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OUTPUT REGULATION OF LINEAR SINGULAR SYSTEMS

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Abstract: In this paper, we discuss the output regulation problem of linear singular systems. Without the assumption that the singular system is regular, the singular system is transformed into a normal state-space system with a smaller order via algebraic elementary transformation. Then we investigate the output regulation problem of the normal state-space system directly to discuss that of the original singular system, and design the controllers which we require. *Copyright ©2002 IFAC*

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1. INTRODUCTION

The output regulation is a very important problem in the control theory and industrial applications. It can be stated as the following: Considering a system with the presence of input disturbance or external signals, it is desirable to find a controller so that the closed-loop system is internally stable and has desired properties such as (asymptotic) disturbance attenuation and signal tracking. The regulation problem for the normal state-space systems attracted much attention in the 1970's, and a rather complete regulation theory of linear state-space system was established during the period. However, there is material difference between singular systems and normal state-space systems, the former are much more complex than the later, especially with the impulse component, so it is more difficult to investigate the singular systems. Nevertheless, it is essential to study them, since there exits many singular systems in the large-scale natural systems, for instance, in power systems, net works (Lewis,1986). Singular systems have attracted attention of many researchers since the later 1970's (Bhattacharyra,1973; Dai,1989; Francis,1977; Wolovich and Ferreira,1979; Y.Chen, *et al.*,1996), and many valuable results are acquired. Especially, Wei Lin and Liyi Dai (Lin and Dai,1996) in 1996 researched the singular systems directly ,and derived a necessary and sufficient condition for this problem to be solvable. In addition, Chuanguo Chen (C.Chen, *et al.*,1996)also discussed it with Weiersstrass form in 1998. They had the assumption that the system is regular, and have to solve equations with a lager order. But this paper doesn't require the regularity of the singular system. Via algebraic elementary transformation, the singular system is converted into a normal state-space system. In this way, the dimension of the original system is reduced, and we can use the known standard results. Consequently, it is easier to solve this problem.

The paper is organized as follows: Section 2 gives the formal statement and transformations of the problem; Section 3 provides the action by the full information feedback; Section 4 by error feedback; in the two former sections, we derive a necessary and sufficient condition for the regulation problem of the resulting system, and design the controller we desire. Section 5 designs the controller of the original system; the brief conclusions are in Section 6.

2. PROBLEM STATEMENT AND TRANSFORMATIONS

Consider a linear singular system described by equations of the form

$$\begin{cases} E\dot{x} = Ax + Bu + Pw \\ \dot{w} = Sw \\ e = Cx + Qw \end{cases} \quad (1)$$

where $x \in R^n, u \in R^r, w \in R^s, e \in R^m$ are the state, the control input, and the exogenous input, the measurable error output respectively; $E, A \in R^{n\times n}, B \in R^{n\times r}, P \in R^{n\times s}, C \in R^{m\times n}, S \in R^{s\times s}, Q \in R^{m\times s}$ are constant matrices, rank$E = p < n$, the paper doesn't require the system(1) is regular, i.e. $\det(sE - A) \not\equiv 0$.

The problem can be stated as follows: as for the system(1), find feedback control $u(t)$ so that the closed-loop is internally stable, and for any $(x(0), w(0))$, the error signal $e(t)$ of the closed-loop system satisfies $\lim_{t\to\infty} e(t) = 0$, that is, achieves the purpose of tracking signal.

In order to convert the problem, we make assumptions as follows:

A1)$[E, A, B]$ is impulsive controllable.

A2)$[E, A, C]$ is impulsive observable.

Because rank$E = p < n$, it is known that there exist invertible matrices $M, N \in R^{n\times n}$, so that

$$MEN = \begin{bmatrix} I_p & 0 \\ 0 & 0 \end{bmatrix}, MAN = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$MB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad MP = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix},$$

$$CN = \begin{bmatrix} C_1 & C_2 \end{bmatrix}, \quad N^{-1}x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Where $A_{11} \in R^{p\times p}$, $B_1 \in R^{p\times r}$, $P_1 \in R^{p\times s}$, $x_1 \in R^p$, the others are matrices with appropriate dimensions. Then the system(1) is r.s.e(restricted system equivalent) the following system:

$$\begin{cases} \dot{x_1} = A_{11}x_1 + A_{12}x_2 + B_1u + P_1w \\ 0 = A_{21}x_1 + A_{22}x_2 + B_2u + P_2w \\ \dot{w} = Sw \\ e = C_1x_1 + C_2x_2 + Qw \end{cases} \quad (2)$$

By the following two different approaches, the system(2) is transformed into normal state-space system.

