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STABILIZATION AND ROBUST CONTROL OF METAL ROLLING MODELED AS A 2D LINEAR SYSTEM

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Abstract: Repetitive processes are a distinct class of 2D linear systems with applications in areas ranging from long-wall coal cutting and metal rolling operations through to iterative learning control schemes. The main feature which makes them distinct from other classes of 2D linear systems is that information propagation in one of the two independent directions only occurs over a finite duration. This, in turn, means that a distinct systems theory must be developed for them, which can then be translated (if appropriate) into efficient routinely applicable controller design algorithms for applications domains. In this paper, we give some new results on LMI based stabilization and robust control of so-called discrete linear repetitive processes and illustrate them by application to a metal rolling process.

Keywords: 2D linear systems, metal rolling, robust control.

1. INTRODUCTION

The essential unique characteristic of a repetitive, or multipass, process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. This, in turn, leads to the unique control problem for these processes in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass to pass direction.

To introduce a formal definition, let $\alpha < +\infty$ denote the pass length (assumed constant). Then in a repetitive process the pass profile y_k, $0 \le t \le \alpha$, generated on pass k acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile $y_{k+1}(t)$, $0 \le t \le \alpha$, $k \ge 0$.

Physical examples of repetitive processes include long-wall coal cutting and metal rolling operations (Edwards, 1974; Smyth, 1992). Also in recent years applications have arisen where adopting a repetitive process setting for analysis has distinct advantages over alternatives. Examples of these so-called algorithmic applications of repetitive process theory include classes of iterative learning control schemes (Amann *et al.*, 1998) and iterative algorithms for solving non-linear dynamic optimal control problems based on the maximum principle (Roberts, 2000).

Attempts to control these processes using standard (or 1D) systems theory/algorithms fail (except in a few very restrictive special cases) precisely because such an approach ignores their inherent 2D systems structure, i.e. information propagation occurs from pass to pass and along a given pass, and the pass initial conditions are reset before the start of each new pass. In seeking a rigorous foundation on which to develop a control theory for these processes it is natural to attempt to exploit structural links which exist between, in particular, the class of so-called discrete linear repetitive processes and 2D discrete linear systems described by the extensively studied Roesser and Fornasini Marchesini state space models (for background on these models see, for example, the relevant references in (Rogers and Owens, 1992; Rogers *et al.*, 2002)). Discrete linear repetitive processes are distinct from such 2D linear systems in the sense that information propagation in one of the two independent directions (along the pass) only occurs over a finite duration.

In this paper, we first introduce the essential unique features of repetitive processes and, in particular, so-called discrete linear repetitive processes which are the subject of this paper by modeling of a simple metal rolling operation. Following this, new results on LMI based stabilization and robust control of this class of repetitive processes will be given and applied to a representative example of the dynamics which can arise in metal rolling operations.

2. METAL ROLLING AS A REPETITIVE PROCESS

Metal rolling is an extremely common industrial process where, in essence, deformation of the workpiece takes place between two rolls with parallel axes revolving in opposite directions. One approach here is to pass the stock (i.e. the metal to be rolled to a pre-specified thickness (also termed the gauge or shape)) through a series of rolls for successive reductions which can be 'costly' in terms of the equipment required. A more economic route is to use a single two high stand, where this process is often termed 'clogging' (see (Edwards, 1974)).

