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EXTENDED CIRCLE CRITERION: STABLE FUZZY CONTROL OF A MILLING PROCESS.

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Abstract: In this work, the Extended Circle Criterion and fuzzy approaches are combined to develop a practical technique for checking stability of Fuzzy Control Systems (FCS). The controller is designed on the basis of skilled operator criteria and engineering knowledge about the process. Afterwards, stability is verified by means of the so-called Extended Circle Criterion, under the assumption that the static characteristic of the controller is sector-bounded (nonlinear portion) and the process (linear portion) can be rendered Strictly Positive Real (SPR). The method is tested in a FCS applied to the milling process. Theoretical and experimental results show that the closed loop system is stable. *Copyright ©2002 IFAC*
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1. INTRODUCTION.

Standard control-system design procedures require information about the dynamic properties of the plant. In case of nonlinear or parameter-uncertain plants it is not always possible to provide suitable mathematical models and to give an exact description of the dynamical behaviour within a wide range. This is the case of the so-called “complex systems”. A remedy when a detailed analytical plant model is missing is to collect the knowledge of skilled experts and to imbed it into a fuzzy controller (Driankov, *et al*., 1993). In fact, during these years several Fuzzy Control Systems (FCS) have been designed taking advantage of the knowledge of expert operators and, after implementation, a satisfactory dynamic performance, exhibited during real tests, has been frequently considered sufficient to ensure the stability.

Nevertheless, a basic requirement that has to be met by any control system is stability, and an analysis of this property needs a plant model. If there is available a linear approximation of the plant, that characterises its main dynamics at least in the vicinity of an operating point, then a test for local asymptotic stability becomes feasible: The feedback interconnection of the plant’s linear transfer function and the nonlinear fuzzy controller leads to a nonlinear standard control loop that can be analysed under the so-called Absolute Stability paradigm, e.g., with the Circle Criterion (Khalil, 1996).

The milling process (Schulz, 1999) is a system that falls into the above mentioned category of complex systems and hence can be treated in the described manner. The present contribution describes a fuzzy controller that is suitable to regulate this process, ensuring absolute stability with a finite domain (i.e., local asymptotic stability). The stability analysis is

performed on the basis of one version of the Circle Criterion: the Extended Circle Criterion reducing the problem to the scalar case (Xu, *et al*., 1996). In order to verify robust stability of the fuzzy control system, the gain of the plant is considered to be uncertain, and the allowed range for this uncertainty is maximised. Thereby it is shown that the stability domain can be maximised facing robustness aspects, and simulations demonstrate how this optimisation affects the control dynamics. A real-time experiment confirms that both, robust stability and suitable performance of the fuzzy control loop are also achievable in an industrial application.
In the next sections the main ideas of the machining process (Section 2), the fuzzy logic control (Section 3) and the absolute stability (Section 4) are outlined. Section 5 describes the application of the Extended Circle Criterion in order to check absolute stability of FCS and to provide suitable scaling factors for a robustly stable performance.

2. MODEL OF THE MILLING PROCESS

The machining process, also known as the metal removal process, is widely used in manufacturing. The reasons why the milling process has been selected as a case study in machining process research are the process's intrinsic nonlinear characteristics and the relatively poor performance of adaptive control systems for this process, in comparison with turning and drilling processes.

The dynamics of the milling process (cutting force response to changes in feed rate) can be approximately modeled using at least a second-order differential equation. A second-order model that relates cutting force F to feed rate f is reported in the literature (Lauderbaugh and Ulsoy, 1989)

$$\frac{d^2F}{dt^2} + 2\xi(t)\omega_n(t)\frac{dF}{dt} + \omega_n^2(t)F(t) = K_n(t)\omega_n^2(t)\cdot f(t) \quad (1)$$

where ξ is the damping ratio, ω_n is the natural frequency, and K_n is the process gain. The damping ratio ξ grows linearly with the depth of cut a and decreases slightly with spindle speed sp. The gain K_n varies non-linearly with the depth of cut and decreases slightly with the cutting speed. The natural frequency ω_n also varies depending on cutting parameters.