Approach 1)(J.Zhu,*et al.*,1999):

For $[E, A, B]$ is impulsive controllable, we know $\begin{bmatrix} A_{22} & B_2 \end{bmatrix}$ has full row rank, so there exist nonsingular matrices $H \in R^{(n-p+r)\times(n-p+r)}$ such that

$$\begin{bmatrix} A_{22} & B_2 \end{bmatrix} H = \begin{bmatrix} I_{n-p} & 0 \end{bmatrix}. \quad (3)$$

With the notation

$$\begin{bmatrix} A_{12} & B_1 \end{bmatrix} H := \begin{bmatrix} \overline{A}_{12} & \overline{B}_1 \end{bmatrix};$$

$$H := \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

where $H_{11} \in R^{(n-p)\times(n-p)}$,$H_{12}$,$H_{21}$,$H_{22}$,$\overline{A}_{12}$,$\overline{B}_1$ are appropriate matrices. we make full rank transformation

$$\begin{bmatrix} x_1 \\ x_2 \\ u \\ w \end{bmatrix} = \begin{bmatrix} I_p & 0 & 0 & 0 \\ 0 & H_{11} & H_{12} & 0 \\ 0 & H_{21} & H_{22} & 0 \\ 0 & 0 & 0 & I_s \end{bmatrix} \begin{bmatrix} x_1 \\ \overline{x}_2 \\ \overline{u} \\ w \end{bmatrix} \quad (4)$$

where $\overline{x}_2 \in R^{n-p}, \overline{u} \in R^r$.

Obviously, by the transformation (4), the system(2) is changed into

$$\begin{cases} \dot{x_1} = (A_{11} - \overline{A}_{12}A_{21})x_1 + \overline{B}_1\overline{u} \\ \quad +(P_1 - \overline{A}_{12}P_2)w & (5.1) \\ \dot{w} = Sw & (5.2) \\ e = (C_1 - C_2H_{11}A_{21})x_1 \\ \quad +(Q - C_2H_{11}P_2)w + C_2H_{12}\overline{u} & (5.3) \\ \overline{x}_2 = -A_{21}x_1 - P_2w & (5.4) \end{cases}$$

that is, the system (5) is a normal state-space system.

Approach 2): As well as $[E, A, C]$is impulsive observable, we know

$$\begin{bmatrix} A_{22} \\ C_2 \end{bmatrix}$$

has full column rank, so there exists a nonsingular matrix $K \in R^{r\times m}$ such that $A_{22} + B_2KC_2$ is invertible. Accordingly, by the output feedback $u = Ke + v$ in advance, the system (2) is reduced to

$$\begin{cases} \dot{x_1} = \overline{A}_{11}x_1 + \overline{B}_1v + \overline{P}_1w \\ \dot{w} = Sw \\ e = \overline{C}_1x_1 + \overline{Q}w - C_2\overline{B}_2v \\ x_2 = -[(\overline{A}_{21} + \overline{B}_2KC_1)x_1 \\ \quad +(\overline{B}_2KQ + \overline{P}_2)w + \overline{B}_2v] \end{cases} \quad (6)$$

where
$\overline{A}_{21} := (A_{22} + B_2KC_2)^{-1}A_{21}$,
$\overline{B}_2 := (A_{22} + B_2KC_2)^{-1}B_2$,
$\overline{P}_2 := (A_{22} + B_2KC_2)^{-1}P_2$,
$\overline{A}_{11} := A_{11} + B_1KC_1 - (A_{12} + B_1KC_2)$
$(\overline{A}_{21} + \overline{B}_2KC_1)$,
$\overline{B}_1 := B_1 - (A_{12} + B_1KC_2)\overline{B}_2$,
$\overline{P}_1 := B_1KQ + P_1 - (A_{12} + B_1KC_2)(\overline{B}_2KQ + \overline{P}_2)$,
$\overline{C}_1 := C_1 - C_2(\overline{A}_{21} + \overline{B}_2KC_1)$,
$\overline{Q} := Q - C_2(\overline{B}_2KQ + \overline{P}_2)$.

