{x}_i = \bar{x}_n + h(\bar{u}_1)\alpha_i \ \ (i \neq j). \quad (31)$$

Define now $\bar{w} = B\bar{x}$, etc.
Model 2, Controllability-observability, linear case, $\delta = 0$.
Let $w = \Delta w + \bar{w}$ and $u = \bar{u} + v$, then a linearized version of Model 2 is

$$\Delta\dot{w} = -BB'\Delta A_g(\bar{w})\Delta w + B\Omega v + Bkf, \quad (32)$$
$$y_g = y - C_w\bar{w} = C_w\Delta w \quad (33)$$

where $A_g(\bar{w})$ is the Jacobian of $G(w)$ at $\bar{w}$.
Linear Control Strategy. If A_g exists then: (i) the pairs $(BB'\Delta A_g, Be_1)$ and $(BB'\Delta A_g, Be_n)$ are contollable; (ii) the pair $(BB'\Delta A_g, C_w)$ is observable. Replacing $g(s)$ by $g_{33}(s)$ in (13) guarantees existence. A local state-feedback controller (LQG) of the form $v = -K\hat{w}$ exists which stabilizes the system about the required equiliibrium $(\bar{w}, \bar{u})$, where $\hat{w}$ is a properly tuned observer estimate of w.
Global nonlinear controller. Consider equation (11) with $\delta = 0$;

$$\dot{x} = -grad(\phi(x)) + \Omega u + kf, \quad (34)$$

Then the function $V = \phi$ satisfies

$$\dot{V} = g_f'(-g_f + \Omega u + kf),$$
$$= -g_f'g_f + g_f'(\Omega u + kf)$$

where $g_f = grad(\phi(x))$. Thus about $x = 0$, $u = 0$; and no faults , the system is globally asymptotically stable. There exists a gain, $R(y_1, y_2)$, of appropriate structure, such that a feedback of the form $u = -R(y_1, y_2)grad(\phi)$, or modifications for different equilibrium operating points, gives $\dot{V} < 0$ $(x \neq 0)$, implying global asymptotic stability. Note u is a function of y so no estimator is needed, but this limits the control performance.

2. TWO OBSERVER DESIGNS.

Design 1. The first design depends on Result 1.
Result 1. System (13), or any of the models (1 to 5) can be transformed to the quadratic polynomial descriptor form;

$$A_1\dot{x}(t) = Ax(t) + E_a d(t) + Kf(t) + Bu(t) + \sum_{i=1}^{m} u^i(t)A_{ux}^i x(t) + \sum_{i=1}^{k} x^i(t)A^i x(t) + \sum_{i=1}^{k} x^i(t)[E^i d(t) + K^i f(t)] \quad (35)$$
$$y(t) = Cx(t) + Qf(t) \quad (36)$$

where $x(t) \in I\!R^{7n}$, $y(t) \in I\!R^{7p}$. Here x, y, represent different states to those in (13). A_1 is nonsingular.
Sketch proof. First define a new state z as $z_i = g(w_i)$ or, using $h_7(s)$, $w_i = h_7(z_i)$. Differentiating, (13) gives the for the ith row,

$$h_7'(z_i)\dot{z}_i = -s_i z + (B\Omega)_i(u+\delta) + (Bk)_i f, \quad (37)$$

where the index refers to rows and $s_i = (BB'\Delta)_i$. For each z_i define six new variables (not necessarily independent) v_i; $v_i = z_i^{i+1}$ $(i = 1,.,6)$. The left-hand side of (37) can now be written as a linear combination of the derivatives of z_i and v_i. Repeating this procedure for each z_i gives the descriptor system (35) of dimension $7n$. This system is equivalent to a system of the same form with $A_1 = I_{7n}$.
Residual design using Result1. For system (35)-(36) a nonlinear time-varying observer can be designed along the lines given in (Shields et al.,2001b);

$$\begin{aligned}\dot{z}(t) &= Fz(t) + Ju(t) + Hy(t)\\ &+ \sum_{i=1}^{m} u^i(t)[H^i_{ux}y(t) + F^i_{ux}z(t)]\\ &+ \sum_{i=1}^{p} y^i(t)[H^i y(t) + F^i z(t)]\end{aligned}$$

where $z(t) \in I\!R^d$, is a linear estimate of $Tx(t)$. A fault residual (detection signal) is defined as

$$\epsilon(t) = L_1 z(t) + L_2 y(t), \quad (38)$$

where $\epsilon(t) \in I\!R^{d_o} (1 \le d_o \le d)$, $k \ge 0$.

