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PRACTICAL STABILIZATION OF A CLASS OF NONLINEAR PLANTS WITH UNCERTAIN INPUT AND OUTPUT NONLINEARITIES.

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AbstractThis paper addresses the stabilization problem of "sandwich systems", i.e. intrinsically nonlinear and uncertain SIMO plants containing simultaneoulsy either backlash and dead zone in the actuator and dead zones in sensors. The proposed controllers, based on sliding mode control, have been shown to achieve state (hence output) boundedness. The rationale followed in control design consists in ensuring the achievement of a sliding motion which, in turn, guarantees the attraction of the state vector towards a boundary layer, whose maximum width depends on the uncertainty on output measurements caused by the presence of unknown deadzones in sensors. Furthermore, the control laws have been designed as to simultaneously guarantee also the avoidance of actuator nonlinearities, ensuring that the 'forbidden region' of dead zones and backlash are never entered, even in the presence of uncertainties. Simulation results show the effectiveness of the proposed controllers.

Keywords: Robust control, Actuator nonlinearities, Sensors Nonlinearities.

1. INTRODUCTION

Control design techniques usually applied in practice do often ignore the presence of nonsmooth nonlinearities both in plant actuators and sensors. Due to physical imperfections, indeed, such nonlinearities are always present in real plants, particularly in mechanical systems. Just to name a few, mechanical connections, hydraulic servovalves and electric servomotors are known to contain backlash: instability or position errors in gear train are often caused by an amount of backlash greater than that necessary to ensure satisfactory meshing of gears (Campos *et al.*, 2000). Moreover, dry friction or stiction is a common source of deadzone nonlinearities in electromechanical systems, and temperature changes on the surface of these components can produce relevant variations of deadzone effects (Tsang and Li, 2001). Robot arms, in particular, have been found to lack in steady-state positioning accuracy as a result of a number of different sources, among which backlash in the joint drive train (Ahmad, 1988). Finally, proportional-derivative controllers have been observed to produce limit cycles if the actuators contain nonlinearities such as backlash and deadzones (Campos *et al.*, 2000).

Although often neglected, these nonlinearities are particularly harmful, because they usually lead to deterioration of system performance. As discussed in (Tao and Kokotovic, 1996), "Actuator and sensor nonlinearities are among the key factors limiting both static and dynamic performance of feedback control systems". They are the causes of oscillations, delays and inaccuracy: for example, servomechanisms usually require complete elimination of backlash to work properly.

A number of techniques are available in the literature to compensate for nonlinearities present in the actuator only. Starting from the pioneering work by

Recker et al. (Recker *et al.*, 1991), the idea of employing an adaptive inverse of the nonlinearity itself in the controller in order to cancel its effects has been widely used to cope with actuator deadzones, backlash and hysteresis with unknown parameters (Tao and Kokotovic, 1996) (Grundelius and Angeli, 1996) (Tao and Kokotovic, 1994*b*) (Tao and Kokotovic, 1995*b*) (Tao and Kokotovic, 1995*a*). These papers all require that the plant is linear. Continuous-time dynamic inversion using neural networks is presented in (Seidl *et al.*, 1998), (Selmic and Lewis, 1999), (Selmic and Lewis, 1998), while fuzzy logic is used in (Kim *et al.*, 1994), (Lewis *et al.*, 1997), (Woo *et al.*, 1998). Variable Structure Control (VSC) has been used as well: in (Azenha and Machado, 1996), a linear plant is considered, and a describing function based model is adopted for the input nonlinearities. In these papers, intrinsically nonlinear plants are addressed. Very recently, the fusion of relay feedback control with robust nominal model following control has been used and experimentally tested to handle actuator deadzone nonlinearities (Tsang and Li, 2001).

Nonsmooth nonlinearities affecting only the plant output have been less widely addressed in the literature. Experimental evidence, however, shows that the performance of control systems is severely affected, as in the case of actuator nonlinearities. For example, errors in robot arms positioning accuracy are due, among other reasons, to the fact that joint sensors are located in the actuator rather than on the joint (Ahmad, 1988), the presence of a backlash gap in gears causing the output motion not to directly follow the input motion. In this framework, results based on adaptive control are available (Tao and Kokotovic, 1994*a*) (Tao and Kokotovic, 1996) as well as compensation techniques by disturbance observers (Shahruz, 2000). Moreover, a RLS algorithm avoiding nonlinearity inversion holding for deadzones in sensors is described in (Wigren and Nordsio, 1999). In all cases, a plant linearity assumption is still required.

