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A NEW LOWER BOUND FOR SCHEDULING OF FMS BASED ON AGV MATERIAL HANDLING

P. Lacomme*, A. Moukrim**, N. Tchernev ***

* *Université Blaise Pascal, Laboratoire d'Informatique, de Modélisation et d'Optimisation des Systèmes (LIMOS), Campus Universitaire des Cézeaux, 63177 Aubière Cedex France, Tel.: +33 4 73 40 75 85, E-mail: lacomme@isima.fr*

** *Université de Technologie de Compiègne, HeuDiaSyC, UMR CNRS 6599, BP 20529 60205 Compiègne cedex (France) Tel.: +33 3 44 23 49 52, Fax: +33 3 44 23 44 77, E-mail: Aziz.Moukrim@hds.utc.fr*

*** *IUP de Management et Gestion des entreprise, Université d'Auvergne, Laboratoire d'Informatique, de Modélisation et d'Optimisation des Systèmes (LIMOS), Campus Universitaire des Cézeaux, 63177 Aubière Cedex France, Tel.: +33 4 73 40 77 71, E-mail: tchernev@sp.isima.fr*

Abstract: This paper addresses the simultaneous job input sequencing and vehicle dispatching problems in flexible manufacturing systems (FMS) using a single device Automated Guided Vehicle System (AGVS). A Branch and Bound approach based on simulation has been previously developed. However, efficient easily computable lower bounds are still required to provide solutions for small and medium size problems. In this paper a new lower bound is proposed and benchmarks proved that this lower bound outperforms the lower bound previously published. *Copyright © 2002 IFAC*

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1. INTRODUCTION

Many Flexible Manufacturing Systems (FMS) employ Automated Guided Vehicle Systems (AGVS) as material handling system. The AGVS effectiveness depends on several factors, among them a well designed vehicle management which functions are: (i) *dispatching*, which is the process of selecting and assigning tasks to material handling devices; (ii) *routing*, which is the process of selecting specific paths taken by material handling devices; (iii) *scheduling*, which is the process of determining of the arrival and departure times of material handling devices. Scheduling encompasses the dispatching and routing issues with the introduction of time in order to reduce the impact of blocking and congestion in meeting a material handling workload. The vehicle management has a significant effect on the travel time, the number of devices used, the AGVS response time, the operating expenses and the initial investment costs.

One of the most difficult operational problems in FMS is the proper coordination of the production sequencing and the allocation in the time of required resources. This paper deals with the simultaneous job input sequencing and vehicle dispatching in FMS using a single vehicle AGVS. The objective is the makespan minimization. However, to obtain really profitable solutions the following constraints must be taken into account: (i) the limited input/output buffer capacity; (ii) the limitation on the number of jobs simultaneously allowed in the shop; (iii) the dynamic behaviour of the system under study and thus the impact of the vehicle blocking and congestion as well as the impact of the machine blocking.

This problem is previously studied by Gourgand *et al.,* (1999) who propose a two stages iterative approach for approximately solving the joint job input sequencing and vehicle dispatching. Their approach considers the constraints defined above.

Large size problems with up to 50 jobs to sequence are solved by an iterative approach.

Lacomme *et al.*, (2000) propose a branch and bound approach coupled with a discrete events simulation model. The discrete events simulation model provides evaluation of the makespan taking into account all the managing constraints of the system. Very efficient lower bound are required to ensure performances to branch and bound approaches. Unfortunately, previous lower bounds dedicated to this problem are not enough efficient and the lower bound used in others scheduling problems required many investigations to take into account the problem constraints.

In the following sections, the problem is described and the Branch and Bound approach proposed by Lacomme *et al.* (2000) and Espinouse *et al*. (2001) is presented. A new efficient Lower Bound is presented in section 4. The section 5 provides a benchmark test. Lastly, the results of the study are summarised and some suggestions for future researches are identified.

2. PROBLEM FORMULATION

The simultaneous job input sequencing and vehicle dispatching in FMS with finite buffers capacity and a single vehicle AGVS can be stated as follows: given a FMS and dispatching rules, determine the job input sequence which minimizes the makespan. Since in FMS the number of jobs simultaneously allowed into the system is limited and the sum of processing times is constant, the only way to minimize the makespan is to reduce the waiting times due to the blocking and non availability of resources needed. Therefore the objective is to find an order in which the jobs enter in the manufacturing system reducing the waiting times due to the blocking and non availability of resources needed as well as minimizes the deadheading time.