Facing both, nonlinear effects and varying parameters, it is not possible to build an exact analytical model of the plant; this is the reason why the control problem will in this contribution be addressed by a fuzzy controller based on an expert's knowledge. However, in order to characterise the main dynamical properties of the milling process, a rough locally valid approximation can be made following the procedure described in (Haber, *et al.*, 2001).

This model relates cutting force F to feed rate f considering an operating point (i.e., f_o=100 mm/min., sp_o=1000 rpm), it can be expressed as:

$$G(s) = \frac{2.614s + 188.3}{s^2 + 3.243s + 24.35} \quad (2)$$

This transfer function provides a coarse characterisation of the dynamic behaviour of the milling process with the aim of investigating machine tool performance, stability analysis and simulations**.**

3. FUZZY LOGIC CONTROL.

In order to obtain best quality of the workpiece's surface and for reducing the operating time, the cutting force should be kept constant during machining. Facing disturbances such as variable depth of cut in the workpiece, among others, the above-mentioned demand can only be met under control. Regarding the complexity of the milling process, the control task is solved applying a controller that is also nonlinear, whose design does not require any analytical model of the plant: a dynamic fuzzy controller.

Input variables for the dynamic fuzzy controller are the cutting force error ΔF in Newtons and the change in cutting force error $\Delta^2 F$ in Newtons. Both are included in the error vector. The output variable is the feed rate increment (Δf in percentage of the initial value).

$$\mathbf{e} = \begin{bmatrix} g_1 \cdot \Delta F & g_2 \cdot \Delta^2 F \end{bmatrix};\ uo = [\Delta f] \quad (3)$$

where g_1, g_2 are the scaling factors for inputs

The fuzzy partitions of the universes of discourse and the creation of the rule basis are drawn from the criteria of skilled experts. Membership functions having the forms of trapezoid-shaped functions are used here. The control surface can be seen in figure 1, considering that
$X_1 = g_1 \cdot \Delta F = [-150,150]$, $X_2 = g_2 \cdot \Delta^2 F = [-150,150]$ and $y = \Delta f = [-13,13]$.

These nonlinearities are essential in order to achieve good performance. Recently, (Ying, 1999) has proved that using trapezoidal membership functions, the resulting system is the sum of a global nonlinear controller (static part) and a local nonlinear PI-like controller (dynamically changing with regard to the input space). Therefore, it is expected that this kind of membership functions can be relevant for dealing with nonlinear process behavior.

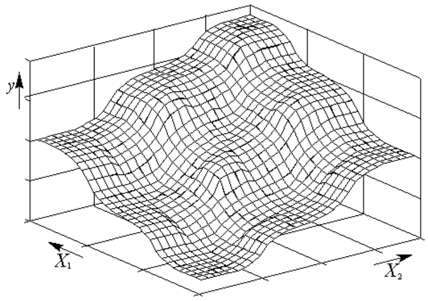

Fig. 1. Static Characteristic of a Two-input/One-output fuzzy controller.

The "Sup-Product" compositional operator and the center of gravity (COG) are selected as compositional rule of inference and defuzzification strategy, respectively (Haber *et al.*, 1998). The strategy used to compute *f* determines what type of fuzzy regulator is to be used, in this case PI-wise.

4. ABSOLUTE STABILITY.

The classical Lur'e problem assumes a stable linear system $G(s)$ in the feedforward path of a feedback scheme (see fig. 2):

$$\dot{x} = \mathbf{A}x + \mathbf{b} \cdot u; y = \mathbf{c}^{\mathbf{T}} x \quad (4)$$

$$u = -\phi(y) \quad (5)$$

$$G(s) = \mathbf{c}^{\mathbf{T}} \cdot (s\mathbf{I} - \mathbf{A})^{-1} \mathbf{b} \quad (6)$$

and a memoryless continuous nonlinear function $\phi(y)$ which fulfils the so called "sector condition"

$$\Phi_K = \{\phi(0) = 0; k_1 y^2 \le y \cdot \phi(y) \le k_2 y^2\} \quad (7)$$

where $x \in R^n, u, y \in R^p$, $(\mathbf{A}, \mathbf{b})$ is controllable, $(\mathbf{A}, \mathbf{c}^{\mathbf{T}})$ is observable, and $\Phi_K : R^1 \to R^1$. The system (6) fedback by k_1 must be asymptotically stable.