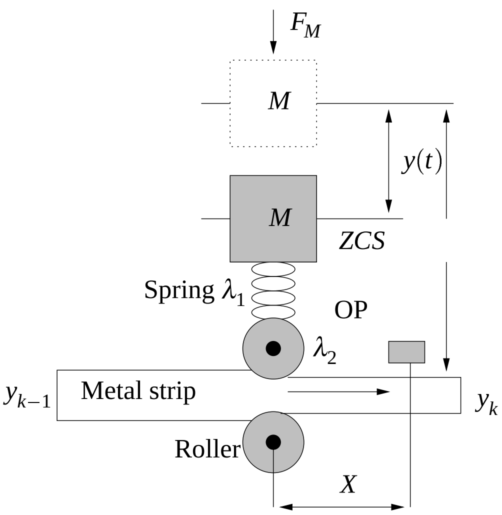

Fig. 1. Metal rolling process

In practice, a number of models of this process can be developed depending on the assumptions made on the underlying dynamics and the particular mode of operation under consideration. Here we consider the dynamics of the case shown schematically in Figure 1, where ZCS denotes the zero compression separation and OP the output sensor. The particular task here is to develop a simplified (but practically feasible) model relating the gauge on the current and previous pass through the rolls. These are denoted here by $y_k(t)$ and $y_{k-1}(t)$ respectively and the other process variables and physical constants are defined as follows:
$F_M(t)$ is the force developed by the motor;
$F_s(t)$ is the force developed by the spring;
M is the lumped mass of the roll-gap adjusting mechanism;
λ_1 is the stiffness of the adjustment mechanism spring;
λ_2 is the hardness of the metal strip;
$\lambda = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$ is the composite stiffness of the metal strip and the roll mechanism.

To model the basic process dynamics, refer to Fig 1 where the force developed by the motor is

$$F_M(t) = F_s(t) + M\ddot{y}(t), \tag{1}$$

(where $y(t)$ is defined in Fig. 1) and the force developed by the spring is given by

$$F_s(t) = \lambda_1[y(t) + y_k(t)]. \tag{2}$$

This last force is also applied to the metal strip by the rolls and hence

$$F_s(t) = \lambda_2[y_{k-1}(t) - y_k(t)]. \tag{3}$$

Hence the following linear differential equation models the relationship between $y_k(t)$ and $y_{k-1}(t)$ under the above assumptions

$$\begin{aligned}\ddot{y}_k(t) + \frac{\lambda}{M}y_k(t) &= \frac{\lambda}{\lambda_1}\ddot{y}_{k-1}(t) + \frac{\lambda}{M}y_{k-1}(t)\\ &\quad - \frac{\lambda}{M\lambda_2}F_M(t).\end{aligned} \tag{4}$$

Suppose now that differentiation in (4) is approximated by backward difference discretization with sampling period T. (For analysis and discussion of the choice of the sampling period to discretize differential linear repetitive processes see, for example, (Galkowski *et al.*, 1999)). Then the resulting difference equation is

$$\begin{aligned}y_k(t) &= a_1 y_k(t-T) + a_2 y_k(t-2T)\\ &\quad + a_3 y_{k-1}(t) + a_4 y_{k-1}(t-T)\\ &\quad + a_5 y_{k-1}(t-2T) + bF_M(t),\end{aligned} \tag{5}$$

where

$$a_1 = \frac{2M}{\lambda T^2 + M},\ a_2 = \frac{-M}{\lambda T^2 + M}$$
$$a_3 = \frac{\lambda}{\lambda T^2 + M}\left(T^2 + \frac{M}{\lambda_1}\right),\ a_4 = \frac{-2\lambda M}{\lambda_1(\lambda T^2 + M)}$$
$$a_5 = \frac{\lambda M}{\lambda_1(\lambda T^2 + M)},\ b = \frac{-\lambda T^2}{\lambda_2(\lambda T^2 + M)}$$

Now set $t = pT$ and $y_k(p) = y_k(pT)$. Then (5) can be written as

$$x_k(p+1) = Ax_k(p) + Bu_k(p) + B_0 y_{k-1}(p)$$
$$y_k(p) = Cx_k(p) + Du_k(p) + D_0 y_{k-1}(p), \quad (6)$$

where $u_k(p) = F_M(p)$ and

$$x_k(p) = \left[y_k(p-1, p-2) \;\; y_{k-1}(p-1, p-2)\right]^T,$$
$$y_k(p-1, p-2) := \left[\, y_k(p-1) \;\; y_k(p-2) \,\right]$$
$$y_{k-1}(p-1, p-2) := \left[\, y_{k-1}(p-1) \;\; y_{k-1}(p-2) \,\right]$$
$$A = \begin{bmatrix} a_1 & a_2 & a_4 & a_5 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$
$$B = \begin{bmatrix} b \\ 0 \\ 0 \\ 0 \end{bmatrix}, \; B_0 = \begin{bmatrix} a_3 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$
$$C = \left[\, a_1 \;\; a_2 \;\; a_4 \;\; a_5 \,\right], \; D = b, \; D_0 = a_3.$$

