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ADAPTIVE ROBUST STABILIZATION FOR A CLASS OF UNCERTAIN LINEAR TIME–VARYING SYSTEMS WITH MULTIPLE TIME DELAYS

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Abstract: The problem of robust stabilization is considered for a class of uncertain linear time–varying systems with multiple time delays. In the paper, the upper bounds of uncertainties and external disturbances are assumed to be unknown. An adaptation law is introduced to estimate such unknown bounds, and by employing the updated values of these unknown bounds a class of memoryless state feedback controllers is proposed. Based on Lyapunov stability theory and Lyapunov–Krasovskii functional, it is shown that by making use of the proposed memoryless state feedback controller, the solutions of the resulting adaptive closed–loop time-delay system can be guaranteed to be uniformly bounded, and the states are uniformly asymptotically stable.

Keywords: Uncertain systems, time delay, robust control, adaptive control, uniform boundedness, asymptotic stability.

1. INTRODUCTION

Many practical control problems, such as those arising in chemical processes, hydraulics, rolling mills, economics, involve time–delay systems, connected with measurement of system variables, physical properties of the equipment, signal transmission (transport delay), and so on (see e.g. (Kolmanovskii and Nosov, 1986), (Manu and Mohammad, 1987)). The existence of delay is frequently a source of instability of the systems. On the other hand, it is not avoidable to include some uncertain parameters and disturbance in practical control systems due to modeling errors, measurement errors, linearization approximations, and so on. Therefore, the problem of robust stabilization of uncertain dynamical systems with time delay has received considerable attention of many researchers (see, e.g. (Cheres *et al.*, 1989), (Wu, 1997), (Wu, 2000), (Wu and Mizukami, 1995), (Wu and Mizukami, 1996), and the references therein).

In the control literature, for uncertain time–delay systems, where the system state vector is available, the upper bounds of the vector norms on the uncertainties are generally supposed to be known, and such bounds are employed either to construct some types of stabilizing state feedback controllers (see, e.g. (Cheres *et al.*, 1989), (Wu and Mizukami, 1996)), or to develop some stability conditions (see, e.g. (Wu, 1997), (Wu and Mizukami, 1995)). However, in a number of practical control problems, such bounds may be unknown, or be partially known. In some cases, it may also be difficult to evaluate their upper bounds. Therefore, for such a class of uncertain time–delay systems whose uncertainty bounds are partially known, adaptive control schemes should be introduced to update these unknown bounds.

For such uncertain systems without time–delay, several types of adaptive robust state feedback controller have been proposed (see, e.g. (Brogliato and Neto, 1995), (Choi and Kim, 1993), (Wu, 1999). But, few efforts are made to consider the problem of adaptive robust control for such uncertain systems with time–delay.

In this paper, the problem of robust stabilization is considered for a class of linear time–varying systems with the delayed state perturbations, uncertainties, and external disturbances. It is assumed that the upper bounds of the delayed state perturbations, uncertainties, and external disturbances, are unknown. In the paper, first some adaptation laws is proposed to estimate such unknown bounds. Then, by making use of the updated values of these unknown bounds a class of memoryless state feedback controllers is constructed. Moreover, on the basis of the Lyapunov stability theory and Lyapunov–Krasovskii functional, it is shown that by employing the proposed memoryless state feedback controller, the solutions of the resulting adaptive closed–loop time-delay system can be guaranteed to be uniformly bounded, and the states are uniformly asymptotically stable.

2. PROBLEM FORMULATION

Consider a class of uncertain linear time–varying systems with multiple time delays described by

$$\begin{aligned}\frac{dx(t)}{dt} &= \Big[A(t) + \Delta A(v,t)\Big]x(t)\\ &+\sum_{j=1}^{r}\Delta A_j(\zeta,t)x(t-h_j)\\ &+\Big[B(t)+\Delta B(\xi,t)\Big]u(t)+q(\nu,t) \qquad (1)\end{aligned}$$

where $t \in R$ is the "time", $x(t) \in R^n$ is the current value of the state, $u(t) \in R^m$ is the control input, $A(t)$, $B(t)$ are continuous matrices of appropriate dimensions, $\Delta A(\cdot)$, $\Delta A_j(\cdot)$, $j = 1, 2, \ldots, r$, and $\Delta B(\cdot)$ represent the system uncertainties and are continuous in all their arguments, and the vector $q(\cdot)$ is the external disturbance vector, which is also assumed to be continuous in all their arguments. Moreover, the uncertain parameters $(v,\ \xi,\ \zeta,\ \nu) \in \Psi \subset R^L$ are Lebesgue measurable and take values in a known compact bounding set Ω. In addition, the time delays h_j, $j = 1, 2, \ldots, r$, are assumed to be any positive constants which are not required to be known for the system designer, as shown in Section 3.