From the above discussion, it follows that the simultaneous presence of nonsmooth nonlinearities both in actuators and in sensors should be accounted for. Hence, the so called "sandwich systems" are, at the moment, a challenging issue for control researchers. Moreover, a noticeable uncertainty should be taken into account in nonlinearities models parameters, in order to match a suf£ciently wide set of real situations.

In this paper, an alternative robust control method, based on Sliding Mode Control (SMC) (Utkin, 1992), is proposed to handle sandwich systems containing simultaneously either backlash and dead zone in the actuator and dead zones in sensors. Nonsmooth nonlinearities have been described using piece-wise linear functions (Tao and Kokotovic, 1996) (Desoer and Shahruz, 1986), and intrinsically nonlinear and uncertain SIMO plants are addressed. Model inversion is not required, to avoid the possible ampli£cation of

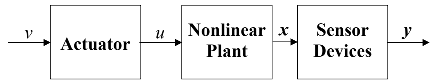

Fig.1 - Block scheme of the plant.

additive measurement disturbances which may result from inversion of the output nonlinearity.

The robust stabilizing sliding mode control laws presented guarantee the boundedness of the system state variable. The control policy adopted, indeed, ensures that the state vector is attracted towards a boundary layer whose maximum width depends on the uncertainty on output measurements caused by the presence of unknown deadzones in sensors. On the contrary, actuator nonlinearities are simply avoided, ensuring that the 'forbidden region' of dead zones and backlash are never entered, even in the presence of uncertainties. Note that the presence of the boundary layer is also crucial for avoiding the high frequency oscillations typical of SMC.

2. PRELIMINARIES AND PROBLEM STATEMENT

An uncertain nonlinear system of the following form is given:

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= h(\mathbf{x},\mathbf{p}) + g(\mathbf{x},\mathbf{p})u \\ \mathbf{y} &= \phi(\mathbf{x}) \end{cases} \tag{2.1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector and $u \in \mathbb{R}$ is the plant input. The vector $\mathbf{p} \in P \subseteq \mathbb{R}^p$, representing plant physical parameters, is assumed to vary within a closed and bounded subset $P \subseteq \mathbb{R}^p$. The function $g(\mathbf{x},\mathbf{p}) : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}$ is the state-input map, $h(\mathbf{x},\mathbf{p}) : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}$ describes the intrinsic plant nonlinearity and possible disturbances. Both functions $g(\mathbf{x},\mathbf{p})$ and $h(\mathbf{x},\mathbf{p})$ are assumed to be smooth with respect to their arguments.

As in many realistic situations, the system state vector is supposed not accessible for direct measurement. The output of n sensors are available instead, the vector $\mathbf{y}$ containing such measurements (see Fig.1).

The nonlinear vectorial function $\phi(\mathbf{x}) = [\phi_1(x_1)\ \phi_2(x_2)\ \ldots\ \phi_n(x_n)]^T$ describes the unknown nonlinearities appearing in sensors. Dead-zone nonlinearities are assumed present, described by the following characteristics:

Sensor Dead Zone:

$$y_i = \phi_i(x_i) = \begin{cases} m_r^{(i)}(x_i - b_r^{(i)}) & if \quad x_i \geq b_r^{(i)} \\ 0 & if \ -b_l^{(i)} < x_i < b_r^{(i)} \\ m_l^{(i)}(x_i + b_l^{(i)}) & if \quad x_i \leq -b_l^{(i)} \end{cases}$$

$$i = 1 \ldots n \tag{2.2}$$

The nonlinear system (2.1) is supposed to be preceded by the actuating device $u = f(v)$ (see Fig.1), u being the plant input not available for control. In this paper dead-zone or backlash nonlinearities have been considered to be present in the actuator:

Actuator Dead Zone:

$$u = f(v) = \begin{cases} \mu_r(v - \beta_r) & if \quad v \geq \beta_r \\ 0 & if \ -\beta_l < v < \beta_r \\ \mu_l(v + \beta_l) & if \quad v \leq -\beta_l \end{cases} \qquad (2.3)$$

Actuator Backlash: A compact description of backlash is (Tao and Kokotovic, 1995*b*):

$$\dot{u} = G(u, v, \dot{v}) = \begin{cases} \nu\dot{v} \ if \ \dot{v} > 0 \ and \ u = \nu(v - c_r), \ or & (2.4a) \\ \quad if \ \dot{v} < 0 \ and \ u = \nu(v - c_l) & \\ 0 \quad otherwise & (2.4b) \end{cases}$$

The idea pursued in this paper is to design sliding mode control laws able to achieve robust performances in the presence of the above non-smooth nonlinearities with uncertainties.