Obviously, the system(6) is also a normal state-space system.

Section 3 and 4 just discuss the output regulation problem of the system (5) and indirectly to solve that of the system(1).

As usual, the control action can be provided by the following forms:

(1) the full information feedback: $u = K_0x_1 + Lw \quad (*1)$, where $K_0 \in R^{r\times p}, L \in R^{r\times s}$ are constant matrices to seek.

(2) the error feedback: $\dot{\xi} = F\xi + Ge, v = R\xi \quad (*2)$, where $\xi \in R^{n_c}$ is the state of the controller, $F \in R^{n_c \times n_c}, G \in R^{n_c \times m}, R \in R^{r \times n_c}$are constant matrices to seek.

So the problem is transformed into: for the full information feedback, find K_0, Lsuch that the closed-loop system with the system(5) is internally stable,and for $(x_1(0), w(0))$, the error signal $e(t)$ satisfies $\lim_{t \to \infty} e(t) = 0$. Moreover, for the error feedback, find F, G, R such that the closed-loop system with the system(6) is internally stable, and for $(x_1(0), \xi(0), w(0))$, the error signal $e(t)$ also satisfies $\lim_{t \to \infty} e(t) = 0$.

To solve the problem, we make the following assumptions aside:

A3)$\sigma(S) \subset \overline{C}^+ =: \{\lambda \in C | \mathrm{Re}(\lambda) \geq 0\}$, it doesn't affect the output regulation, since the affection on the closed-loop system with internal stability caused by the disturbance can be neglected in the output regulation problem.

A4)$[E, A, B]$is R-stabilizable, i.e. $\mathrm{rank}[\lambda E - A \quad B] = n, \forall \lambda \in \overline{C}^+$.

A5)$\left(\begin{bmatrix} \overline{A}_{11} & \overline{P}_1 \\ 0 & S \end{bmatrix} \begin{bmatrix} \overline{C}_1 & \overline{Q} \end{bmatrix} \right)$ is detectable, $\forall \lambda \in \overline{C}^+$.

A6)rank $\begin{bmatrix} \lambda E - A & B \\ C & 0 \end{bmatrix} = n + m.$

The combination of A4), A5) and A6) is prerequisite condition to solve the very problem.

3. FULL INFORMATION FEEDBACK

This section uses the feedback control law of the form: $\overline{u} = K_0 x_1 + Lw$, the first approach is exploited here(of course, the second one done too), then putting this control law and the system (5)together, the closed-loop system is

$$\begin{cases} \dot{x_1} = (A_{11} - \overline{A}_{12} A_{21} + \overline{B}_1 K_0) x_1 \\ \quad +(\overline{B}_1 L + P_1 - \overline{A}_{12} P_2) w \\ \dot{w} = Sw \\ e = (C_1 - C_2 H_{11} A_{21} + C_2 H_{12} K_0) x_1 \\ \quad +(Q - C_2 H_{11} P_2 + C_2 H_{12} L) w \\ \overline{x}_2 = -A_{21} x_1 - P_2 w \end{cases} \tag{7}$$

Above all, a lemma is introduced.

Lemma 1(Dai,1989): Consider the system

$$\begin{cases} \dot{x} = Ax + Pw \\ \dot{w} = Sw \\ e = Cx + Qw \end{cases}$$

where $A \in R^{n \times n}, P \in R^{n \times s}, C \in R^{m \times n}, S \in R^{s \times s}, Q \in R^{m \times s}, x \in R^n, w \in R^s, e \in R^m$ as before.