Under such assumptions, the Lur'e problem consist of studying stability of the closed loop system for any $\phi \in \Phi_K$. If (7) holds for all $y \in [-\infty, \infty)$ then the sector condition is said to hold globally. Nevertheless, if $y \in [b_1, b_2]$ then the sector condition holds only on a finite domain (locally). Therefore, the system (4)-(6) is absolutely stable if the origin is globally asymptotically stable for any nonlinearity in the given sector, or it is *absolute stable with a finite domain* if the origin is asymptotically stable for any nonlinearity that holds the sector condition in a finite domain (Khalil, 1996).

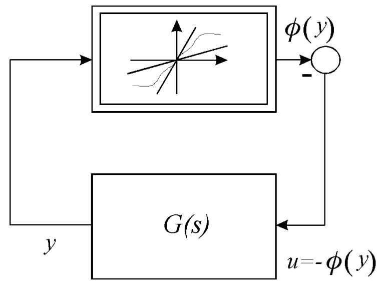

Fig. 2 Simplified structure of a closed-loop system.

4.1 Extended Circle Criterion.

The implementation of the Circle Criterion for single- input single-output systems is straightforward and it is closely related to the Nyquist criterion (Ray and Majumder, 1984). However, this not often the case and many fuzzy controllers (e.g., typically error and change in error) single-output controller. For this case it is necessary to make some modifications in order to preserve the absolute stability conditions, ensuring $\Phi_K : R^p \to R^p$.

Let the focus be on a MISO fuzzy controller with two-inputs and one-output. In this case, in order to hold absolute stability premises, there are two choices. One way is to modify the nonlinear portion such that $\Phi_K : R^2 \to R^2$ (Schmitt, 1999). Another way is to introduce some modifications in the control loop in order to guarantee that $y \in R^1$, and thus reducing the problem to the scalar case.

Let us consider the second option and r=0 for simplicity. Fig. 3b shows the scheme of a PI-like FCS resulting from the modification of the scheme of Fig. 3a by introducing the following term

$$G_m(s) = (g_1 + g_2 s)^{-1} \cdot \kappa \quad (8)$$
$$\kappa = \sqrt{g_1^2 + g_2^2}$$

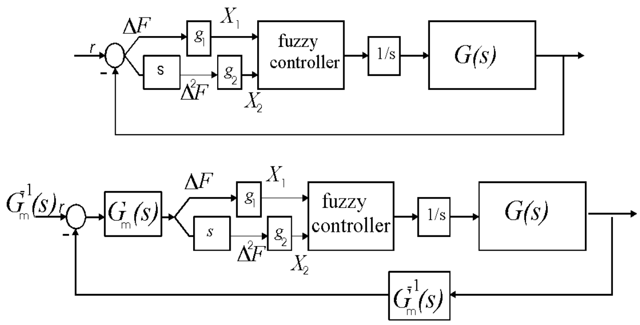

Fig. 3. a) PI FCS scheme b) modified scheme

An "extended sector condition" can be derived from (7) for the case of two-input one-output nonlinear function

$$k_1\sigma^2 \le \sigma\,\phi(g_1\Delta F, g_2\Delta^2 F) \le k_2\sigma^2$$
$$\sigma = \frac{g_1\Delta F + g_2\Delta^2 F}{\kappa}, \kappa = \sqrt{g_1^2 + g_2^2} \quad (9)$$