De£ne the following sliding surface:

$$s(\mathbf{x}) = c_n \frac{d^{n-1}x_1}{dt^{n-1}} + c_{n-1}\frac{d^{n-2}x_1}{dt^{n-2}} + \ldots + c_1 x_1 = = \sum_{i=1}^{n} c_i x_i = 0 \qquad (2.5)$$

where the coef£cients c_i, $i = 1 \ldots n$ are such that the polynomial $\sum_{i=1}^{n} c_i \lambda^{i-1}$ is Hurwitz. Due to this latter condition, without loss of generality it can be assumed that $c_i > 0 \ \forall i = 1 \ldots n$.

The following Assumptions are introduced:

Assumption 2.1. The function $g(\mathbf{x}, \mathbf{p})$ is such that:

$$g(\mathbf{x}, \mathbf{p}) \neq 0 \quad \forall \mathbf{x} \in \mathbb{R}^n, \forall \mathbf{p} \in P \qquad (2.6)$$

Due to the smoothness of $g(\mathbf{x}, \mathbf{p})$, it can be assumed, without loss of generality, that $g(\mathbf{x}, \mathbf{p}) > 0 \quad \forall \mathbf{x} \in \mathbb{R}^n, \forall \mathbf{p} \in P$.

Assumption 2.2. The coef£cients of the actuator dead zone and backlash nonlinearities are uncertain, with uncertainties bounded by known constants, i.e.:

- Actuator Dead zone:

$$\begin{aligned} \mu_r &= \hat{\mu}_r + \Delta\mu_r, \ |\Delta\mu_r| \leq \rho_{\mu r} \\ \mu_l &= \hat{\mu}_l + \Delta\mu_l, \ |\Delta\mu_l| \leq \rho_{\mu l} \\ \beta_r &= \hat{\beta}_r + \Delta\beta_r, \ |\Delta\beta_r| \leq \rho_{\beta r} \\ \beta_l &= \hat{\beta}_l + \Delta\beta_l, \ |\Delta\beta_l| \leq \rho_{\beta l} \end{aligned} \qquad (2.7a)$$

- Actuator Backlash:

$$\begin{aligned} \nu &= \hat{\nu} + \Delta\nu, \ |\Delta\nu| \leq \rho_\nu \\ c_r &= \hat{c}_r + \Delta c_r, \ |\Delta c_r| \leq \rho_{cr} \\ c_l &= \hat{c}_l + \Delta c_l, \ |\Delta c_l| \leq \rho_{cl} \end{aligned} \qquad (2.7b)$$

Assumption 2.3. The coef£cients $b_r^{(i)}$, $b_l^{(i)}$, $i = 1 \ldots n$ of the sensor dead zone nonlinearities are uncertain, with uncertainties bounded by known constants, i.e.:

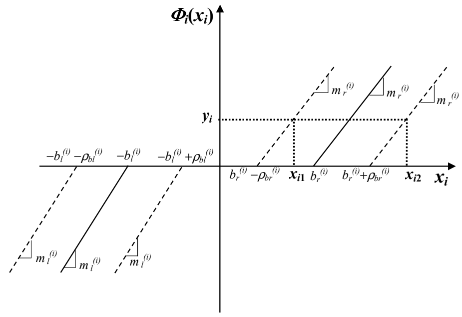

Fig.2 - Variation bounds of x_i.

$$\begin{aligned} b_r^{(i)} &= \hat{b}_r^{(i)} + \Delta b_r^{(i)}, \ |\Delta b_r^{(i)}| \leq \rho_{br}^{(i)} \\ b_l^{(i)} &= \hat{b}_l^{(i)} + \Delta b_l^{(i)}, \ |\Delta b_l^{(i)}| \leq \rho_{bl}^{(i)} \\ i &= 1 \ldots n \end{aligned} \qquad (2.8)$$

Note that the above Assumptions are not restrictive, and do hold for a wide class of nonlinear plants, especially mechanical systems.