The initial condition for system (1) is given by

$$x(t) = \chi(t), \qquad t \in [t_0 - \bar{h},\ t_0] \qquad (2)$$

where $\chi(t)$ is a continuous function on $[t_0 - \bar{h},\ t_0]$, and $\bar{h} := \max\{h_j,\ j = 1, 2, \ldots, r\}$

Provided that all current values of the states are available, the memoryless state feedback controller $u(t)$ can be represented by a function:

$$u(t) = p(x(t), t) \qquad (3)$$

where $p(\cdot)\ :\ R^n \times R \to R^m$ is a continuous function.

Now, the question is to how to synthesize a memoryless state feedback controller $u(t)$ that can guarantee the stability of uncertain time–delay system (1).

In the paper, the following standard assumptions are introduced.

Assumption 2.1. The pair $\{A(\cdot),\ B(\cdot)\}$ given in system (1) is uniformly completely controllable.

Assumption 2.2. For all $(x,\ t) \in R^n \times R$ there exist continuous matrix functions $H(\cdot)$, $H_j(\cdot)$, $j = 1, 2, \ldots, r$, $E(\cdot)$, $w(\cdot)$ of appropriate dimensions such that

$$\begin{aligned}\Delta A(\cdot) &= B(t)H(\cdot), \quad \Delta A_j(\cdot) = B(t)H_j(\cdot)\\ \Delta B(\cdot) &= B(t)E(\cdot), \quad q(\cdot) = B(t)w(\cdot)\end{aligned}$$

For convenience, the following notations are introduced, which represent the bounds of the uncertainties and external disturbances.

$$\begin{aligned}\rho(t) &:= \max_{v}\big\|H(v,t)\big\|\\ \rho_j(t) &:= \max_{\zeta}\big\|H_j(\zeta,t)\big\|\\ \mu(t) &:= \min_{\xi}\left[\frac{1}{2}\lambda_{\min}\left(E(\xi,t)+E^{\top}(\xi,t)\right)\right]\\ \rho_q(t) &:= \max_{\nu}\big\|w(\nu,t)\big\|\end{aligned}$$

where $\|\cdot\|$ is the spectral norm of a matrix "$\cdot$", and $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of the matrix "$\cdot$", respectively. Moreover, the uncertain $\rho(t)$, $\rho_j(t)$, $\mu(t)$, $\rho_q(t)$ are assumed to be continuous and bounded for any $t \in R^+$.

By employing the notations given above, we introduce for system (1) the following standard assumption.

Assumption 2.3. For every $t \geq t_0$, $\mu(t) > -1$.

Remark 2.1. It is well known that *Assumption 2.1* is standard and denotes the internally stabilizability of the nominal system, i.e., the system in the absence of the delayed state perturbations, uncertainties, and external disturbances. *Assumption 2.2* defines the matching condition about the uncertainties, and is a rather standard assumption for robust control problem (see, e.g. (Brogliato and Neto, 1995), (Cheres *et al.*, 1989), (Wu, 1999), (Wu, 2000), (Wu and Mizukami, 1993), (Wu and

Mizukami, 1996)). In general, for system with matched uncertainties, one may always design some types of stabilizing feedback controllers. However, this assertion is not valid for systems with unmatched uncertainties. For such uncertain systems, one must find some conditions such that some types of stability can be guaranteed (see, e.g. (Barmish and Leitmann, 1982), (Chen and Leitmann, 1987)).