Assume $\sigma(A) \subset C^-, \sigma(S) \subset \overline{C}^+$, then for $(x(0), w(0)), e(t)$ satisfies $\lim_{t \to \infty} e(t) = 0$ if and only if there exists a matrix $\Lambda \in R^{m \times n}$ such that

$$\begin{cases} \Lambda S = A\Lambda + P \\ 0 = C\Lambda + Q \end{cases}$$

Lemma 2: A4) holds if and only if $[A_{11} - \overline{A}_{12} A_{21} \;\; \overline{B}_1]$ is stabilizable.

Theorem 1: If A4)holds, the output regulation problem of the system (5) is solvable via the full information feedback if and only if there exist two matrices $\Lambda \in R^{p \times s}$ and $\Gamma \in R^{r \times s}$such that

$$\begin{cases} \Lambda S = (A_{11} - \overline{A}_{12} A_{21}) \Lambda + \overline{B}_1 \Gamma \\ \quad +P_1 - \overline{A}_{12} P_2 & (8) \\ 0 = (C_1 - C_2 H_{11} A_{21}) \Lambda + C_2 H_{12} \Gamma \\ \quad +Q - C_2 H_{11} P_2 & (9) \end{cases}$$

Proof: (Sufficiency)By Lemma 2, $[A_{11} - \overline{A}_{12} A_{21} \;\; \overline{B}_1]$ is stabilizable, so there exists K_0 such that

$$\sigma(A_{11} - \overline{A}_{12} A_{21} + \overline{B}_1 K_0) \subset C^-.$$

Using this K_0 and the solutions Λ, Γ to (8)-(9),construct the control law:

$$\overline{u} = K_0 x_1 + (\Gamma - K_0 \Lambda) w \tag{10}$$

(5) and (10) form the closed-loop system which is:

$$\begin{cases} \dot{x_1} = (A_{11} - \overline{A}_{12} A_{21} + \overline{B}_1 K_0) x_1 \\ \quad +[\overline{B}_1 (\Gamma - K_0 \Lambda) + P_1 - \overline{A}_{12} P_2] w \\ \dot{w} = Sw \\ e = (C_1 - C_2 H_{11} A_{21} + C_2 H_{12} K_0) x_1 \\ \quad +[Q - C_2 H_{11} P_2 + C_2 H_{12} (\Gamma - K_0 \Lambda)] w \end{cases}$$

Apparently,the matrices above satisfy:

$$\begin{cases} \Lambda S = (A_{11} - \overline{A}_{12} A_{21} + \overline{B}_1 K_0) \Lambda \\ \quad +\overline{B}_1 (\Gamma - K_0 \Lambda) + P_1 - \overline{A}_{12} P_2 \\ 0 = (C_1 - C_2 H_{11} A_{21} + C_2 H_{12} K_0) \Lambda + Q \\ \quad -C_2 H_{11} P_2 + C_2 H_{12} (\Gamma - K_0 \Lambda) \end{cases}$$

By Lemma 1, for any$(x_1(0), w(0)), \lim_{t \to \infty} e(t) = 0$.

Thus the control law (10)solves the output regulation problem of the system (5).

(Necessity): by Lemma 1, it holds.

4. ERROR FEEDBACK

In this section, we deal with the error feedback control law: $\dot{\xi} = F\xi + Ge, v = R\xi$, the second approach can only be adopted.Under this control law, the closed-loop system for the system(6) is:

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{\xi} \end{bmatrix} = A_c \begin{bmatrix} x_1 \\ \xi \end{bmatrix} + P_c w \\ \dot{w} = Sw \\ e = \begin{bmatrix} \overline{C}_1 & C_2 \overline{B}_2 R \end{bmatrix} \begin{bmatrix} x_1 \\ \xi \end{bmatrix} + \overline{Q} w \end{cases} \tag{11}$$

With the notion

$$A_c := \begin{bmatrix} \overline{A}_{11} & \overline{B}_1 R \\ G\overline{C}_1 & F - GC_2\overline{B}_2 R \end{bmatrix},$$

$$P_c := \begin{bmatrix} \overline{P}_1 \\ G\overline{Q} \end{bmatrix} \tag{12}$$