In this paper the following problem is addressed:

Problem 1. The problem here considered consists in £nding a feedback controller guaranteeing bounded state variables (hence bounded outputs) for the system (2.1), in the case where the nonlinearity of the actuating device is either a dead zone or a backlash, and dead zone sensor nonlinearities are present, too.

3. PLANT WITH SANDWICH DEAD ZONE/DEAD ZONE NONLINEARITIES.

In this section we consider the plant (2.1) with the dead zone nonlinearity appearing in both actuator and sensors.

De£ne the following quantities for $i = 1 \ldots n$:

$$\hat{b}_i = \begin{cases} \hat{b}_r^{(i)} & if \ y_i > 0 \\ -\hat{b}_l^{(i)} & if \ y_i < 0 \end{cases} \qquad (3.1a)$$

$$m_i = \begin{cases} m_r^{(i)} & if \ y_i > 0 \\ m_l^{(i)} & if \ y_i < 0 \end{cases} \qquad (3.1b)$$

$$\Delta b_i = \begin{cases} \Delta b_r^{(i)} & if \ y_i > 0 \\ -\Delta b_l^{(i)} & if \ y_i < 0 \end{cases} \qquad (3.2)$$

$$\rho_{b1}^{(i)} = \begin{cases} -\rho_{br}^{(i)} & if \ y_i > 0 \\ -\rho_{bl}^{(i)} & if \ y_i < 0 \end{cases} \qquad \rho_{b2}^{(i)} = -\rho_{b1}^{(i)} \qquad (3.3)$$

$$x_{i1} = \begin{cases} \dfrac{y_i}{m_i} + \hat{b}_i + \rho_{b1}^{(i)} & if \ y_i \neq 0 \\ -\rho_{bl}^{(i)} - \hat{b}_l^{(i)} & if \ y_i = 0 \end{cases} \qquad (3.4)$$

$$x_{i2} = \begin{cases} \dfrac{y_i}{m_i} + \hat{b}_i + \rho_{b2}^{(i)} & if \ y_i \neq 0 \\ \rho_{br}^{(i)} + \hat{b}_r^{(i)} & if \ y_i = 0 \end{cases} \qquad (3.5)$$

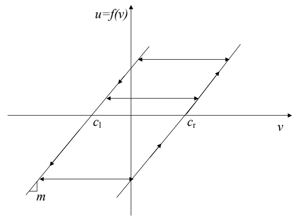

Fig.3 - Backlash model.

Note that the expressions (3.4), (3.5) are the variation bounds (due to the bounded uncertainties in the dead-zone parameters $b_l^{(i)}$, $b_r^{(i)}$) of the i-th component of the state vector $\mathbf{x}$, obtained in correspondence of the measured i-th entry y_i of the sensors output vector $\mathbf{y}$ (see Fig.2).

Let $Q \subseteq \mathrm{I\!R}^n$ be the box delimited by the above bounds, i.e.:

$$Q = \{\mathbf{x} \in \mathrm{I\!R}^n : x_i \in [x_{i1}\ x_{i2}]\}, \quad i = 1 \dots n \quad (3.6)$$

Taking into account (2.2), (2.8), expressions (3.4) and (3.5) become, respectively:

$$x_{i1} = \begin{cases} x_i - \Delta b_i + \rho_{b1}^{(i)} & if \ \ y_i \neq 0 \\ -\rho_{bl}^{(i)} - b_l^{(i)} & if \ \ y_i = 0 \end{cases} \quad (3.7)$$

$$x_{i2} = \begin{cases} x_i - \Delta b_i + \rho_{b2}^{(i)} & if \ \ y_i \neq 0 \\ \rho_{br}^{(i)} + b_r^{(i)} & if \ \ y_i = 0 \end{cases} \quad (3.8)$$

De£ne also the following sets: $\mathcal{I} = \{1 \dots n\}$, $\mathcal{J} = \{j \in \mathcal{I} : y_j \neq 0\}$, $\mathcal{K} = \{k \in \mathcal{I} : y_k = 0\}$. The following Lemma can now be proved.