Remark 2.2. Assumption 2.3 is also standard, and can be regarded as a necessary condition for robust stability of uncertain systems (see, e.g. (Cheres *et al.*, 1989), (Choi and Kim, 1993), (Wu and Mizukami, 1996), and the references relative to robust stabilization of uncertain systems).

Remark 2.3. The stabilizing state feedback controllers proposed in the control literature are based on the fact that the bounds of the uncertainties are known (see, e.g. (Cheres *et al.*, 1989), (Wu and Mizukami, 1996) for time–delay systems). When such bounds are unknown, some updating laws to such unknown bounds must be introduced to construct adaptive robust controllers. In a recent paper (Wu, 2000), a memoryless adaptive robust state feedback controller is proposed for a class of uncertain systems with multiple delayed state perturbations. However, the uncertain time–delay systems considered in (Wu, 2000) do not involve the uncertainty of input gain and external disturbances, and the adaptive robust controllers proposed in (Wu, 2000) stabilize the systems only in the sense of uniform ultimate boundedness. Moreover, it seems that even for uncertain systems without delayed state perturbations, the problem of stabilization has not well been discussed yet when the systems include the uncertainty on input gain and its bound is not exactly known. In this paper, we want to propose a class of memoryless adaptive robust state feedback controllers for uncertain time–delay system (1) where the upper bounds of the delayed state perturbations, uncertainties, and external disturbances, are unknown.

On the other hand, it follows from *Assumption 2.1* that for any symmetric positive definite matrix $Q \in R^{n\times n}$, and any positive constant η, the matrix Riccati equation of the form

$$\frac{dP(t)}{dt} + A^{\top}(t)P(t) + P(t)A(t) - \eta P(t)B(t)B^{\top}(t)P(t) = -Q \quad (4)$$

has a solution $P(t) \in R^{n\times n}$, which satisfies

$$\alpha_1 I \le P(t) \le \alpha_2 I \quad (5)$$

for all $t \in R^{+}$, where α_1 and α_2 are two positive numbers (see, e.g., (Ikeda *et al.*, 1972)).

3. MAIN RESULTS

Since the bounds $\rho(t)$, $\rho_j(t)$, $j = 1, 2, \ldots, r$, $\mu(t)$, $\rho_q(t)$ have been assumed to be continuous and bounded for any $t \in R^{+}$, it can be supposed that there exist some positive constants ρ^*, ρ_j^*, $j = 1, 2, \ldots, r$, μ^*, ρ_q^*, which are defined by

$$\rho^* := \max\{\, \rho(t) \;:\; t \in R^{+} \,\} \quad (6a)$$

$$\rho_j^* := \max\{\, \rho_j(t) \;:\; t \in R^{+} \,\} \quad (6b)$$

$$\mu^* := \min\{\, \mu(t) \;:\; t \in R^{+} \,\} > -1 \quad (6c)$$

$$\rho_q^* := \max\{\, \rho_q(t) \;:\; t \in R^{+} \,\} \quad (6d)$$

Here, it is worth pointing out that the constants ρ^*, ρ_j^*, $j = 1, 2, \ldots, r$, μ^*, ρ_q^*, are still unknown. Therefore, such unknown bounds can not be directly employed to construct the stabilizing state feedback controllers.

Without loss of generality, we also introduce the following definition:

$$\psi^* := \frac{1}{1+\mu^*}\left(1 + (\rho^*)^2 + \sum_{j=1}^{r} (\rho_j^*)^2\right) \quad (7a)$$

$$\phi^* := \frac{\rho_q^*}{1+\mu^*} \quad (7b)$$

where ψ^* and ϕ^* are obviously unknown positive constants. Moreover, a function $\bar{\mu}(x,t)$ is defined by

$$\bar{\mu}(x,t) := B^{\top}(t)P(t)x(t)$$

Now, for the uncertain time–delay system described by (1) we propose the following nonlinear memoryless adaptive robust state feedback controller:

$$u(t) = p_1(x(t),t) + p_2(x(t),t) \quad (8a)$$

where $p_1(\cdot)$ and $p_2(\cdot)$ are given by the following functions:

