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APPROACH OF FUZZY MODELING WITH BOUNDED DATA UNCERTAINTIES

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Abstract: Some measured data matrices in projects are subjected to (not necessarily small) deterministic perturbations. Different from former Robust Fuzzy Tree model (RFT) which studies structured uncertainties, we propose another RFT to deal with unstructured uncertainties. This RFT uses fuzzy tree model (FT) and unstructured robust least squares (RLS) solution to work on the nonlinear modeling problem with unstructured bounded data uncertainties. The RFT not only keeps the features that FT can deal with high dimensional problem, has less computation load and has high precise, but also decreases drastically the sensitivity of FT to bounded uncertainties. *Copyright © 2002 IFAC*

Keywords: robust least-squares estimation, fuzzy tree model, Takagi-Sugeno model

1. INTRODUCTION[1]

Some perturbations (not necessarily small) have effect on measured data in projects. The given input-output data is not a single pair (A, b), but a family of matrices $(A + \Delta A, b + \Delta b)$, where $\Delta = [\Delta A, \Delta b]$ is an unknown-but-bounded matrix. Linear modeling problem is to find a solution x to an overdetermined set of equations $Ax \cong b$. Least squares solution (LS) minimizes the residual $\|\Delta b\|$ while total least squares solution (TLS) is to find the smallest error $\|[\Delta A \ \Delta b]\|_F$. They both do not consider the bounded data uncertainties problem that is studied in robust least squares solution (RLS). Ghaoui and Lebret discussed structured robust least squares solution (SRLS) which minimizes a worst-case residual error $r(A, b, \rho, x) = \max_{\|\delta\| \le \rho} \left\| (A_0 + \sum \delta_i A_i)x - (b_0 + \sum \delta_i b_i) \right\|$ (L Ghaoui and H Lebret, 1997). Independently, S. Chandrasekaran and G.H.Golub formulated unstructured RLS that minimizes another error $r(A, b, \eta, \eta_b) = \max_{\|\Delta A\| \le \eta, \|\Delta b\| \le \eta_b} \left\| (A + \Delta A)x - (b + \Delta b) \right\|$ (S. Chandrasekaran, G.H.Golub, et.al.1998). In this paper we work on a nonlinear modeling problem with unstructured bounded data uncertainties, so only the latter RLS solution is used, which can guarantee that the effect of uncertainties will never be over-estimated and that the computation has less load.

Now, consider the nonlinear modeling problem with data uncertainties whose input data vector and output

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value are denoted, respectively, as x and y, with perturbations denoted as Δx and Δy correspondingly. Within the scope of our knowledge, no result has been shown on this problem. Thanks to the fact that the consequent parts of a Takagi-Sugeno fuzzy logic model (TS) (T. Takagi, M. Sugeno, 1985) describe linear relations between input vector and output value, we can employ RLS solution on these parts to form a nonlinear robust TS model. Fuzzy tree model (FT) (Zhang J, Mao J, 1999) is one of TS models, which uses a growing binary tree to divide sample data space. Compared with some other TS fuzzy logic models, such as ANFIS (J.Shing, R Jang, 1993), fuzzy subtractive clustering model (S. Chiu, 1994), FT model is of less computation, higher accuracy, and insensitivity to the dimension of the input space. Zhang (Zhang J, 1999) in his dissertation proposed a nonlinear robust fuzzy tree model (RFT) to deal with structured bounded data uncertainties problem with $\Delta x = rk$, $|\gamma| \le \rho$ and k being a constant vector. Different from the former, this paper proposes another RFT model to work on unstructured bounded data uncertainties problem with $\|\Delta x\|_\infty \le \zeta, |\Delta y| \le \zeta_b$.