The issue of joint optimization of job and AGV schedules is a complex problem which has been formulated as a non-linear integer program see (Bilge and Ulusoy, 1995), or linear see (Anwar and Nagi, 1988). Due to its intractability the literature contains numerous heuristic approaches (Gourgand *et al.*, 1999). For the literature review the reader can refer to (Ganesharajah *et al.*, 1998). However, the joint use of mathematical programming techniques or heuristics and simulation has considerable potential to improve the state of knowledge in this area.

In the next section we present the simulation based on Branch and Bound approach taking into account the following constraints:

- the limited input/output buffer capacity;
- the limitations on the number of jobs simultaneously allowed in the system;
- the dynamic behaviour of the system under study and thus the impact of the vehicle blocking and congestion as well as the impact of the machine blocking.

3. THE BRANCH AND BOUND APPROACH

Let us define the following notations which will be used to describe the Branch and Bound:

n number of jobs to sequence

m the number of different job types

N the number of jobs simultaneously allowed in the FMS

M_0 the input station of the system

M_S the output station of the system

m_s the number of machine in the system

m_s' the number of machine in the system except the input/output station

i job type index $i \in I = \{1,2,..,m\}$

n_i number of jobs of type i such that $\sum_{i=1}^{m} n_i = n$

x_p partial jobs input sequence in which jobs in the first p positions have been fixed

x_n^* optimal job input sequence

U_{n-p} the set of the unscheduled jobs associated to the partial job input sequence x_p

$H(x_p)$ the makespan associated to the partial jobs input sequence x_p. $H(x_p)$ is evaluated using the discrete events simulation model.

The problem is to compute x_n^* for which $\forall x_n\ H(x_n^*) \le H(x_n)$. The feasible set of solutions of the job input sequencing problem from a combinatorial point of view is given by $X = \{$set of all n-jobs schedules$\}$. Although complete enumeration would permit to obtain the optimal solution, this approach is unpractical due to computational time problems. For example for 20 jobs to schedule (5 types of jobs and 4 jobs of each type) there is $(20!/(4!4!4!4!4!)) \approx 3.10^{11}$ different jobs input sequences of 20 jobs. Dominated solutions (which can be excluded from consideration) permit to reduce the search effort.

General framework

The studied problem can be decomposed into two subproblems and solved by an iterative search procedure that tries to accommodate the combinatorial nature in finding the solution. Given a solution for the job input sequencing subproblem by the branch and bound algorithm, it remains to find a vehicle schedule based on a given vehicle dispatching rule and to evaluate the system performance and the makespan using a dedicated discrete events simulation model. In other words at each node of the search tree the current solution is evaluated by a dedicated discrete events simulation model (figure 1). Iterations are carried out in order to improve the initial job input sequence and the vehicle schedule and to find a better solution. After a given number of iterations the procedure will terminate with the optimal solution, if a feasible solution exists.

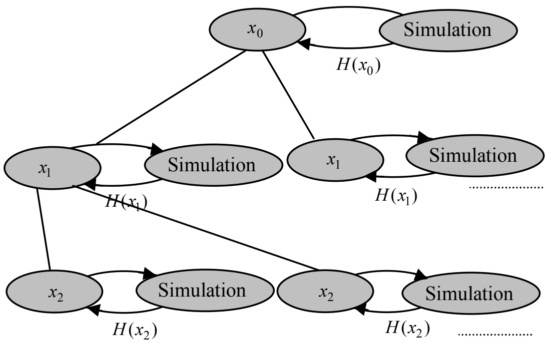

Fig.1. Template of the general framework

At the top of the search tree, v_0, the initial solution $H(x_0)$ is evaluated using the dedicated discrete events simulation model. In the job sequence x_0, the jobs are ordered in a cyclic manner. The upper bound, noted UB_0, for this first iteration is set to $H(x_0)$. At each iteration t a node v_t is chosen, according to a depth-first search, in the search tree. The lower bound for node v_t, noted LB^t, is calculated. Whenever LB^t is greater than the current upper bound UB, the branch is pruned. At the bottom of the search tree, the upper bound is updated, if the current upper bound UB is greater than LB^t, a backtrack is performed.

3. PREVIOUS LOWER BOUNDS

To provide more details of the procedure the following additional definitions are introduced. Assume a specific node is reached in the search process where:

$\overline{H}^p(x_n)$ the lower bound of x_n with only first p positions which have been fixed. It means that only $H(x_p)$ of the x_p partial jobs input sequence has been evaluated by simulation.