$$p_1(x(t),t) = -\frac{1}{2}\,\eta\hat{\psi}(t)B^{\top}(t)P(t)x(t) \quad (8b)$$

$$p_2(x(t),t) = -\frac{\hat{\phi}^2(t)\bar{\mu}(x,t)}{\|\bar{\mu}(x,t)\|\,\hat{\phi}(t) + \varepsilon e^{-\beta(t-t_0)}} \quad (8c)$$

and where ε and β are any positive constants, and η is a positive constant which is chosen such that

$$Q - (1+r)\eta^{-1}I > 0 \quad (8d)$$

In particular, $\hat{\psi}(\cdot)$ and $\hat{\phi}(\cdot)$ are respectively the estimate of the unknown ψ^* and ϕ^*, which are updated by the following adaptive laws:

$$\frac{d\hat{\psi}(t)}{dt} = \eta\gamma_1 \left\| B^\top(t)P(t)x(t)\right\|^2 \quad (9a)$$

$$\frac{d\hat{\phi}(t)}{dt} = 2\gamma_2 \left\| B^\top(t)P(t)x(t)\right\| \quad (9b)$$

where γ_1 and γ_2 are any positive constants, $\hat{\psi}(t_0)$ and $\hat{\phi}(t_0)$ are finite.

Thus, applying (8) to (1) yields an uncertain closed–loop system of the form:

$$\begin{aligned}\frac{dx(t)}{dt} &= \left[A(t) - \frac{1}{2}k_1(t)B(t)B^\top(t)P(t)\right]x(t)\\ &+\left[\Delta A(\cdot) - \frac{1}{2}k_1(t)\Delta B(\cdot)B^\top(t)P(t)\right]x(t)\\ &+\sum_{j=1}^{r}\Delta A_j(\cdot)x(t-h_j)\\ &+\left[B(t)+\Delta B(\cdot)\right]p_2(x(t),t)+q(\nu,t) \quad (10)\end{aligned}$$

where

$$k_1(t) = \eta\,\hat{\psi}(t), \quad k_2(t) = \hat{\phi}^2(t)$$

On the other hand, letting $\tilde{\psi}(t) = \hat{\psi}(t) - \psi^*$ and $\tilde{\phi}(t) = \hat{\phi}(t) - \phi^*$, we can rewrite (9) as the following error system

$$\frac{d\tilde{\psi}(t)}{dt} = \eta\gamma_1 \left\| B^\top(t)P(t)x(t)\right\|^2 \quad (11a)$$

$$\frac{d\tilde{\phi}(t)}{dt} = 2\gamma_2 \left\| B^\top(t)P(t)x(t)\right\| \quad (11b)$$

In the following, by $(x, \tilde{\psi}, \tilde{\phi})(t)$ we denote a solution of the uncertain closed–loop system and the error system. Then, the following theorem can be obtained which shows the globally boundedness of the solutions of (10) and (11).

Theorem 3.1. Consider the uncertain adaptive closed–loop time–delay dynamical system described by (10) and (11), which satisfies *Assumptions 2.1* to *2.3*. Then, the solutions $(x, \tilde{\psi}, \tilde{\phi})$ $(t;\, t_0, x(t_0), \tilde{\psi}(t_0), \tilde{\phi}(t_0))$ of (10) and (11) are globally bounded and

$$\text{(i)} \lim_{t\to\infty} x(t;\, t_0, x(t_0), \tilde{\psi}(t_0), \tilde{\phi}(t_0)) = 0 \quad (12a)$$

$$\text{(ii)} \lim_{t\to\infty}\frac{d\tilde{\psi}(t)}{dt} = 0, \quad \lim_{t\to\infty}\frac{d\tilde{\phi}(t)}{dt} = 0 \quad (12b)$$

Proof : For (10) and (11), we fist define a Lyapunov–Krasovskii functional candidate as follows.

$$\begin{aligned}V(x,\Psi) &= x^\top(t)P(t)x(t)\\ &+\sum_{j=1}^{r}\eta^{-1}\int_{t-h_j}^{t} x^\top(\tau)x(\tau)d\tau\\ &+\frac{1}{2}(1+\mu^*)\Psi^\top(t)\Gamma^{-1}\Psi(t) \quad (13)\end{aligned}$$

where Γ^{-1} and $\Psi(\cdot)$ are defined as

$$\Psi(\cdot) := \begin{bmatrix}\tilde{\psi}(\cdot)\\ \tilde{\phi}(\cdot)\end{bmatrix}, \quad \Gamma^{-1} := \begin{bmatrix}\gamma_1^{-1} & 0\\ 0 & \gamma_2^{-1}\end{bmatrix}$$