Lemma 2. The sign of $s(\mathbf{x})$ can be uniquely determined from the sensors output vector $\mathbf{y}$ at any time instant when $s(\mathbf{x})$ is outside a bounded region described by the following inequalities:

$$\begin{cases} s(\mathbf{x}) > -\epsilon_2 \\ s(\mathbf{x}) < \epsilon_1 \end{cases} \quad (3.9)$$

with:

$$\epsilon_1 = 2 \sum_{j \in \mathcal{J}} c_j |\rho_{b1}^{(j)}| + \sum_{k \in \mathcal{K}} c_k \left(b_r^{(k)} + \rho_{br}^{(k)} + b_l^{(k)} + \rho_{bl}^{(k)} \right) \quad (3.10)$$

$$\epsilon_2 = 2 \sum_{j \in \mathcal{J}} c_j \rho_{b2}^{(j)} + \sum_{k \in \mathcal{K}} c_k \left(b_r^{(k)} + \rho_{br}^{(k)} + b_l^{(k)} + \rho_{bl}^{(k)} \right) \quad (3.11)$$

The proof is omitted for brevity.

The main result about the practical stabilization of the plant (2.1) with dead zone nonlinearities in both actuation and sensing devices can now be stated after the introduction of the following de£nitions.

Given a measurement vector $\mathbf{y}$ and the corresponding box $Q \subseteq \mathrm{I\!R}^n$ de£ned in (3.6), the following quantities can be introduced:

$$s^{min}(\mathbf{x}) \stackrel{\text{def}}{=} \sum_{i=1}^{n} c_i x_{i1} \quad s^{max}(\mathbf{x}) \stackrel{\text{def}}{=} \sum_{i=1}^{n} c_i x_{i2}$$

$$h_M \stackrel{\text{def}}{=} \max_{\mathbf{x} \in Q,\ \mathbf{p} \in P} |h(\mathbf{x}, \mathbf{p})| \quad (3.12)$$

$$\sigma_M \stackrel{\text{def}}{=} \max_{\mathbf{x} \in Q} \sum_{i=1}^{n-1} |c_i x_{i+1}| \quad (3.13)$$

$$g_M \stackrel{\text{def}}{=} \max_{\mathbf{x} \in Q,\ \mathbf{p} \in P} g(\mathbf{x}, \mathbf{p}), \quad g_m \stackrel{\text{def}}{=} \min_{\mathbf{x} \in Q,\ \mathbf{p} \in P} g(\mathbf{x}, \mathbf{p})$$

Finally, using (3.12), (3.13) de£ne:

$$\rho_M = \sigma_M + c_n h_M \quad (3.14)$$

Theorem 3. The system described by (2.1),(2.2),(2.3) is given, under Assumptions 2.1, 2.2 and 2.3. The following control law:

$$v = \begin{cases} \theta_1 \cdot \dfrac{\eta + \rho_M + c_n g_M (\hat{\mu}_r + \rho_{\mu r})(\hat{\beta}_r + \rho_{\beta r})}{c_n g_m (\hat{\mu}_r - \rho_{\mu r})} \\ \qquad if \ s^{max}(\mathbf{x}) < 0 \qquad (3.15a) \\ -\theta_2 \cdot \dfrac{\eta + \rho_M + c_n g_M (\hat{\mu}_l + \rho_{\mu l})(\hat{\beta}_l + \rho_{\beta l})}{c_n g_m (\hat{\mu}_l - \rho_{\mu l})} \\ \qquad if \ s^{min}(\mathbf{x}) > 0 \qquad (3.15b) \\ 0 \qquad otherwise \qquad (3.15c) \end{cases}$$

where $\eta > 0$ and $\theta_1 > 1$, $\theta_2 > 1$, ensures the achievement of a sliding motion on (2.5) with a boundary layer whose maximum width is given by (3.9)-(3.11), i.e. guarantees the boundedness of the system state variables.

The proof is omitted for brevity.

4. PLANTS WITH SANDWICH BACKLASH/DEAD ZONE NONLINEARITIES.

In this section it is considered the plant (2.1) having a backlash nonlinearity in the actuator and dead zones in sensors.

The following model of backlash (Desoer and Shahruz, 1986) (Corradini and Orlando, 2002), equivalent to (2.4), will be used for design in the following.