$E(x_p)$ the p^{th} job entry date in the system. $E(x_p)$ is computed by simulation

$DS(x_p)$ the p^{th} job end date in the system. Represent the output station reached date

t_{ij} the loaded vehicle travel time from machine i to machine j including loading and unloading time

v_{ij} the unloaded vehicle travel time from machine i to machine j

p_{ik} the processing time of the job k on the machine i

m_k the number of machines that job k has to visit

$M(k,i)$ the i^{th} machine that has to be visited by job k where $M(k,0)$ is the system input station and $M(k,m_k)$ is the system output station

The challenging problem consists in computing a powerful estimation of $H(x_p)$. This estimation is noted $\overline{H}^p(x_n)$ (see above) because only p jobs have been already sequenced (figure 2).

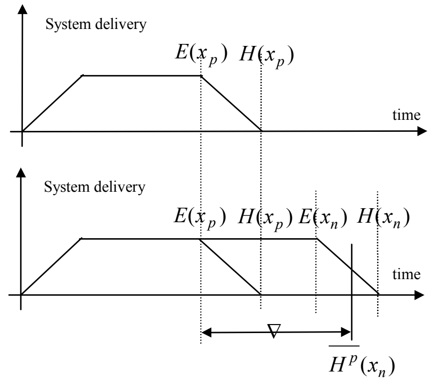

Fig.2. The Lower Bound with regard to $H(x_p)$

A detailed presentation of some lower bounds are proposed in (Espinouse *et al.*, 2001). The authors report:

- a basic lower bound;
- a lower bound dedicated to problem with an important number of jobs based on the same types;
- a lower bound taking into account the limited number of jobs simultaneously allowed in the system;
- a probabilistic estimation of the vehicle unloaded transportation time.

The benchmark of (Espinouse *et al.*, 2001) proves that no lower bound is better than other ones. The basic lower bound presented below provides solutions with lowest computational time requirements see (Espinouse *et al.*, 2001):

$$\overline{H}^p(x_n) = E(x_p) + Max\begin{pmatrix} \sum_{k\in U_{n-p}} \sum_{i=0}^{m_k-1} t_{M(k,i),M(k,i+1)} \\ Max_i \sum_{k\in U_{n-p}} p_{i,k} \end{pmatrix}$$

is a lower bound of $H(x_n)$ denoted LB_1.

4. NEW LOWER BOUND

A lower bound is defined as follows: $\overline{H}^p(x_n) = \alpha_P + \nabla$, where:

α_P the evaluation of $E(x_{p+1})$

- ∇ the evaluation of the time required to treat unscheduled jobs of U_{n-p}

4.1 Entry date of the $(p+1)^{th}$ job

The entry date of the $(p+1)^{th}$ job highly depends on the number of jobs simultaneously allowed in the system and indeed of the p value. Two cases should be distinguished:

<u>First case:</u> $p < N$

As long as possible and as long as the number of jobs in the system is inferior than N the managing rule of the system consists in serviced first the entry station. So the vehicle serviced the entry station as long as possible when $p < N$**.**

The entry date of the $(p+1)^{th}$ job is $E(x_p)$ added to the loaded vehicle travel time required to go from the entry station to the first station used by the p^{th} job and the unloaded travel time to go from the first station of the p^{th} job to the entry station. The entry date of the $(p+1)^{th}$ job is delayed (figure 3) due to the transportation times of p^{th} job.

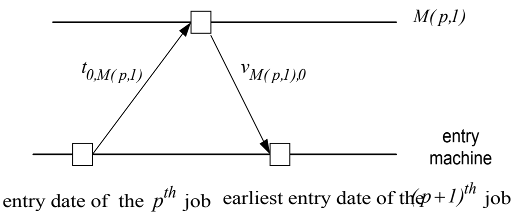

Fig. 3. Transportation time required between entry dates of job p and job $p+1$

So when $p < N$**,** the entry date of the $(p+1)^{th}$ job is grater than or equal to: $E(x_p) + t_{M(p,0),M(p,1)} + v_{M(p,1),M(p,0)}$.

<u>Second case:</u> $p = N$

Let us consider first that the ordered set of jobs at the system entrance is equal to the ordered set of jobs at the output machine.

The entry date of $(p+1)^{th}$ job is grater than or equal to the end date of the $(p+1-N)^{th}$ job. So $E(x_{p+1}) \geq DS(x_{p+1-N})$. Figure 4 provides an example for $N = 4, p = 5$.

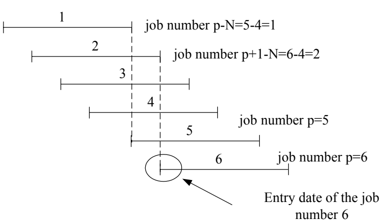

Fig. 4. Example for $N = 4, p = 5$

More complex situations can occur because each job has one type defining a list of machines with a processing time associated to the job and the machine. So the ordered set of output jobs could be different than the input ordered set of job at the system entrance.