Lemma 3: For the closed-loop system(11), suppose $\sigma(A) \subset C^-$, $\sigma(S) \subset \overline{C^+}$, then for any $(x_1(0), \xi(0), w(0)) \in R^p \times R^{n_c} \times R^s$, $\lim_{t\to\infty} e(t) = 0$ if and only if there exists $\Lambda \in R^{(p+n_c)\times s}$ such that

$$\begin{cases} \Lambda S = \begin{bmatrix} A_{11} - A_{12}(\overline{A}_{21} + \overline{B}_2 KC_1) & (B_1 - A_{12}\overline{B}_2)R \\ 0 & F \end{bmatrix}\Lambda + \begin{bmatrix} P_1 - A_{12}(\overline{B}_2 KQ + \overline{P}_2) \\ 0 \end{bmatrix} \\ 0 = \begin{bmatrix} \overline{C}_1 & C_2\overline{B}_2 R \end{bmatrix}\Lambda + \overline{Q}. \end{cases} \tag{13}$$

Lemma 4: A4) holds if and only if $(\overline{A}_{11}, \overline{B}_1)$is stabilizable.

Theorem 2: If A4), A5) hold, it is solvable of the output regulation problem of the system (6) via error feedback if and only if there exist two matrices $\Lambda \in R^{p\times s}$ and $\Gamma \in R^{r\times s}$ such that:

$$\begin{cases} \Lambda S = [A_{11} - A_{12}(\overline{A}_{21} + \overline{B}_2 KC_1)]\Lambda + \\ (B_1 - A_{12}\overline{B}_2)\Gamma + P_1 - A_{12}(\overline{B}_2 KQ + \overline{P}_2) & (14) \\ 0 = \overline{C}_1\Lambda_1 - C_2\overline{B}_2\Gamma + \overline{Q} & (15) \end{cases}$$

Proof: (Sufficiency) By Lemma 4, the system(6) is stabilizable, so there exists a matrix K_0 such that

$$\sigma(\overline{A}_{11} + \overline{B}_1 K_0) \subset C^- \tag{i}$$

In addition, A5) holds, there exist $G = \begin{bmatrix} G_0^\tau & G_1^\tau \end{bmatrix}^\tau$, $G_0 \in R^{p\times m}, G_1 \in R^{s\times m}$such that

$$\sigma\left(\begin{bmatrix} \overline{A}_{11} & \overline{P}_1 \\ 0 & S \end{bmatrix} - \begin{bmatrix} G_0 \\ G_1 \end{bmatrix}\begin{bmatrix} \overline{C}_1 & \overline{Q} \end{bmatrix}\right) \subset C^-. \tag{ii}$$

Using the matrices K, G and the solutions Λ, Γ to(14)-(15), we construct the controller

$$\dot{\xi} = F\xi + Ge, v = R\xi. \tag{16}$$

where $F := \begin{bmatrix} \overline{A}_{11} - G_0\overline{C}_1 + \overline{B}_1 K_0 + G_0 C_2\overline{B}_2 K_0 & \overline{P}_1 - G_0\overline{Q} + \overline{B}_1(\Gamma - K_0\Lambda) + G_0 C_2\overline{B}_2(\Gamma - K_0\Lambda) \\ -G_1\overline{C}_1 + G_1 C_2\overline{B}_2 K_0 & S - G_1\overline{Q} + G_1 C_2\overline{B}_2(\Gamma - K_0\Lambda) \end{bmatrix}$,

$R := \begin{bmatrix} K_0 & \Gamma - K_0\Lambda \end{bmatrix}$.

then the closed-loop system together the system(6) is

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{\xi} \end{bmatrix} = \overline{A}_c\begin{bmatrix} x_1 \\ \xi \end{bmatrix} + \overline{P}_c w \\ \dot{w} = Sw & (iii) \\ e = \begin{bmatrix} \overline{C}_1 & C_2\overline{B}_2 R \end{bmatrix}\begin{bmatrix} x_1 \\ \xi \end{bmatrix} + \overline{Q}w \end{cases}$$

where $\overline{A}_c := \begin{bmatrix} \overline{A}_{11} & \overline{B}_1 K_0 & \overline{B}_1(\Gamma - K_0\Lambda) \\ G_0\overline{C}_1 & \overline{A}_{11} - G_0\overline{C}_1 + \overline{B}_1 K_0 & \overline{P}_1 - G_0\overline{Q} + \overline{B}_1(\Gamma - K_0\Lambda) \\ G_1\overline{C}_1 & -G_1\overline{C}_1 & S - G_1\overline{Q} \end{bmatrix}$, $\overline{P}_c := \begin{bmatrix} \overline{P}_1 \\ G_0\overline{Q} \\ G_1\overline{Q} \end{bmatrix}$