Let $(x(t),\, \Psi(t))$ be the solution of (10) and (11) for $t \ge t_0$. Then by taking the derivative of $V(\cdot)$ along the trajectories of (10) and (11), and from Assumption 2.2 it can be obtained that for any $t \ge t_0$,

$$\begin{aligned}\frac{dV(x,\Psi)}{dt} &= x^\top(t)\left[\frac{dP(t)}{dt} + A^\top(t)P(t)\right.\\ &\left.+P(t)A(t) - k_1(t)P(t)B(t)B^\top(t)P(t)\right]x(t)\\ &+2\bar{\mu}^\top(x,t)\sum_{j=1}^{r}H_j(\zeta,t)x(t-h_j)\\ &+2\bar{\mu}^\top(x,t)H(\upsilon,t)x(t)\\ &-k_1(t)\bar{\mu}^\top(x,t)\left[\frac{1}{2}\left(E(\cdot)+E^\top(\cdot)\right)\right]\bar{\mu}(x,t)\\ &-\frac{2\,k_2(t)\bar{\mu}^\top(x,t)\left[I_m + E(\xi,t)\right]\bar{\mu}(x,t)}{\|\bar{\mu}(x,t)\|\,\hat{\phi}(t) + \varepsilon e^{-\beta(t-t_0)}}\\ &+2\bar{\mu}^\top(x,t)w(\nu,t)\\ &+\sum_{j=1}^{r}\eta^{-1}\left[\|x(t)\|^2 - \|x(t-h_j)\|^2\right]\\ &+(1+\mu^*)\Psi^\top(t)\Gamma^{-1}\frac{d\Psi(t)}{dt} \quad (14)\end{aligned}$$

Thus, from (4), (6), and (14) it can be obtained that for any $t \ge t_0$,

$$\begin{aligned}\frac{dV(x,\Psi)}{dt} &\le -x^\top(t)Qx(t) + \sum_{j=1}^{r}\eta^{-1}\|x(t)\|^2\\ &+\eta\left\|B^\top(t)P(t)x(t)\right\|^2\\ &-k_1(t)(1+\mu^*)\left\|B^\top(t)P(t)x(t)\right\|^2\\ &+2\rho^*\left\|B^\top(t)P(t)x(t)\right\|\,\|x(t)\|\\ &+2\sum_{j=1}^{r}\rho_j^*\|x(t-h_j)\|\,\left\|B^\top(t)P(t)x(t)\right\|\\ &-\sum_{j=1}^{r}\eta^{-1}\|x(t-h_j)\|^2\\ &-\frac{2\,k_2(t)(1+\mu^*)\left\|B^\top(t)P(t)x(t)\right\|^2}{\left\|B^\top(t)P(t)x(t)\right\|\hat{\phi}(t) + \varepsilon e^{-\beta(t-t_0)}}\\ &+2\rho_q^*\left\|B^\top(t)P(t)x(t)\right\|\\ &+(1+\mu^*)\Psi^\top(t)\Gamma^{-1}\frac{d\Psi(t)}{dt} \quad (15)\end{aligned}$$