Computational details and detectability theorems can be used from (Shields et al., 2001a) to design the matrices F, J, H, T, L_1, L_2, $H^i_{ux}(i = 1,\cdots,m)$, $F^i_{ux}(i = 1,\cdots,m)$, $H^i(i = 1,\cdots,p)$,$F^i(i = 1,\cdots,p)$. The obsever error and residual then satisfy the forms

$$\dot{e}(t) = W^e(t)e(t) + W^*(t)f(t), \quad (39)$$

$$\epsilon(t) = L_1\left[e(t) - T\Phi Q f(t)\right]. \quad (40)$$

Error convergeance for $f = 0$ is ensured in the design.
Design 2. Firsly an algorithm is given to derive the relationship between the inputs (u_1, u_2) and the single output $Y = d_n y_2 = q_n$. The specific form of Model 5, equations (22)-(26), is used. Firstly functions ϕ_i $(i = 1,.,n)$ are defined as;

$$\begin{aligned}\phi_1 &= q_n = Y,\\ \phi_2 &= h[\dot{\phi}_1 + k_{n,n}g(Y) - \lambda_n u_2],\\ \phi_3 &= \\ &+h[\dot{\phi}_2 + k_{n-1,n-1}g(\phi_2) - k_{n-1,n}g(\phi_1)\\ &-\lambda_{n-1}u_2 - \gamma_{n-1}f],\\ \phi_{i+1} &= \\ &+h[\dot{\phi}_i + k_{ii,ii}g(\phi_i) - k_{ii,ii+1}g(\phi_{i-1})\\ &-\gamma_{ii}f],\\ &(i = 3,..n-1), (ii = n-i+1). \quad (41)\end{aligned}$$

Here ϕ_{i+1} is a function of Y and derivatives of Y to order i, of u_2 and derivatives of u_2 to order $i-1$, and of f and derivatives of f to order $i-2$. Note that the influence of δ has been dropped for simplicity and can be recovered by noting the influence of u. From the form of model 5 and these definitions;

$$q_{n-i} = \phi_{i+1}, \ \ i = 0,.,n-1. \quad (42)$$

Thus, now $q_{n-1} = \phi_n$, and by substitution into (22) the following result holds:
Result 2. An input-output relationship from u_1 and u_2 to Y is

$$\begin{aligned}\dot{\phi}_n &+ k_{1,1}g(\phi_n) - k_{1,2}g(\phi_{n-1})\\ &-\lambda_1 u_1 - \gamma_1 f = 0 \quad (43)\end{aligned}$$

By considereing the first output y_1 in Model 5 a lower order model involving both inputs and outputs is obtained:
Result 3. An input-output relationship from u_2 to y_1 and Y is

$$\sum_{i=1}^{n-1} \frac{1}{d_i}\phi_{n-i+1} = y_1 - \frac{1}{d_n}Y \quad (44)$$

Define now the vector Φ by its components: $(\Phi)_i = \phi_i$, $(i = 1,..,n)$. After some analysis (43) can be expressed in the form;
Result 4. The input-output relationship in Result 2, from u_1 and u_2 to Y, is expressable as

$$Y^{(n)} = A_1(\Phi)[u_1 + \frac{\gamma_1}{\lambda_1}f + A_2(.)], \quad (45)$$

where A_2 in full functional form is $A_2(Y,.,Y^{(n-1)},u_2,.,u_2^{(n-1)},f,.,f^{n-2})$, where $Y^{(n)}$ is the nth derivative of Y w.r.t. t and where,

$$\begin{aligned}A_1 &= (h'(g(\phi_n))h'(g(\phi_{n-1}))...h'(g(\phi_1))^{-1}\\ &= g'(\phi_n)g'(\phi_{n-1})...g'(\phi_1) \quad (46)\end{aligned}$$