The upper and lower sector constants k_1, k_2 can be calculated as

$$k_1 = \inf_{\Delta F, \Delta^2 F} \frac{\kappa \cdot \phi(g_1\Delta F, g_2\Delta^2 F)}{g_1\Delta F + g_2\Delta^2 F}, k_2 = \sup_{\Delta F, \Delta^2 F} \frac{\kappa \cdot \phi(g_1\Delta F, g_2\Delta^2 F)}{g_1\Delta F + g_2\Delta^2 F} \quad (10)$$

It means to introduce a summing point at the controller input such that the input of the nonlinear portion (fuzzy controller) is considered as scalar and the linear part would be

$$\hat{G}(s) = (g_1 + g_2 G_2(s)) \cdot \frac{1}{s} \cdot G(s) \cdot \frac{1}{\kappa} \quad (11)$$

Theorem 1(Kitamura and Kurozumi, 1992): Taking the system of figure 3 ($r=0$) with the fuzzy controller satisfying the condition (9), let $\hat{G}(s)$ from (11) be completely controllable and observable and $\hat{G}(s)\cdot\left[1 + k_1 \cdot \hat{G}(s)\right]^{-1}$ be Hurwitz (k_1>0), and $g_1 > 0, g_2 > 0$. The system is absolutely stable if the Nyquist plot of $\hat{G}(s)$ satisfies the Circle Criterion

$$\mathrm{Re}\left[\frac{\hat{G}(j\omega)}{1 + k_1\hat{G}(j\omega)}\right] + \frac{1}{\gamma\cdot(k_2 - k_1)} \ge \varepsilon > 0 \quad (12)$$

where γ is the scaling factor for the sector size.

The *absolute stability with a finite domain* (i.e., local asymptotic stability) is claimed for due to the finite and bounded characteristics of physical variables (e.g., saturation in actuators). The intersection between the lower sector bound (line) and the saturation of the nonlinear portion defines the *finite domain*. On the other hand, a variation of the scaling factors g_1, g_2 modifies the control surface and consequently the lower and upper sector bounds.

4.2 Optimization Based on Extended Circle Criterion.

The first step is to verify whether a FCS is absolute stable with a finite domain or not. Another issue is to find appropriate scaling factor g_1, g_2 such that absolute stability with a finite domain with a maximum allowable sector size for any nonlinearity (γ_{MAX}) can be proved. Therefore, the value of γ can be optimized varying g_1, g_2, seeking for a maximum allowable sector size $\gamma\cdot(k_2 - k_1)$ for the control surface. Hence one can use this enlarged sector as a region of stability for uncertainty in the plant gain.

If it is considered that

$$G(s) = \frac{\alpha \cdot N(s)}{D(s)} \quad (13)$$
$$\alpha = \Gamma \cdot K_n$$

where, *G(s)* is the transfer function, *N(s)* is the numerator polynomial, *D(s)* is the denominator polynomial, K_n is the nominal process gain and $\Gamma \ge 1$ is an uncertainty in a loop gain meaning a variant plant gain.

Now, it is possible to put Γ in the nonlinear portion, so that the maximum allowed γ (γ_{MAX}) yields the upper bound of uncertainty in the loop gain. γ_{MAX} implies all uncertain loop gains such that absolute stability is given for

$$\forall \Gamma \in [1, \gamma_{MAX}] \Leftrightarrow 1 \le \Gamma \le \gamma_{MAX} \quad (14)$$

Considering the need to obtain a robustly stable performance of FCS, the scaling factors can be determined on the basis of some criterion. A variation of the scaling factors g_1 and g_2 modifies the control surface and consequently the lower and upper sector bounds k_1 and k_2. So the following algorithm is implemented in order to improve the impact of the input scaling factors on robust stability.

- Choose $0<g_1<1$ and $0<g_2<1$.
- Calculate k_1 and k_2 that depend on g_1 and g_2 - according to Eq. (10)
- Calculate

$$\varepsilon_{inf} = \inf_{\omega}\left(Re\left\{\frac{\hat{G}(j\omega)}{1 + k_1\hat{G}(j\omega)}\right\} - \varepsilon\right) \quad (15)$$

and check if $\frac{\hat{G}(s)}{1 + k_1\hat{G}(s)}$ is asymptotically stable.