The model of (6) is a particular example of that for discrete linear repetitive processes where, in the general case on pass k, $x_k(p)$ is the $n \times 1$ state vector, $y_k(p)$ is the $m \times 1$ pass profile vector and $u_k(p)$ is the $l \times 1$ control input vector. To complete the process description it is necessary to specify the pass length and the initial, or boundary, conditions, i.e. the pass state initial vector sequence and the initial pass profile. Here the boundary conditions will be specified in the following section where controller design is the subject. In these design studies, the data used is $\lambda_1 = 600$, $\lambda_2 = 2000$, $M = 100$ and $T = 0.1$. This yields $\lambda = 461.54$ and the following matrices in (6)

$$A = \begin{bmatrix} 1.9118 & -0.0047 & -1.4706 & 0.7353 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$
$$B = \begin{bmatrix} -2.2059 \times 10^{-5} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \; B_0 = \begin{bmatrix} 0.7794 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$
$$C = \left[\, 1.9118 \;\; -0.0047 \;\; -1.4706 \;\; 0.7353 \,\right],$$
$$D = 2.2059 \times 10^{-5}, \; D_0 = 0.7794.$$

3. STABILIZATION

The stability theory (Rogers and Owens, 1992; Rogers *et al.*, 2002) for linear repetitive processes is based on an abstract model of the underlying dynamics in a Banach space setting which includes all such processes as special cases. In effect, this consists of two distinct concepts termed asymptotic stability and stability along the pass respectively where the former is a necessary condition for the latter. Noting again the unique control problem for these processes, asymptotic stability demands that bounded input sequences produce bounded output sequences (in a well defined sense) over the finite pass length and stability along the pass demands that this property holds independent of this parameter.

When applying this theory to discrete linear repetitive processes it is necessary to properly model the initial conditions, termed the boundary conditions in repetitive process theory, i.e. the pass state initial vector sequence and the initial pass profile. (In particular, it is known (see (Owens and Rogers, 1999) (for differential linear repetitive processes with a direct extension to their discrete counterparts) that if the pass state initial vector sequence is an explicit function of the previous pass profile then this alone can cause instability.) Here these are taken to be of the form

$$x_{k+1}(0) = d_{k+1}, \; k \geq 0$$
$$y_0(p) = y(p), \; 0 \leq p \leq \alpha, \quad (7)$$

where d_{k+1} is an $n \times 1$ vector with constant entries and $y(p)$ is an $m \times 1$ vector whose entries are known functions of p. With these boundary conditions, the following set of necessary and sufficient conditions for stability along the pass is the starting point for the results in the remainder of this paper.

Theorem 1. Discrete linear repetitive processes described by (6) and (7) are stable along the pass if, and only if, the 2D characteristic polynomial

$$C\left(z_1, z_2\right) := \det \begin{bmatrix} I_n - z_1 A & -z_1 B_0 \\ -z_2 C & I_m - z_2 D_0 \end{bmatrix}, \quad (8)$$

satisfies

$$C\left(z_1, z_2\right) \neq 0, \forall \left(z_1, z_2\right) \in \overline{U}^2, \quad (9)$$

where

$$\overline{U}^2 = \left\{ \left(z_1, z_2\right) : \left|z_1\right| \leq 1, \left|z_2\right| \leq 1 \right\}. \quad (10)$$

Note that (9) gives the necessary conditions that $r(D_0) < 1$ (asymptotic stability) and $r(A) < 1$ which should be verified before proceeding further with any stability analysis.

Now define the following matrices from the state space model (6)

$$\widehat{A}_1 = \begin{bmatrix} A & B_0 \\ 0 & 0 \end{bmatrix}, \quad \widehat{A}_2 = \begin{bmatrix} 0 & 0 \\ C & D_0 \end{bmatrix}. \quad (11)$$

Then we have the following sufficient condition for stability along the pass of processes described by (6) and (7) (for a proof see (Rogers *et al.*, 2001)).