Firstly, to verify the stability of (iii),because

$$\det(\lambda I - \overline{A}_c) = c\det(\lambda I - (\overline{A}_{11} + \overline{B}_1 K_0))\bullet \det\left(\lambda I - \begin{bmatrix} \overline{A}_{11} - G_0\overline{C}_1 & \overline{P}_1 - G_0\overline{Q} \\ -G_1\overline{C}_1 & S - G_1\overline{Q} \end{bmatrix}\right)$$

and (i), (ii) get $\sigma(\overline{A}_c) \subset C^-$, where c are constant scalars. So (iii) is stable.

Secondly, to prove that for any $(x_1(0), \xi(0), w(0))$, $\lim_{t\to\infty} e(t) = 0$.

To this end, setting $\Lambda_0 = \begin{bmatrix} \Lambda^\tau & I_m \end{bmatrix}^\tau$, Λ, Γ satisfy (14)-(15), then

$$F\Lambda_0 = \begin{bmatrix} \overline{A}_{11}\Lambda + \overline{P}_1 + \overline{B}_1\Gamma \\ -G_1\overline{C}_1\Lambda_1 + S - G_1\overline{Q} + G_1 C_2\overline{B}_2\Gamma \end{bmatrix}$$

$$= \begin{bmatrix} \Lambda S \\ S \end{bmatrix} = \Lambda_0 S,$$

By the definition of R, (14)-(15) is

$$\begin{cases} \Lambda S = [A_{11} - A_{12}(\overline{A}_{21} + \overline{B}_2 KC_1)]\Lambda + P_1 \\ +(B_1 - A_{12}\overline{B}_2)R\Lambda_0 - A_{12}(\overline{B}_2 KQ + \overline{P}_2) \\ 0 = \overline{C}_1\Lambda_1 - C_2\overline{B}_2 R\Lambda_0 + \overline{Q} \end{cases}$$

therefore

$$\begin{cases} \begin{bmatrix} \Lambda \\ \Lambda_0 \end{bmatrix} S = \begin{bmatrix} P_1 - A_{12}(\overline{B}_2 KQ + \overline{P}_2) \\ 0 \end{bmatrix} + \begin{bmatrix} \overline{A} & (B_1 - A_{12}\overline{B}_2)R \\ 0 & F \end{bmatrix}\begin{bmatrix} \Lambda \\ \Lambda_0 \end{bmatrix} \\ 0 = \begin{bmatrix} \overline{C}_1 & C_2\overline{B}_2 R \end{bmatrix}\begin{bmatrix} \Lambda \\ \Lambda_0 \end{bmatrix} + \overline{Q}. \end{cases}$$

where $\overline{A} := A_{11} - A_{12}(\overline{A}_{21} + \overline{B}_2 KC_1)$, noting $\Lambda^* = \begin{bmatrix} \Lambda^\tau & \Lambda_0^\tau \end{bmatrix}^\tau$, Λ^* satisfy(13). By Lemma 3, for any $(x_1(0), \xi(0), w(0)) \in R^p \times R^{n_c} \times R^s$, $\lim_{t\to\infty} e(t) = 0$.

(Necessity): By Lemma 3, it holds.

Remark: 1) Theorem 1 and Theorem 2 offer the sufficient and necessary conditions to solve the output regulation problem of the system(5) and (6) via full information feedback and error feedback respectively, that is, solving two algebraic equations. So another way is given to deal with the problem.

2) In the sufficiency of Theorem 1 and Theorem 2, we construct the form of controllers, the right controllers of the system(5) and (6) can be designed according to them.