- Choose

$$\frac{1}{\gamma} = -(k_2 - k_1)\varepsilon_{\inf} \quad (16)$$

this choice guarantees that (12) holds.

In order to get a maximum enlargement of the sector size γ_{MAX} the following constrained optimization over g_1 and g_2 can be made:

Goal: 1/γ→min **for** $\gamma \ge 1, 0 < g_1 < 1, g_2 = \sqrt{1 - g_1^2}$

The procedure for applying the Extended Circle Criterion in this work can be summarized as follows. Initially is checked that *G(s)* is completely controllable and completely observable and then, it is verified whether the resulting closed-loop system is stable or not. Considering unitary scaling factors $g_1 = g_2 = 1 \rightarrow \kappa = 1$ and using (9)-(10), the sector constants $k_1 = 0.0398$ and $k_2 = 0.1950$ are calculated. The maximum admissible values for the loop gain can be obtained from the condition (14). FCS is absolute stable with a finite domain for all $1 \le \Gamma \le 1.47$. The optimization gives $g_{1\text{opt}} = 0.977$, $g_{2\text{opt}} = 0.213$ and $\gamma = 2.57$. For this case one obtains $k_1 = 0.0266$ and $k_2 = 0.1946$, and absolute stability with a finite domain holds for all $1 \le \Gamma \le 2.57$.

5. VERIFICATION OF THE STABILITY.

In this section is verified the absolute stability of FCS using the Circle Criterion and a graphical method. Moreover, simulations and real time experiments are developed to evaluate the controller performance.

The procedure for applying the Extended Circle Criterion can be summarized as follows. Using the model of the milling dynamics (2) and applying the Extended Circle Criterion, the closed-loop stability can be verified. It is possible to verify whether the Nyquist plot lies outside the circle whose center is $C_e = \left(\frac{-(k_2 + k_1)}{2k_1}, 0 \right)$ and radius $r_a = \frac{(k_2 - k_1)}{2k_1}$. If it does, then the closed-loop system is absolutely stable. For both cases, all the Nyquist plots lie outside the corresponding circle (see figure 4).

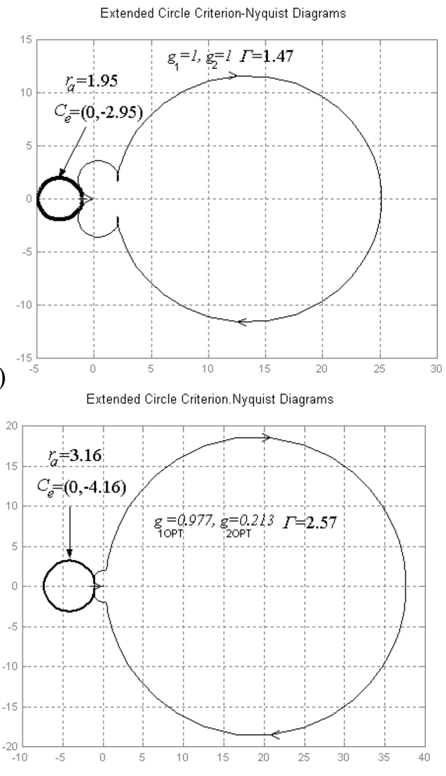

Fig. 4 Nyquist plots showing the application of Extended Circle Criterion.

5.1 Simulations and Real Time Experiments.

The model (2) is also used to simulate the closed-loop fuzzy control system with MATLAB/SIMULINK. The desired force level used in simulation is $F_r = 450\text{N}$. Corresponding F vs. t patterns are shown in figure 5.

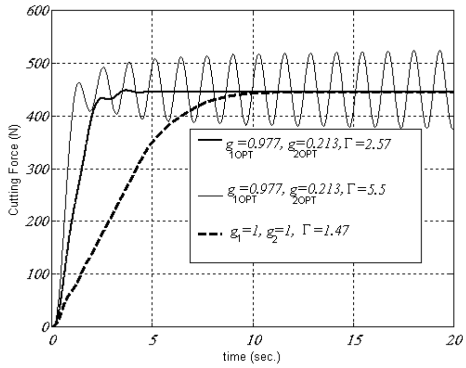

Fig. 5. Cutting-force step responses.