Theorem 2. Discrete linear repetitive processes described by (6) and (7) are stable along the pass if $\exists$ matrices $P = P^T > 0$ and $Q = Q^T > 0$ satisfying the following LMI

$$\begin{bmatrix} \widehat{A}_1^T P \widehat{A}_1 + Q - P & \widehat{A}_1^T P \widehat{A}_2 \\ \widehat{A}_2^T P \widehat{A}_1 & \widehat{A}_2^T P \widehat{A}_2 - Q \end{bmatrix} < 0. \quad (12)$$

In terms of the design of control schemes for discrete linear repetitive processes, most work has been done in the iterative learning control area (see, for example, (Amann *et al.*, 1998)). Here it has become clear that a particularly powerful control action comes from using (state) feedback action on the current pass augmented by feedforward action from the previous pass. Here we consider a control law of the following form over $0 \leq p \leq \alpha$, $k \geq 0$

$$\begin{aligned} u_{k+1}(p) &= K_1 x_{k+1}(p) + K_2 y_k(p) \\ &:= K \begin{bmatrix} x_{k+1}(p) \\ y_k(p) \end{bmatrix}, \end{aligned} \tag{13}$$

where K_1 and K_2 are appropriately dimensioned matrices to be designed. This results in the following condition for closed loop stability along the pass

$$C_c(z_1, z_2) \neq 0, \forall (z_1, z_2) \in \overline{U}^2 \tag{14}$$

where

$$C_c(z_1, z_2) := \det \begin{bmatrix} I_n - z_1 \tilde{A} & -z_1 \tilde{B}_0 \\ -z_2 \tilde{C} & I_m - z_2 \tilde{D}_0 \end{bmatrix}. \tag{15}$$

with $\tilde{A} = A + BK_1$, $\tilde{B}_0 = B_0 + BK_2$, $\tilde{C} = C + DK_1$, $\tilde{D}_0 = D_0 + DK_2$.

Now introduce the matrices

$$\widehat{B}_1 = \begin{bmatrix} B \\ 0 \end{bmatrix}, \ \widehat{B}_2 = \begin{bmatrix} 0 \\ D \end{bmatrix}. \tag{16}$$

Theorem 3. Closed loop stability along the pass holds if $\exists$ matrices $Y = Y^T > 0$, $Z = Z^T > 0$, and N such that the following LMI holds.

$$\begin{bmatrix} Z - Y & 0 & Y\widehat{A}_1^T + N^T \widehat{B}_1^T \\ 0 & -Z & Y\widehat{A}_2^T + N^T \widehat{B}_2^T \\ \widehat{A}_1 Y + \widehat{B}_1 N & \widehat{A}_2 Y + \widehat{B}_2 N & -Y \end{bmatrix} < 0. \tag{17}$$

Also if this condition holds then a stabilizing K for the control law (13) is given by

$$K = NY^{-1}. \tag{18}$$

In the particular example considered here, the underlying LMI test is feasible and the following K gives stability along the pass closed loop

$$K = 1 \times 10^4 \begin{bmatrix} K_1 & K_2 \end{bmatrix}, \tag{19}$$

where

$$\begin{aligned} K_1 &= \begin{bmatrix} 8.5536 & -0.0046 & -6.0744 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} 2.7369 & 3.4478 \end{bmatrix}. \end{aligned} \tag{20}$$

In this numerical example, the resulting closed loop system is again of the form (6) where B and D are as before but now

$$A = \begin{bmatrix} 0.0249 & -0.0057 & -0.1307 & 0.1316 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0.0189 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \tag{21}$$

$$\begin{aligned} C &= \begin{bmatrix} 0.0249 & -0.0057 & -0.1307 & 0.1316 \end{bmatrix}, \\ D_0 &= 0.0189. \end{aligned} \tag{22}$$

In the design of control laws for discrete linear repetitive processes, stability along the pass will often only be the minimal requirement. In particular, a key task will be to ensure that the example under consideration retains this stability property in the presence of process parameter variations. The analysis which follows in the remainder of this paper produces new results on this general area and again uses the metal rolling model as an illustrative example.

4. ROBUST CONTROL

In this section, we develop the first major results on an LMI approach to stability analysis in the presence of uncertainty in the process definition. To begin, introduce the so-called augmented process and input matrices as

$$\Phi = \begin{bmatrix} A & B_0 \\ C & D_0 \end{bmatrix}, \ \Psi = \begin{bmatrix} B \\ D \end{bmatrix}. \tag{23}$$

Then here we treat the case when these matrices are subject to additive perturbations defined as follows

$$\Phi_p := \Phi + \Delta\Phi \tag{24}$$

$$\Psi_p := \Psi + \Delta\Psi, \tag{25}$$

where

$$\Delta\Phi = \begin{bmatrix} \Delta A & \Delta B_0 \\ \Delta C & \Delta D_0 \end{bmatrix}, \ \Delta\Psi = \begin{bmatrix} \Delta B \\ \Delta D \end{bmatrix}. \tag{26}$$

Also we assume that the uncertainties here have the following typical structure

$$\begin{bmatrix} \Delta\Phi & \Delta\Psi \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F \begin{bmatrix} E_1 & E_2 \end{bmatrix}, \tag{27}$$

where the matrices on the right-hand side are of compatible dimensions and also $F^T F \leq I$.