Now, the problem is changed into that of discussing the solvable condition of the two algebraic equations.

Lemma 5(C.Chen,*et al.*,1999): There exist matrices $\Lambda \in R^{n\times s}$ and $\Gamma \in R^{r\times s}$ such that

$$\begin{cases} \Lambda S &= A\Lambda + B\Gamma + P \\ 0 &= C\Lambda + D\Gamma + Q \end{cases}$$

if and only if

$$\text{rank}\begin{bmatrix} \lambda I - A & B \\ C & D \end{bmatrix} = n+m, \forall\lambda \in \sigma(S),$$

i.e. $\begin{bmatrix} \lambda I - A & B \\ C & D \end{bmatrix}$ has full row rank.

where $A \in R^{n\times n}, B \in R^{n\times r}, P \in R^{n\times s}, C \in R^{m\times n}, S \in R^{s\times s}, Q \in R^{m\times s}, D \in R^{m\times s}$.

It is evident that by Lemma 5, the sufficient condition of the output regulation problem of the system(5)is

$\text{rank}\begin{bmatrix} \lambda I - (A_{11} - \overline{A}_{12}A_{21}) & \overline{B}_1 \\ C_1 - C_2H_{11}A_{21} & C_2H_{12} \end{bmatrix} p+m, \forall\lambda \in \sigma(S)$.

That of the system(6) is

$$\text{rank}\begin{bmatrix} \lambda I - (A_{11} - A_{12}(\overline{A}_{21} + \overline{B}_2KC_1)) & B_1 - A_{12}\overline{B}_2 \\ \overline{C}_1 & -C_2\overline{B}_2 \end{bmatrix} = p+m, \forall\lambda \in \sigma(S).$$

Lemma 6: For any $\forall\lambda \in \sigma(S)$.

$$\begin{bmatrix} \lambda I - (A_{11} - \overline{A}_{12}A_{21}) & \overline{B}_1 \\ C_1 - C_2H_{11}A_{21} & C_2H_{12} \end{bmatrix}$$

has full row rank or

$$\begin{bmatrix} \lambda I - (A_{11} - A_{12}(\overline{A}_{21} + \overline{B}_2KC_1)) & B_1 - A_{12}\overline{B}_2 \\ \overline{C}_1 & -C_2\overline{B}_2 \end{bmatrix}$$

has full row rank if and only if A6)holds.

We conclude from the above proof that the sufficient condition of the output regulation problem of the system(5) and (6)is

Theorem 3: If A1), A4), A6)hold, the output regulation problem of the system(5) is solvable via the full information feedback.

Theorem 4: If A1), A2), A4), A5), A6) hold, the output regulation problem of the system (6) is solvable via error feedback.

Remark: By the above discussion, we settle down the output regulation problem of the system (5) and (6). The following problem is that how to design the control u according to $\overline{u}$ and v.

5.TO DESIGN THE CONTROL u

The previous sections have solved the output regulation problem of the inverted systems, and got the control law in the sufficiency of Theorem 1 and Theorem 2. In this section, we obtain the control u of the system(1) by some equivalent transformations.

1) Full information feedback: By (4), (5.4), (10), we get

$$\begin{aligned} u &= \begin{bmatrix} H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \overline{x}_2 \\ \overline{u} \end{bmatrix} \\ &= \begin{bmatrix} H_{21} & H_{22} \end{bmatrix} \\ &\left(\begin{bmatrix} A_{21} & 0 \\ K_0 & 0 \end{bmatrix} N^{-1}x + \begin{bmatrix} P_2 \\ \Gamma - K_0\Lambda \end{bmatrix} w \right) \\ &:= K_1x + L_1w. \end{aligned}$$

2) Error feedback: By $u = Ke + v$and(16), we have

$$\begin{cases} u &= Ke + R\xi \\ \dot{\xi} &= F\xi + Ge \end{cases}$$

where F, G, R are chosen in Theorem 2.

Now, we verify the above feedback really realizes the output regulation of the system (1).

First of all, we testify the error feedback controller is the anticipant one.