Notice the fact that for any positive constant $c > 0$,

$$2ab \le \frac{1}{c}a^2 + cb^2, \quad \forall\, a,\, b > 0$$

Then, from (15) it can further be obtained that for any $t \geq t_0$,

$$\begin{aligned}\frac{dV(x,\Psi)}{dt} &\leq -x^\top(t)\Big[Q-\eta^{-1}(1+r)I\Big]x(t)\\&+(1+\mu^*)\Bigg[-k_1(t)\left\|B^\top(t)P(t)x(t)\right\|^2\\&+\frac{\eta}{1+\mu^*}\left(1+(\rho^*)^2+\sum_{j=1}^{r}(\rho_j^*)^2\right)\\&\cdot\left\|B^\top(t)P(t)x(t)\right\|^2\\&-\frac{2\,k_2(t)\left\|B^\top(t)P(t)x(t)\right\|^2}{\left\|B^\top(t)P(t)x(t)\right\|\hat{\phi}(t)\;+\;\varepsilon\mathrm{e}^{-\beta(t-t_0)}}\\&+2\frac{\rho_q^*}{1+\mu^*}\left\|B^\top(t)P(t)x(t)\right\|\Bigg]\\&+(1+\mu^*)\Psi^\top(t)\Gamma^{-1}\frac{d\Psi(t)}{dt}\\&=-x^\top(t)\tilde{Q}x(t)\\&+(1+\mu^*)\Bigg[-k_1(t)\left\|B^\top(t)P(t)x(t)\right\|^2\\&+\eta\psi^*\left\|B^\top(t)P(t)x(t)\right\|^2\\&-\frac{2\,k_2(t)\left\|B^\top(t)P(t)x(t)\right\|^2}{\left\|B^\top(t)P(t)x(t)\right\|\hat{\phi}(t)\;+\;\varepsilon\mathrm{e}^{-\beta(t-t_0)}}\\&+2\phi^*\left\|B^\top(t)P(t)x(t)\right\|\Bigg]\\&+(1+\mu^*)\left(\gamma_1^{-1}\tilde{\psi}(t)\frac{d\tilde{\psi}(t)}{dt}+\gamma_2^{-1}\tilde{\phi}(t)\frac{d\tilde{\phi}(t)}{dt}\right)\end{aligned} \tag{16}$$

where

$$\tilde{Q} := Q-\eta^{-1}(1+r)I > 0$$

Notice that the facts that

$$\hat{\psi}(t) = \tilde{\psi}(t)+\psi^*, \quad \hat{\phi}(t) = \tilde{\phi}(t)+\phi^*$$

it follows from (16) that for all $(t,x,\Psi) \in R \times R^n \times R^2$,

$$\begin{aligned}\frac{dV(x,\Psi)}{dt} &\leq -\lambda_{\min}(\tilde{Q})\|x(t)\|^2\\&+2(1+\mu^*)\varepsilon\exp\{-\beta(t-t_0)\}\end{aligned} \tag{17}$$

Moreover, letting

$$\tilde{x}(t) := \begin{bmatrix} x^\top(t) & \Psi^\top(t)\end{bmatrix}^\top, \quad \tilde{\varepsilon} := 2(1+\mu^*)\varepsilon$$

it can be obtained from (17) that for any $t \geq t_0$,

$$\begin{aligned}\frac{dV(\tilde{x}(t))}{dt} &\leq -\lambda_{\min}(\tilde{Q})\|x(t)\|^2\\&+\tilde{\varepsilon}\exp\{-\beta(t-t_0)\}\end{aligned} \tag{18}$$

On the other hand, in the light of the definition, given in (13), of Lyapunov function, there always exist two positive constants $\delta_{\min}$ and $\delta_{\max}$ such that for any $t \geq t_0$,

$$\tilde{\gamma}_1(\|\tilde{x}(t)\|) \leq V(\tilde{x}(t)) \leq \tilde{\gamma}_2(\|\tilde{x}(t)\|) \tag{19}$$

where

$$\tilde{\gamma}_1(\|\tilde{x}(t)\|) := \delta_{\min}\|\tilde{x}(t)\|^2 \tag{20a}$$

and

$$\begin{aligned}\tilde{\gamma}_2(\|\tilde{x}(t)\|) &:= \delta_{\max}\|\tilde{x}(t)\|^2\\&+\sum_{j=1}^{r}\eta^{-1}h_j\sup_{\tau\in[t-h_j,\,t]}\|x_j(\tau)\|^2\end{aligned} \tag{20b}$$

Now, from (18)–(20), we want to show that the solutions $\tilde{x}(t)$ of (10) and (11) are uniformly bounded, and that the state $x(t)$ converges asymptotically to zero.

By the continuity of (10) and (11), it is obvious that any solution $(x,\tilde{\psi},\tilde{\phi})\,(t;t_0,x(t_0),\tilde{\psi}(t_0),\tilde{\phi}(t_0))$ of the system is continuous.