Now introduce the following matrices.

$$\begin{aligned} \widehat{\Delta\Phi}_1 &= \begin{bmatrix} \Delta A & \Delta B_0 \\ 0 & 0 \end{bmatrix} & \widehat{\Delta\Phi}_2 &= \begin{bmatrix} 0 & 0 \\ \Delta C & \Delta D_0 \end{bmatrix} \\ \widehat{\Delta\Psi}_1 &= \begin{bmatrix} \Delta B \\ 0 \end{bmatrix} & \widehat{\Delta\Psi}_2 &= \begin{bmatrix} 0 \\ \Delta D \end{bmatrix}. \end{aligned} \tag{28}$$

Then we can write $\Delta\Phi$ and $\Delta\Psi$ in the form

$$\Delta\Phi = \widehat{\Delta\Phi}_1 + \widehat{\Delta\Phi}_2 = \widehat{H}_1 F E_1 + \widehat{H}_2 F E_1 \quad (29)$$

$$\Delta\Psi = \widehat{\Delta\Psi}_1 + \widehat{\Delta\Psi}_2 = \widehat{H}_1 F E_2 + \widehat{H}_2 F E_2 \quad (30)$$

where

$$\widehat{H}_1 = \begin{bmatrix} H_1 \\ 0 \end{bmatrix}, \ \widehat{H}_2 = \begin{bmatrix} 0 \\ H_2 \end{bmatrix}. \quad (31)$$

The LMI sufficient condition for stability along the pass given in Theorem 2 applied in this case is equivalent to the existence of matrices $P = P^T > 0$ and $Q = Q^T > 0$ such that

$$\widehat{A}^T P \widehat{A} + \widehat{Q} < 0, \quad (32)$$

where

$$\widehat{A} = \begin{bmatrix} \widehat{A}_1 & \widehat{A}_2 \end{bmatrix}, \ \widehat{Q} = \begin{bmatrix} P-Q & 0 \\ 0 & -Q \end{bmatrix}, \quad (33)$$

and we now have the following result.

Theorem 4. Discrete linear repetitive processes described by (6) and (7) whose defining matrices have the uncertainty structure (28)–(30) are stable along the pass if $\exists$ matrices $P = P^T > 0$, $Q = Q^T > 0$ such that

$$\left(\widehat{A} + \widehat{H}\widehat{F}\widehat{E}_1\right)^T P \left(\widehat{A} + \widehat{H}\widehat{F}\widehat{E}_1\right) + \widehat{Q} < 0, \quad (34)$$

where

$$\widehat{H} = \begin{bmatrix} \widehat{H}_1 & \widehat{H}_2 \end{bmatrix}, \ \widehat{F} = I_2 \otimes F, \ \widehat{E}_1 = I_2 \otimes E_1, \quad (35)$$

and $\otimes$ denotes the matrix Kronecher product. Also it is well known that, for any choice of Q, $F^T F < I$, $\exists$ $P > 0$ such that (34) holds if, and only if, $\exists$ a scalar $\varepsilon > 0$ such that

$$\begin{bmatrix} -P^{-1} + \varepsilon \widehat{H}\widehat{H}^T & \widehat{A} \\ \widehat{A}^T & \varepsilon^{-1}\widehat{E}_1^T \widehat{E}_1 + \widetilde{Q} \end{bmatrix} < 0. \quad (36)$$

Now we have the following result which gives a sufficient condition, expressed in terms of an LMI, for stability along the pass under the uncertainty structure defined above.