Digital simulations show that the closed-loop system is stable. However, limit cycles appear as Γ increases, in this case for $\Gamma > 3.1$. Theoretical results $1 < \Gamma \le 1.47$ are naturally conservative with regard to simulations, as the former assumptions for the stability for the arbitrary nonlinear function contained in the sector (9). Similar results are obtained for $g_{1opt} = 0.977, g_{2\text{opt}} = 0.213, \gamma = 2.57$, the system is stable and limit cycles appear for $\Gamma > 5.5$.

In order to measure F and to perform the generation of f values, a personal computer is added to the control scheme. The CNC-PLC, essential in any machine tool, guides the sequence of tool positions and/or the path of the tool during machining. Additionally, the fuzzy controller, implemented in the PC using C/C++ programming language, determines the proper feed rate (f). Other measurement devices such as sensors and an acquisition card (A/D device, filters etc.) were installed.

The experimental tests are conducted on a 20HP-4 axes milling machine, which is interfaced with a personal computer by a RS-232 communication link at 9600 bauds. The nominal cutting conditions are $sp = 1000\ rpm$ and $f = 100\ mm/min$ with one step disturbance of $8\,mm$ in the DOC. The set point is $F_r = 450\,N$. Spatial position of the cutting tool is fixed manually by the operator and for our experiments it is always at a constant vertical tool position (constant radial DOC). Machining is supposed to be done in one direction only. For the current experimental works, new milling cutters are used. A two-flutted milling tool 25*mm* in diameter is also chosen and the fuzzy controller is run on 7075 aluminum alloy.

In order to verify whether the behavior of the actual system is improved or not, another run is performed using the optimum scaling factors above obtained. The same cutting conditions are used.

Figure 6 shows the cutting force and feed rate responses for unitary input scaling factors, as well as for the input scaling factors attained after optimization. As can be seen, the fuzzy control system actually exhibits closed-loop stability, and the response is made significantly faster yielding a robust stable performance.

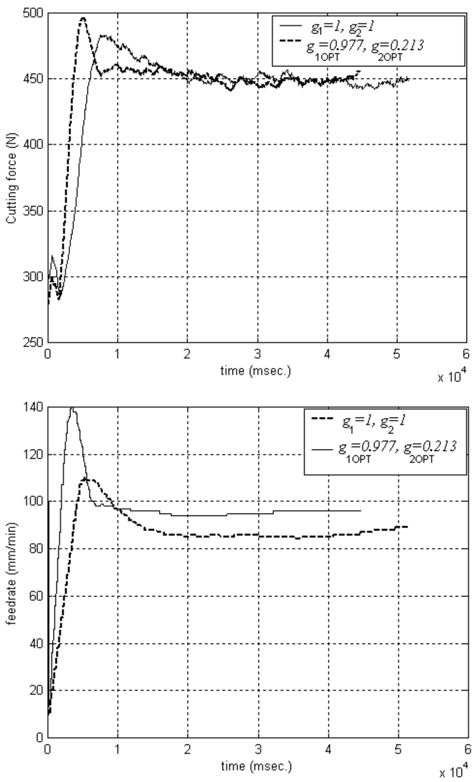

Fig. 6. Step response of the FCS.

6. CONCLUSIONS

In this paper sufficient conditions are determined for the absolute stability of a FCS, having the configuration illustrated in figure 2. The stability analysis is performed with the Extended Circle Criterion that allows dealing with multiple-input single-output fuzzy controllers.

The optimization of input scaling factors enables an enlargement of the sector size and therefore an increase of the range allowed for an uncertain plant gain. Furthermore this improvement concerning robust stability turns out to yield also a better performance of the fuzzy controller, and this was verified during experiments.

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