2. RFT MODEL WITH UNSTRUCTURED BOUNDED DATA UNCERTAINTIES

2.1 Fuzzy tree model (FT)

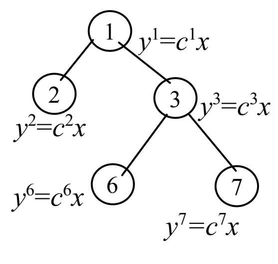

Figure 1. Structure sketch of a FT model

A FT model uses leaf nodes of a growing binary tree to divide sample data space. For every leaf node in a leaf nodes set , a fuzzy rule is defined by,

$L^{(t)}$: if x belongs $\chi_t\,(t \in \tilde{T})$, then

$$\hat{y}^t = c_0^t + c_1^t x_1 + \cdots + c_n^t x_n \qquad (2.1)$$

where $\tilde{T}$ denotes the leaf nodes set of a binary tree, χ_t denotes the fuzzy set for the leaf node t. If the membership function on the leaf node t is denoted by $\mu_t(x)$ and the input vector is denoted by $x = [1 \;\; x_1 \;\; x_2 \;\; \cdots \;\; x_n]^T$, then the output of the fuzzy tree model is constructed as,

$$\hat{y}(x) = \sum_{t\in\tilde{T}} \mu_t(x)\hat{y}^t \Big/ \sum_{t\in\tilde{T}} \mu_t(x) \qquad (2.2)$$

Every node $t \in T$ presents a linear relation $\hat{y}^t = (c_t)^T x$ with $c_t = [c_0^t \; c_1^t \cdots c_n^t]^T$. The $\hat{y}^t$ in (2.2) is constructed by $\hat{y}^t = (c_t)^T x$ of all leaf nodes. Thus $\hat{y}$ and all $c_t\,(t \in \tilde{T})$ construct a linear relation. Hence, if other parameters in (2.1) and K sampling data are given, all $c_t\,(t \in \tilde{T})$ can be estimated by solving a least squares solution (LS) and the following error arrives minimum,

$$E = \sum_{i=1}^{K} \frac{1}{2}(y^i - \hat{y}^i)^2 \qquad (2.3)$$

The membership function of root node is defined by $\mu_{r(T)}(x) = 1$ and the membership function of any other node is defined by (2.6), where the corresponding auxiliary membership function is defined by (2.5), $p(\cdot)$ denotes parent node. α_t is positive for every left child node and is negative for every right child node.

$$\theta_t = \sum_{i=1}^{K} \mu_{p(t)}(x^i)(c_{p(t)} x^i) \Big/ \sum_{i=1}^{K} \mu_{p(t)}(x^i) \qquad (2.4)$$

$$\hat{\mu}_t(x) = \frac{1}{1 + \exp[-\alpha_t (c_{p(t)}^T x - \theta_t)]} \qquad (2.5)$$

$$\mu_t(x) = \mu_{p(t)}(x)\hat{\mu}_t(x) \qquad (2.6)$$

Define node ($t \in \tilde{T}$) error by e_t and divided node error by e_t^c. If $e_t > e_t^c$ holds, then the node t shall be divided.

$$e_t = \sum_{i=1}^{K} [\mu_t(x^i)(y^i - c_t^T x^i)]^2 \qquad (2.7)$$

$$e_t^c = \sum_{i=1}^{K} [\mu_t(x^i)(y^i - \frac{\hat{\mu}_{l(t)}(x^i) c_{l(t)} x^i + \hat{\mu}_{r(t)}(x^i) c_{r(t)} x^i}{\hat{\mu}_{l(t)}(x^i) + \hat{\mu}_{r(t)}(x^i)})]^2 \qquad (2.8)$$

FT model is not only insensitivity to the dimension of the input space, but also can make a compromise between the amount of model fuzzy rules and the accuracy of model parameters.