Theorem 5. Discrete linear repetitive processes described by (6) and (7) whose defining matrices have the uncertainty structure defined by (28)–(30) are stable along the pass if $\exists$ matrices $Y = Y^T > 0$ and $Z = Z^T > 0$ such that the following LMI holds

$$\begin{bmatrix} -Y & \widehat{A}_1 Y & \widehat{A}_2 Y & \varepsilon\widehat{H}_1 & \varepsilon\widehat{H}_2 & 0 & 0 \\ Y\widehat{A}_1^T & Z-Y & 0 & 0 & 0 & YE_1^T & 0 \\ Y\widehat{A}_2^T & 0 & -Z & 0 & 0 & 0 & YE_1^T \\ \varepsilon\widehat{H}_1^T & 0 & 0 & -\varepsilon I & 0 & 0 & 0 \\ \varepsilon\widehat{H}_2^T & 0 & 0 & 0 & -\varepsilon I & 0 & 0 \\ 0 & E_1 Y & 0 & 0 & 0 & -\varepsilon I & 0 \\ 0 & 0 & E_1 Y & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0. \quad (37)$$

and the dimension of the identity matrix here is as required.

Proof: First make the substitutions $Y = P^{-1}$ and $Z = Q^{-1}$, apply the Schur's complement formula to (36), and follow this by an obvious congruence transformation. The result then follows immediately.

The following result gives a solution of stabilization problem under the uncertainty structure defined above.

Theorem 6. Discrete linear repetitive processes described by (6) and (7) whose defining matrices have the uncertainty structure defined by (28)–(30) are stable along the pass under the control law (13) if $\exists$ a scalar $\varepsilon > 0$ and matrices $Y = Y^T > 0$, $Z = Z^T > 0$, and N such that the following LMI holds. In which case the stabilizing controller K is given by (18).

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{12}^T & M_{22} & M_{23} \\ M_{13}^T & M_{23}^T & M_{33} \end{bmatrix} < 0, \quad (38)$$

where

$$M_{11} = \begin{bmatrix} -Y & G_{12} & G_{13} \\ G_{12}^T & Z-Y & 0 \\ G_{13}^T & 0 & -Z \end{bmatrix},$$

$$G_{12} = \widehat{A}_1 Y + \widehat{B}_1 N,$$

$$G_{13} = \widehat{A}_2 Y + \widehat{B}_2 N,$$

$$M_{12} = \begin{bmatrix} \varepsilon\widehat{H}_1 & \varepsilon\widehat{H}_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$M_{13} = \begin{bmatrix} 0 & 0 \\ YE_1^T + N^T E_2^T & 0 \\ 0 & YE_1^T + N^T E_2^T \end{bmatrix},$$

$$M_{22} = \begin{bmatrix} -\varepsilon I & 0 \\ 0 & -\varepsilon I \end{bmatrix},$$

$$M_{23} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$M_{33} = \begin{bmatrix} -\varepsilon I & 0 \\ 0 & -\varepsilon I \end{bmatrix}.$$

In the numerical example considered in this paper, we consider the case when in (29)-(31)

$$H_1 = \begin{bmatrix} 0.2311 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ H_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.6068 \end{bmatrix}$$

$$E_1^T = 1.0 \times 10^{-5} \begin{bmatrix} 0.4859 \\ 0.8912 \\ 0.7620 \\ 0.4564 \\ 0.0185 \end{bmatrix},$$

$$E_2 = 8.2140 \times 10^{-6}$$

In this case the LMI of Theorem 6 is feasible and this yields the following controller matrix

$$K^T = 1.0 \times 10^4 \begin{bmatrix} 8.50241493 \\ -0.0207 \\ -6.3857 \\ 3.2705 \\ 3.1746 \end{bmatrix}. \quad (39)$$

5. CONCLUSIONS

Linear repetitive processes are a distinct class of 2D systems of both theoretical and practical interest. The feature which distinguishes them from, in particular, other extensively studied classes of 2D linear systems is that information propagation in one of the two independent directions (along the pass) only occurs over a finite duration. This means that a distinct systems/controller design theory must be developed for them.

Previous work has focused, in the main, on stability theory and associated tests. This has resulted in a rigorous stability theory supported by computationally feasible stability tests. A key feature of this work is that, unlike the 1D linear systems case, even the 'Nyquist like' stability tests which can be employed do not provide a solid basis on which to base even initial controller design studies. The major novel feature in this paper is that (building on the work in (Rogers *et al.*, 2001)) the use of LMI based tools provides such a basis and here we have illustrated the underlying theoretical developments in this area using a metal rolling model as an example. In particular, it has been shown this approach enables us to begin the study of robust control for these processes.

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