Clearly, the input-output map (45) exists provided $h'(g(\phi_i)) \neq 0$, for all i, near the operating point (for local existence). Note that the detailed form of A_2 in (45) can easily be derived given n and the definitions of ϕ_i. To obtain a smooth input-output map for fault detection the following procedure is now proposed:
Step 1; in the definitions of ϕ_i replace $h(s)$ and $g(s)$ by $h_7(s)$ and $g_{33}(s)$, respectively. From (10), since also the devivative $h_7'(s) > 0$, the function $A_1(\Phi)$ in the (45) is well defined and $A_1(\Phi) \neq 0$

over finite values of ϕ_i.
Step 2. Define new states $x_i = Y^{(i-1)}$ then (45) can be written in the following, well defined, state-space canonical form of dimension minimal degree n (see (Isidori, 1995) for definitions on relative degree)

$$\dot{x} = Jx + e_n F_c(x, U, f_f) \tag{47}$$
$$Y(t) = e_1' x \tag{48}$$

where from (45)
$F_c = A_1(\Phi)[u_1 + \frac{\gamma_1}{\lambda_1} f + A_2(.)]$,
and U and f_f are extended vectors;
$U = [u_1, u_2, ..., u_2^{(n-1)}]'$,
$f_f = [f, f', ..., f_{(n-2)}]'$.
Note that F_c is a known polynomial scalar function in the components of x, U and f_f. Here J has as the form and property
$J = [0, e_1, e_2, .., e_{n-1}]$; $J^n = 0$.

Using the same steps on the input-output model (44) of Result 3 an $(n-1)$ dimensional canonical form can be obtained (details are omitted here) where x has components $Y^{(i-1)}$ $(i = 1, ., n-1)$;

$$\dot{x} = Jx + e_{n-1} F_{c2}(y_1, x, U, f_f) \tag{49}$$
$$Y(t) = e_1' x \tag{50}$$

where F_{c2} is a scalar and U and f_f are now different extended vectors;
$U = [u_2, ..., u_2^{(n-2)}]'$,
$f_f = [f, f', ..., f_{(n-3)}]'$.
Here F_c depends on measurment y_1 and $J = [0, e_1, e_2, .., e_{n-2}]$, $J^{n-1} = 0$.

Both canonical forms can be used for fault detection but here only the first form will be considered to show the main steps.
Fault detection observer design. Let $x = \bar{x} + z$, $U = \bar{U} + V$ in the region of the operating point $(\bar{x}, \bar{U})$ where;

$$0 = J\bar{x} + F_c(\bar{x}, \bar{U}, 0) \tag{51}$$
$$y(t) = Y(t) - e_1'\bar{x} = e_1' z, \tag{52}$$

then (47)-(48) has the form

$$\dot{z} = \hat{J} z + e_n F_{c3}(z, V, f_f) \tag{53}$$
$$y(t) = e_1' z \tag{54}$$

where $\hat{J} = J + e_n a'$ and
$F_{c2}(x, U, f_f) - F_{c2}(\bar{x}, \bar{U}, 0) = F_{c3} + a'z$.
Here $a'z$ is the linear term in z. Assume there exists a uniformly bounded positive function $\alpha_1(\| V \|) > 0$ such that;

$$\| F_{c3}(z_1, V, 0) - F_{c3}(z_2, V, 0) \| \le \alpha_1(\| V \|) \| z_1 - z_2 \| \tag{55}$$

Define the observer estimate of z as $\bar{z}$ where

$$\dot{\bar{z}} = \hat{J}\bar{z} + e_n F_{c3}(\bar{z}, V, 0) + S^{-1} e_1 (y - \bar{y}) \tag{56}$$
$$\bar{y}(t) = e_1' \bar{z}, \tag{57}$$

and fault detection residual $r(t)$ as

$$r(t) = W(y(t) - \bar{y}) = W e_1' e \tag{58}$$

where W is a scale factor and where S is the solution of the matrix equation

$$0 = -\theta S - (\hat{J}' S + S\hat{J}) + e_1 e_1', \tag{59}$$

and where θ is a positive gain to be chosen. The error $\epsilon = z - \bar{z}$ satisfies

$$\epsilon = \hat{J}\epsilon + e_n D_{c3}(t) - S^{-1} e_1 e_1' \epsilon, \tag{60}$$
$$D_{c3}(t) = F_{c3}(z, V, f_f) - F_{c3}(\bar{z}, V, 0).$$