2.2 RFT model with unstructured bounded data uncertainties

Given $\left\{(x^i, y^i) \mid x^i \in R^{n+1}, y^i \in R, i = 1,2,\cdots K\right\}$

and perturbations $\Delta x^i = [0 \quad \Delta x_1^i \quad \Delta x_2^i \quad \cdots \quad \Delta x_n^i]^T$, Δy^i where $x^i = [1, x_1^i, \cdots, x_n^i]^T$. The RFT model with unstructured bounded data uncertainties is to work with the bounded data uncertainties of $\left\|\Delta x^i\right\|_\infty \le \zeta, \left|\Delta y^i\right| \le \zeta_b$. A key point is to identify all the linear coefficients $v_t\,(t \in \widetilde{T})$ of the robust linear relations $y_\zeta = v_t^T(x + \Delta x)$ on all leaf nodes, where $v_t = [v_0^t \quad v_1^t \quad \cdots \quad v_n^t]^T$.

Definition1: Given $\left\{t_i \in \widetilde{T} \mid i = 1, \cdots, p\right\}$ and $\left\|\Delta x^i\right\|_\infty \le \zeta, \left|\Delta y^i\right| \le \zeta_b$, the worst-case error of a RFT model with unstructured bounded data uncertainties is defined by

$$r\left(\left\{(x^i, y^i) \mid i = 1,2\cdots K\right\}, \zeta, v_{\widetilde{T}}\right) = \max_{\|\Delta x\| \le \zeta, |\Delta y| \le \zeta_b} \left\|\hat{Y}_\zeta - Y\right\|$$

where

$$v_{\widetilde{T}} = [v_{t_1}^T \; v_{t_2}^T \cdots v_{t_p}^T]^T, \; \hat{Y}_\zeta = [\hat{y}_\zeta^1 \cdots \hat{y}_\zeta^K]^T$$

$$X = [\widetilde{X}^1 \cdots \widetilde{X}^K]^T, \; \Delta X = [\Delta\widetilde{X}^1 \cdots \Delta\widetilde{X}^K]^T \quad (2.9)$$

$$Y = [y^1 \quad \cdots \quad y^K]^T, \Delta Y = [\Delta y^1 \quad \cdots \quad \Delta y^K]^T \quad (2.10)$$

$$(Y + \Delta Y) \approx \hat{Y}_\zeta = v_{\widetilde{T}}^T (X + \Delta X)$$

$$\widetilde{X}^i = \left[\frac{\mu_{t_1}(x^i + \Delta x^i)}{\sum_{t \in \widetilde{T}} \mu_t(x^i + \Delta x^i)}(x^i)^T \quad \cdots \quad \frac{\mu_{t_p}(x^i + \Delta x^i)}{\sum_{t \in \widetilde{T}} \mu_t(x^i + \Delta x^i)}(x^i)^T\right]^T$$

$$\Delta\widetilde{X}^i = \left[\frac{\mu_{t_1}(x^i + \Delta x^i)}{\sum_{t \in \widetilde{T}} \mu_t(x^i + \Delta x^i)}(\Delta x^i)^T \quad \cdots \quad \frac{\mu_{t_p}(x^i + \Delta x^i)}{\sum_{t \in \widetilde{T}} \mu_t(x^i + \Delta x^i)}(\Delta x^i)^T\right]^T$$

$$\hat{y}_\zeta^i = \frac{\sum_{t \in \widetilde{T}} [\mu_t(x^i + \Delta x^i)\hat{y}_\zeta^{t,i}]}{\sum_{t \in \widetilde{T}} \mu_t(x^i + \Delta x^i)}.$$

Definition2: The robust least squares problem of a RFT model with unstructured bounded data uncertainties is defined by (2.11), and the correspondingly robust least squares solution $\left\{v_t, t \in \widetilde{T}\right\}$ is denoted by (2.12). There, "arg" is used to get the corresponding variable value when the function reaches minimum.