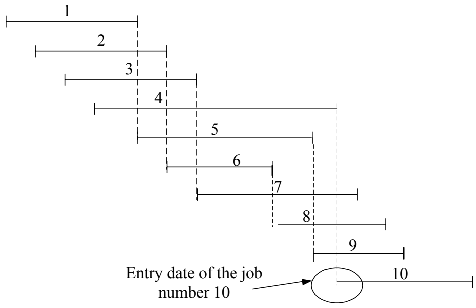

Fig. 5. Example for $N = 4, p = 9$

So the entry date of the $(p+1)^{th}$ is greater or equal to the lowest end date of the jobs which are in the system. So $DE(x_{p+1}) \geq \underset{\{i/DS(x_i)>DE(x_p)\}}{Min} DS(x_i)$.

Obviously, after the job p the unloaded and loaded vehicle travel time must be taken into account. Hence:

$$DE(x_{p+1}) \geq Max \begin{pmatrix} \underset{\{i/DS(x_i)>DE(x_p)\}}{Min} DS(x_i), \\ E(x_p) + t_{M(p,0),M(p,1)} + \\ \qquad v_{M(p,1),M(p,0)} \end{pmatrix}$$

Figure 5 provides an example for $N = 4, p = 9$. The entry date of the job number 10 is the end date of the job number 4.

Therefore, depending on the values of N and p, one can use one of two expressions presented above to the entry date of the job number $(p+1)$. A more concise expression for α_p is available taking into account the number of jobs in the system at the date $E(x_p)$. Let us note: $S_p = \{q / DS(x_q) > DE(x_p)\}$.

$|S_p|$ is the number of job in the system when the entry date of jobs p is reached. According to the limited input/output buffers capacity, $|S_p|$ can be less than or equal to N .

$$\alpha_p = \begin{cases} E(x_p) + t_{M(p,0),M(p,1)} + v_{M(p,1),M(p,0)} & \text{if } |S_p| < N \\ Max \begin{pmatrix} \underset{i \in S_p}{Min}\, DS(x_i), \\ E(x_p) + t_{M(p,0),M(p,1)} + \\ \qquad v_{M(p,1),M(p,0)} \end{pmatrix} & otherwise \end{cases}$$

4.2 Estimation of the time required to treat the remaining jobs of U_{n-p}

The total loaded time required to transport one job between the entry machine and the output station is: $\sum_{i=0}^{m_k-1} t_{M(k,i),M(k,i+1)}$. The total processing time required to treat the job is $\sum_{i=0}^{m_k} p_{i,k}$. So the minimal time required to treat the job is: $D_k = \sum_{i=0}^{m_k} p_{i,k} + \sum_{i=0}^{m_k-1} t_{M(k,i),M(k,i+1)}$. The following characterization of the makespan can be given. The time required to treat all the jobs of U_{n-p} can not be less than or equal to $\underset{i \in U_{n-p}}{Max}\ D_i$. The makespan is superior or equal to the total amount of time required to treat all the jobs of U_{n-p} (including loaded transportation and processing time) divided by the maximal number of tasks executable in parallel. The maximal number of parallel tasks depends of m_s' and N . But the transportation time have been included in the D_k expression. So the vehicle can be considered as a machine providing also operations in parallel. If $N > m_s' + 1$ then the number of machines (increased of one) limits the number of executable jobs in parallel, N limits the number of jobs executable in parallel. Therefore the makespan is greater than or equal to $\frac{\sum_{i \in U_{n-p}} D_i}{Min(N, m_s' + 1)}$ and

$$\nabla = Max\left(\frac{\sum_{i \in U_{n-p}} D_i}{Min(N, m_s' + 1)}, \underset{i \in U_{n-p}}{Max}\ D_i \right)$$ is an estimation of the time required to treat the jobs of U_{n-p} . It follows that:

<u>Lower Bound Theorem:</u>

$\overline{H}^p(x_n) = \alpha_P + \nabla$ with

$$\alpha_p = \begin{cases} E(x_p) + t_{M(p,0),M(p,1)} + v_{M(p,1),M(p,0)} & \text{if } |S_p| < N \\ Max\left(\begin{array}{l} \underset{i \in S_p}{Min}\ DS(x_i), \\ E(x_p) + t_{M(p,0),M(p,1)} + v_{M(p,1),M(p,0)} \end{array} \right) & otherwise \end{cases}$$

and $\nabla = Max\left(\frac{\sum_{i \in U_{n-p}} D_i}{Min(N, m_s' + 1)}, \underset{i \in U_{n-p}}{Max}\ D_i \right)$

is a lower bound of $H(x_n)$. This lower bound is noted LB_2 .