By assumption, if $f_f = 0$,

$$D_{c3} \le \alpha_1(\| V \|) \| \epsilon \|. \tag{61}$$

Result 5. For the case $f_f = 0$, the Lyapunov function, $V(t) = \epsilon' S \epsilon$, satisfies

$$\dot{V} \le -V(\theta - \alpha(t)), \tag{62}$$

where $\alpha(t) = 2\alpha_1(\| V \|)(\frac{\| S \|}{\| S^{-1} \|})^{\frac{1}{2}}$. By assumption a θ exists satisfying

$$\theta > \alpha(t), \tag{63}$$

and hence the observer error is asymptotically stable. The residual (58) is asymptotically zero for no faults and is dependent upon f otherwise.*End result.*
Detection analysis. Equation (60) can be written

$$\dot{\epsilon} = \bar{J}\epsilon + e_n D_{c3}(\epsilon, \bar{z}, V, f_f), \tag{64}$$

where $\bar{J} = \hat{J} - S^{-1} e_1 e_1$. Due to the form of (60) the n-th derivative of ϵ, $\epsilon^{(n)}$, satisfies

$$\epsilon^{(n)} = (\bar{J})^n \epsilon + \sum_0^{n-1} (\bar{J})^i e_n D_{c3}^{(n-1-i)}. \tag{65}$$

and $r^{(n)} = W e_1' \epsilon^{(n)}$. Given that $\epsilon(0) = 0$, initially for example, and $f_f(t) \ne 0$ $(t > 0)$ then a fault will be reflected in the nth derivative if
$W e_1' [\sum_0^{n-1} (\bar{J})^i e_n D_{c3}^{(n-1-i)}] \ne 0$.
In terms of objective 3, a single fault in tank i of equation (6) $(k_i = 1, k_j = 0, (j \ne i))$ will be reflected in the vector f_f and hence $r(t)$.
Application example. The following example illustrates the two designs for a three-tank system. Consider the input $u = [u_1, u_2]'$,
where $u_1 = 2 * 10^{-5}, 0 \le t < 1000$;

$$u_2 = \begin{cases} 3 * 10^{-5} & 0 \le t < 100 \\ 0 & 100 \le t < 800 \\ 3 * 10^{-5} & 800 \le t < 900 \\ 0 & 900 \le t < 1000 \end{cases}$$

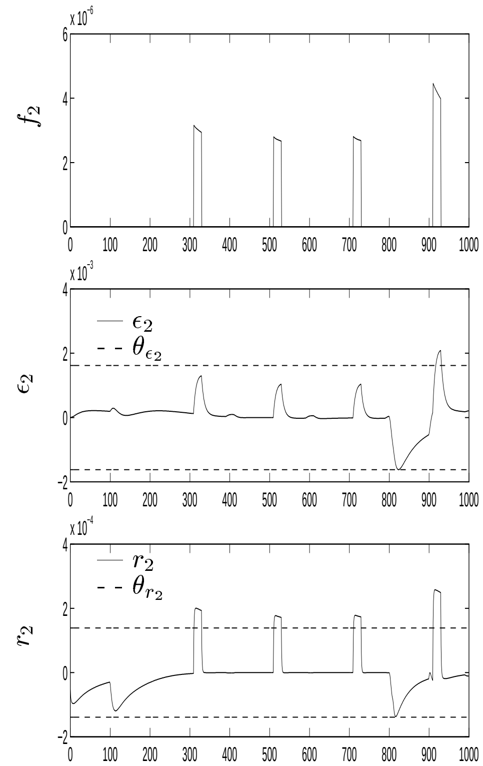

Fig. 1. Residual Performances for fault f_2

Here, u_2 is chosen as a pulse function with 0 values in intervals $[100, 800]$ and $[900, 1000]$. A sinusoidal disturbance was input to tank 3: $\delta_1 = 0$, $\delta_2 = 0.3 * 10^{-5} sin(0.3t)$. Desired objective tank levels where $x_1 = 0.42$, $x_2 = 0.35$, $x_3 = 0.28$.

Figure 1 shows the performance of residuals for the first and second designs; shown as r_2 and ϵ_2, respectively. Here f_2, is a simulated leak in tank 2. The thresholds $\theta_{r_2}(= 0.139 \times 10^{-3})$ and $\theta_{\epsilon_2}(= 1.62 \times 10^{-3})$ are chosen for the residuals r_2 and ϵ_2, respectively. Residual r_2 picks up more fault information than the ϵ_2 for this demanding fault.

4 CONCLUSION

This paper derives several useful transformations and input-output maps for satisfying the three objectives concerning control and fault detection. Two alternative approaches for designing an observer-based residual are given for detecting any fault in the set of inaccessible tanks. Existence of smooth nonlineararites is assured by using polynomial approximations. For Design 2, assumption (55) must hold for global convergeance of the obsever. By contrast Design 1 assumes a polynomial model of degree 2 and several conditions must be satisfied for existence (Shields et al., 2001a). A limited comparison of residual performances is given for a 3-tank system (case $n = 3$). The analysis in the results is useable for many other hydraulic problems with similar flow nonlinearities.

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