De£ne Σ_B as the set of states of the backlash model, i.e. the set of all the points in or between the lines of slope m (see Fig.3). For any point $p_k = (v_k, u_k) \in \Sigma_B$ at any time t_k de£ne two functions $F_i(\cdot, v_k, u_k, t_k)$: $[v_k, \infty) \rightarrow [u_k, \infty)$ and $F_d(\cdot, v_k, u_k, t_k)$: $(-\infty, v_k] \rightarrow (-\infty, u_k]$:

$$F_i(v, v_k, u_k, t_k) = \begin{cases} u_k & v_k < v < \dfrac{u_k}{\nu} + c_r \\ \nu(v - c_r) & \dfrac{u_k}{\nu} + c_r \leq v \end{cases} \quad (4.1a)$$

$$F_d(v, v_k, u_k, t_k) = \begin{cases} \nu(v - c_l) & v \leq \dfrac{u_k}{\nu} + c_l \\ u_k & \dfrac{u_k}{\nu} + c_l < v \leq v_k \end{cases} \quad (4.1b)$$

The characteristic of backlash can be de£ned as follows: for any state $(v_{k-1}, u_{k-1}) \in \Sigma_B$ at any time t_{k-1} and for any input v monotone over $[t_{k-1}, t_k]$ the output $u(t)\ \forall t \in [t_{k-1}, t_k]$ is given by:

$$u(t) = \begin{cases} F_i(v(t); v_{k-1}, u_{k-1}; t_{k-1}) \\ F_d(v(t); v_{k-1}, u_{k-1}; t_{k-1}) \end{cases} \tag{4.2}$$

according to whether $v(\cdot)$ is monotonically increasing or decreasing respectively. Therefore, for any initial state, at any time t and for piecewise continuous input v, the backlash output is uniquely determined.

De£ne the following quantities:

$$v_l = \frac{u_k}{\nu} + c_l \qquad v_r = \frac{u_k}{\nu} + c_r \tag{4.3}$$

Under Assumption 2.2, a minimum value for v_l and a maximum value for v_r can be found in the time interval $t \in [t_{k-1}, t_k)$. Denote these values with v_l^{min} and v_r^{max} respectively:

$$v_l^{min} = \min_{t\in[t_{k-1},t_k)} v_l(t) \qquad v_r^{max} = \max_{t\in[t_{k-1},t_k)} v_r(t)$$

The following Theorem can now be stated.

Theorem 4. The system described by (2.1),(2.2),(2.4) is given, under Assumptions 2.1, 2.2 and 2.3. The following control law:

$$v = \begin{cases} \theta_1 \cdot \dfrac{\eta + \rho_M + c_n g_M(\hat{\nu} + \rho_\nu)(\hat{c}_r + \rho_{cr})}{c_n g_m(\hat{\nu} - \rho_\nu)} \\ \quad if\ s^{max}(\mathbf{x}) < 0 \qquad (4.4a) \\ -\theta_2 \cdot \dfrac{\eta + \rho_M + c_n g_M(\hat{\nu} + \rho_\nu)(\hat{c}_l + \rho_{cl})}{c_n g_m(\hat{\nu} - \rho_\nu)} \\ \quad if\ s^{min}(\mathbf{x}) > 0 \qquad (4.4b) \\ 0 \qquad otherwise \qquad (4.4c) \end{cases}$$

where:

$$\theta_1 \geq \max\left\{1, \frac{(\hat{\nu} - \rho_\nu) c_n g_m v_r^{max}}{\eta + \rho_M + c_n g_M(\hat{\nu} + \rho_\nu)(\hat{c}_r + \rho_{cr})}\right\} \tag{4.5a}$$

$$\theta_2 \geq \max\left\{1, \frac{(\hat{\nu} - \rho_\nu) c_n g_m v_l^{min}}{\eta + \rho_M + c_n g_M(\hat{\nu} + \rho_\nu)(\hat{c}_l + \rho_{cl})}\right\} \tag{4.5b}$$

ensures the achievement of a sliding motion on (2.5) with a boundary layer whose maximum width is given by (3.9)-(3.11), i.e. guarantees the boundedness of the system state variables.

The proof is omitted for brevity.

5. SIMULATION RESULTS

To support the theoretical discussion with simulation data, the described control algorithm has been applied by simulation on the mechanical system proposed in (Lewis *et al.*, 1997) representing a robot-like system with one link [1]. Due to the large presence of mechanical components, indeed, robotics can be in fact considered a key £eld where the robust compensation of actuator and sensors nonlinearities should be pursued, particularly if the intrinsically uncertain knowledge of the plant model parameters is also considered.