5. COMPUTATIONAL EXPERIMENTS

5.1 Network and data

For the procedure evaluation, four different layouts and five job sets taken from (Ulusoy and Bilge, 1993) have been studied.

For each layout four machines and one Input/Output station are used. Each machine has an input and output buffers with a limited capacity. In order to limit the magnitude of this study it is assumed that there are an equal number of places at each buffer. The buffer capacity is limited to two places. The Input/Output machine is assumed to have a sufficient capacity to store all jobs to be scheduled. The other characteristics of these systems are available in (Espinouse *et al.*, 2001).

5.2 Benchmark

Only results with the FIFO rule, Job-Set 1 and Topology 1 are details hereafter. The number of jobs simultaneously allowed in the system is 4, 6 and 8 jobs. The results are provided for 8 and 10 jobs to schedule at the system entrance. To evaluate the performances of the lower bounds, the following criteria have been taken into account:

- the total number of nodes generated;
- the number of nodes pruned;
- the level in the branch and bound process in which the nodes have been pruned;
- the computational times required for optimal solution computation.

All experiments have been performed on a Pentium III 600 MHz personal computer under Windows 95 Operating System using Delphi 5.0 programming environment.

<u>Table 1. Results obtained with LB_2</u>

n	N	Optimal solution	Number of nodes Generated	Pruned	Computational time
10	2	505	92 781	62 277	0'53
10	4	496	105 218	67 742	1'58
10	6	514	112 968	71 430	3'54
10	8	558	138 983	88 225	4'03
12	2	614	1 191 183	793 938	13'00
12	4	566	715 239	483 179	8'13
12	6	598	1 211 421	774 174	13'06
12	8	634	1 398 407	875 062	14'08

Table 1 and 2 prove the efficiency of LB_2 which permit to compute optimal solution in lowest computation time than LB_1 . The total number of generated nodes is less important using LB_2 . Table 3 provides a comparison of the Branch and Bound process under the two lower bounds. 12 jobs have to be scheduled and only 4 jobs are simultaneously allowed in the system.

Table 2 Results obtained with LB_1.

n	N	Optimal solution	Number of nodes Generated	Pruned	Computational time
10	2	505	116 388	74 617	1'50s
10	4	496	105 827	67 823	2'04s
10	6	514	112 968	71 430	3'54s
10	8	558	138 983	88 225	5'04s
12	2	614	1 719 281	1 065 834	27'50
12	4	566	812 022	535 594	10'48s
12	6	598	1 222 788	777 256	12'47s
12	8	634	1 399 509	875 129	15'12s

Table 3. Comparative study of LB_1 and LB_2 with 12 jobs to schedule

Level	LB_2	LB_1
1	0	0
2	0	0
3	0	0
4	0	0
5	0	0
6	0	0
7	**932**	0
8	**21 991**	0
9	107 948	5 765
10	195 995	298 090
11	145 437	221 185
12	10 876	10 554

LB_2 is highly efficient because nodes are pruned in highest level during the branch and bound process. In table 3 LB_2 prune nodes in level 7 (only 6 jobs have been scheduled before pruning) and LB_1 prunes nodes only at level 9. The lower bounds efficiency highly depends on the level at which the prune process is performed. The lower bound LB_2 locate efficiently the optimal solution after a few number of nodes. Table 4 highlights that a great percent of computational time is consumed to prove the solution optimality.

Table 4. Results obtained with LB_2

n	*N*	Total number of nodes generated	Localisation
10	2	92 781	10
10	4	105 218	14 582
10	6	112 968	87 225
10	8	138 983	19 043
12	2	1 191 183	64
12	4	715 239	87 633
12	6	1 211 421	887 184
12	8	1 398 407	573 074

6. CONCLUDING REMARKS

The problem of integrated job input sequencing and device dispatching in a general job-shop, where both buffer and resource capacity are limited, has been addressed using the branch and bound approach proposed by (Lacomme *et al.*, 2000). The proposed approach permits to take into account all the constraints of the problem. Previous lower bounds dedicated to this problem are not enough efficient and the lower bounds used in others scheduling problems required many investigations to take into account the problem characteristics.

The work presented here is a step to optimally solve large scale problems. This work has future useful extensions including:

- Heuristic and metaheuristic utilisation for upper bound computation
- Determination of lower bounds for 2 vehicles.

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