It follows from (18) and (19) that for any $t \geq t_0$,

$$\begin{aligned}0 &\leq \tilde{\gamma}_1(\|\tilde{x}(t)\|) \leq V(\tilde{x}(t))\\&= V(\tilde{x}(t_0))+\int_{t_0}^{t}\dot{V}(\tilde{x}(\tau))d\tau\\&\leq \tilde{\gamma}_2(\|\tilde{x}(t_0)\|)-\int_{t_0}^{t}\tilde{\gamma}_3(\|x(\tau)\|)d\tau\\&+\int_{t_0}^{t}\tilde{\varepsilon}\exp\{-\beta(\tau-t_0)\}d\tau\end{aligned} \tag{21}$$

where the scalar function $\tilde{\gamma}_3(\|x(t)\|)$ is defined as

$$\tilde{\gamma}_3(\|x(t)\|) := \lambda_{\min}(\tilde{Q})\,\|x(t)\|^2 \tag{22}$$

Therefore, from (21) it can be obtained the following two results. First, taking the limit as t approaches infinity on both sides of inequality (21) yields

$$\lim_{t\to\infty}\int_{t_0}^{t}\tilde{\gamma}_3(\|x(\tau)\|)d\tau \leq \tilde{\gamma}_2(\|\tilde{x}(t_0)\|)+\tilde{\varepsilon}\beta^{-1} \tag{23}$$

On the other hand, from (21) we also have

$$0 \leq \tilde{\gamma}_1(\|\tilde{x}(t)\|) \leq \tilde{\gamma}_2(\|\tilde{x}(t_0)\|)+\tilde{\varepsilon}\beta^{-1} \tag{24}$$

which implies that $\tilde{x}(t)$ is uniformly bounded. Since $\tilde{x}(t)$ has been shown to be continuous, it follows that $\tilde{x}(t)$ is uniformly continuous, which implies that $x(t)$ is uniformly continuous. Therefore, it follows from the definition that $\tilde{\gamma}_3(\|x(t)\|)$

is also uniformly continuous. Applying the Barbalat lemma to inequality (23) yields that

$$\lim_{t\to\infty} \tilde{\gamma}_3(\|x(t)\|) = 0 \qquad (25)$$

Furthermore, since $\tilde{\gamma}_3(\cdot)$ is a positive definite scalar function, it is obvious from (25) that we can have

$$\lim_{t\to\infty} \|x(t)\| = 0$$

which implies that (12a) is satisfied. On the other hand, from (11) and (12a) we can easily obtain (12b). ■

Remark 3.1. In the paper, we have considered uncertain systems with multiple constant delays. That is, the delays h_j, $j = 1, 2, \ldots, r$, have been assumed to be any positive constants. However, by employing the method presented in this paper, one can easily extend the results of this paper to such a class of uncertain systems with the time–varying delays $h_j(t)$, $j = 1, 2, \ldots, r$. In fact, if assuming that the delays $h_j(t)$, $j = 1, 2, \ldots, r$, are any continuous and bounded nonnegative functions, and their derivatives are less than one, i.e. $\dot{h}_j(t) < 1$, we can use the same Lyapunov–Krasovskii functional as the one given in (13) for dynamical systems with time–varying delays to obtain similar results.

Remark 3.2. In order to illustrate the validity of the results obtained in the paper, a numerical example is also given, and the simulation is carried out. It is known from the results of the simulation that the proposed adaptive robust state feedback controllers stabilize indeed asymptotically the uncertain time–delay systems. (The details of the illustrative numerical example and the figures of the simulation will be displayed in the presentation.)

4. CONCLUDING REMARKS

The problem of robust stabilization for a class of systems with the delayed state perturbations, uncertainties, and external disturbances, has been discussed. Here, the upper bounds of the delayed state perturbations, uncertainties, and external disturbances, are assumed to be unknown. Based on the updated values of these unknown bounds, a class of memoryless state feedback controllers have been constructed. It have been shown that by employing the proposed controller, the solutions of the resulting adaptive closed–loop time-delay system can be guaranteed to be uniformly bounded, and the states are uniformly asymptotically stable in the presence of multiple delayed state perturbations, uncertainties, and external disturbances.

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