Proof : First, to prove the stability of the closed-loop system.

$$\begin{aligned} &\det\begin{pmatrix} \lambda E - (A + BKC) & -BR \\ -GC & \lambda I - F \end{pmatrix} \\ &= \det\begin{pmatrix} \lambda I_p - (A_{11} + B_1KC_1) & -(A_{12} + B_1KC_2) & -B_1R \\ -(A_{21} + B_2KC_1) & -(A_{22} + B_2KC_2) & -B_2R \\ -GC_1 & -GC_2 & \lambda I - F \end{pmatrix} \\ &= \delta\det(\lambda I - (\overline{A}_{11} + \overline{B}_1K_0)) \\ &\det\left(\lambda I - \begin{bmatrix} \overline{A}_{11} - G_0\overline{C}_1 & \overline{P}_1 - G_0\overline{Q} \\ -G_1\overline{C}_1 & S - G_1\overline{Q} \end{bmatrix}\right) \end{aligned}$$

where δ is constant,

$\sigma\left(\begin{bmatrix} \overline{A}_{11} & \overline{P}_1 \\ 0 & S \end{bmatrix} - \begin{bmatrix} G_0 \\ G_1 \end{bmatrix} \begin{bmatrix} \overline{C}_1 & \overline{Q} \end{bmatrix}\right) \subset C^-$ and $\sigma(\overline{A}_{11} + \overline{B}_1K_0) \subset C^-$.

so $\sigma\left(\begin{bmatrix} E & 0 \\ 0 & I_{p+s} \end{bmatrix}, \begin{bmatrix} A + BKC & BR \\ GC & F \end{bmatrix}\right) \subset C^-$.
By[3], for any $(x(0), \xi(0), w(0))$, $\lim\limits_{t\to\infty} e(t) = 0$.

It suffices to prove there exist Σ, Ω such that

$$\begin{cases} E\Sigma S &= A\Sigma + P + BR\Omega \\ \Omega S &= F\Omega \\ 0 &= C\Sigma + Q \end{cases}$$

i.e. to verify there exist Σ, Ω such that

$$\begin{cases} MENN^{-1}\Sigma S &= MANN^{-1}\Sigma + MP + MBR\Omega \\ \Omega S &= F\Omega \\ 0 &= CNN^{-1}\Sigma + Q \end{cases}$$

Defining $N^{-1}\Sigma := \begin{bmatrix} \Sigma_1 \\ \Sigma_2 \end{bmatrix}$, The above are

$$\begin{cases} \Sigma_1 S &= A_{11}\Sigma_1 + A_{12}\Sigma_2 + P_1 + B_1R\Omega & (a) \\ 0 &= A_{21}\Sigma_1 + A_{22}\Sigma_2 + P_2 + B_2R\Omega & (b) \\ \Omega S &= F\Omega & (c) \\ 0 &= C_1\Sigma_1 + C_2\Sigma_2 + Q & (d) \end{cases}$$

Setting $\Sigma_1 = \Lambda, \Sigma_2 = -(\overline{A}_{21} + \overline{B}_2KC_1)\Lambda - \overline{B}_2 - (\overline{B}_2KQ + \overline{P}_2), \Omega = \begin{bmatrix} \Lambda^\tau & I_m \end{bmatrix}^\tau$, where Λ, Γ

satisfy (14)-(15). It is apparent that (a),(c),(d)are (14),(16),(15) respectively.

Next, to prove (b) holds. By (d), it is only need to prove

$$(A_{21} + B_2KC_1)\Sigma_1 + (A_{22} + B_2KC_1)\Sigma_2$$
$$+P_2 + B_2R\Omega + B_2KQ = 0$$

Using the expression of Σ_2, the above holds. So (b) holds. Up to now, the above equations hold in turn.

Analogously, the full information feedback control law is also the desired one.

In this way, we solve the output regulation problem of the singular system (1) thoroughly.

6.CONCLUSIONS

This paper discuss the output regulation problem of the linear singular system. The important idea here is that under the immaterial assumptions, the singular system can be transformed into the normal state-space system with a lower order via the algebraic elementary transformation, thus we can use the existing results. So the output regulation problem of the original system can be solved indirectly by investigating that of the normal state-space system directly. At last,the problem is solved.

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