Considering as state variables the angular displacement $x_1 = \theta$ and its time derivative $x_2 = \dot{\theta}$, the following system model is obtained:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\alpha_1 x_2 + \alpha_2 x_2^2 \cos(x_1) - \alpha_3 \sin(x_1) + u \end{cases}$$

where $\alpha_i = \hat{\alpha}_i + \Delta\alpha_i$, $i = 1, \ldots, 3$ are uncertain parameters whose nominal values are given by $\hat{\alpha}_1 = \dfrac{1}{T}$, $\hat{\alpha}_2 = \bar{m}a$, $\hat{\alpha}_3 = \bar{m}ga$, being $\bar{m}$ the load mass, T the motor time constant, a the length and g the gravitational constant. As in (Lewis *et al.*, 1997) the following nominal values have been used: $T = 1$ s, $\bar{m} = 1$ kg, $a = 3.5$ m. Note that in (Lewis *et al.*, 1997) the system coef£cients have been considered exactly known, and no uncertain terms have been added. The system is supposed to be driven by a control input u, which, in turn, is the output of a block containing an actuator nonlinearity (see Fig.1). The variable actually available for control is therefore the input v of this latter block. As far as sensors are concerned, the outputs of n sensors, containing dead zone nonlinearities, are assumed available (see Fig.1), the system state vector being not accessible for direct measurement. The Assumptions 2.2, 2.3 are supposed satis£ed, with $\hat{\mu}_r = 1$, $\hat{\mu}_l = 0.7$, $\hat{\beta}_r = 0.1 = \hat{\beta}_l = 0.1$, $\hat{\nu} = 1$, $\hat{c}_r = 0.1$, $\hat{c}_l = -0.1$, $m_r^{(i)} = 1$, $m_l^{(i)} = 0.7$, $\hat{b}_r^{(i)} = \hat{b}_l^{(i)} = 0.1$, $i = 1, 2$. A 15% variation has been applied both to the parameters α_i, $i = 1 \ldots 3$ and to all the coef£cients appearing in the mathematical description of the considered actuator and sensors nonlinearities, except for $m_r^{(i)}$, $m_l^{(i)}$ (according to Assumption 2.3). Simulations have been performed choosing as initial conditions $x_1(0) = 0.5$, $x_2(0) = 0$. With reference to the case of backlash appearing in the actuator and deadzone affecting sensors (*backlash/deadzone*), the controller of Theorem 4 has been used, with $\eta = 0.01$, $c_1 = 0.1$, $c_2 = 1.5$. The results are reported in Fig. 4-5 (state variables)and in Fig.6 (variable v).

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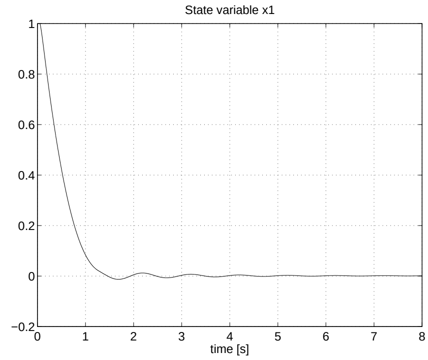

Fig.4 - Backlash / Dead zone: State variable x_1.

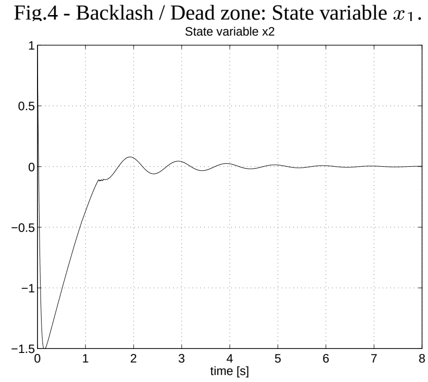

Fig.5 - Backlash / Dead zone: State variable x_2.

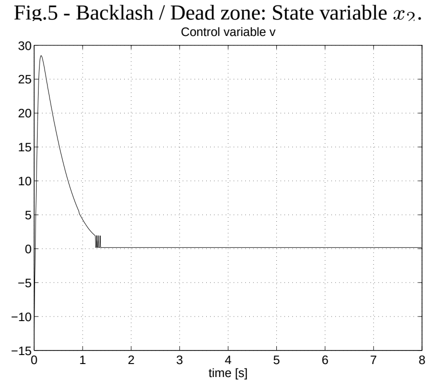

Fig.6 - Backlash / Dead zone: control variable v.

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