$$\phi\left(\left\{(x^i, y^i) \mid i = 1,2\cdots K\right\}, \zeta, v_{\widetilde{T}}\right) = \min_{\left\{v_t, t \in \widetilde{T}\right\}} r\left(\left\{(x^i, y^i) \mid i = 1,2\cdots K\right\}, \zeta, v_{\widetilde{T}}\right) \quad (2.11)$$

$$v_{\widetilde{T}} = \arg \phi\left(\left\{(x^i, y^i) \mid i = 1,2\cdots K\right\}, \zeta, v_{\widetilde{T}}\right) \quad (2.12)$$

The key problem is to get $v_{\widetilde{T}}$ from $(Y + \Delta Y) = v_{\widetilde{T}}^T (X + \Delta X)$. Theorem 1 gives a transformation relation from the known bounded data uncertainties $\left\|\Delta x^i\right\|_\infty \le \zeta, \left|\Delta y^i\right| \le \zeta_b$ into $\|\Delta X\|_2 \le \eta, \|\Delta Y\|_2 \le \eta_b$. Then the approach to get the $v_{\widetilde{T}}$ by using $\|\Delta X\|_2 \le \eta, \|\Delta Y\|_2 \le \eta_b$ is given in theorem 2 and two supplements.

Theorem 1: Given $\left\{t_i \in \widetilde{T} \mid i = 1, \cdots, p\right\}$ and a known input-output data set $\left\{(x^i, y^i) \mid x^i \in R^{n+1}, y^i \in R, i = 1,2,\cdots K\right\}$ with bounded data uncertainties $\left\|\Delta x^i\right\|_\infty \le \zeta, \left|\Delta y^i\right| \le \zeta_b$, the solution $v_{\widetilde{T}}$ of the robust least squares problem of a RFT model can be transformed into solving the following minimization problem,

$$\min_{v_{\widetilde{T}}} \max \left\{\|(X + \Delta X)\hat{x} - (Y + \Delta Y)\|_2 : \|\Delta X\|_2 \le \eta, \|\Delta Y\|_2 \le \eta_b\right\} \quad (2.13)$$

where η and η_b are selected as $\eta = \zeta\sqrt{(n+1)p}$ and $\eta_b = \zeta_b\sqrt{K}$ respectively.

Proof: Denote a $(n+1)p$ row vector by z. According to definitions and properties of the 2-induced norm and the ∞-induced norm of ΔX, the following deduction holds.

$$\|\Delta X\|_2 = \sup_{\|z\|_2 = 1} \|(\Delta X)z\|_2 \le \sup_{\|z\|_2 = 1} \|(\Delta X)z\|_\infty \le \|\Delta X\|_\infty \left(\sup_{\|z\|_2 = 1} \|z\|_\infty\right) \quad (2.14)$$

According to definitions of the ∞-induced norm of ΔX and Δx^i and $\left\|\Delta x^i\right\|_\infty \le \zeta$, the following deduction holds.

$$\|\Delta X\|_\infty = \max_j \sum_{l=1}^{n+1} \sum_{i=1}^{p} \left[\frac{\mu_{t_i}(x^j + \Delta x^j)}{\sum_{t \in \widetilde{T}} \mu_t(x^j + \Delta x^j)} \left|\Delta x_l^j\right|\right] \le \max_j \left\|\Delta x^j\right\|_\infty \le \zeta \quad (2.15)$$

For every $(n+1)p$ row vector z, there is the following inequation.

$$\|z\|_\infty \le \sqrt{(n+1)p}\,\|z\|_2 \quad (2.16)$$

Introduce the above two inequations into (2.14), hence we get the following expression.

$$\|\Delta X\|_2 \le \zeta\sqrt{(n+1)p} \sup_{\|z\|_2 = 1} \|z\|_2 = \zeta\sqrt{(n+1)p}$$

Similarly, according to the definition of the 2 norm of ΔY and the known bounded uncertainties $\left|\Delta y^i\right| \le \zeta_b$, there is the following expression.

$$\|\Delta Y\|_2 \le \sqrt{K}\,|\Delta y| = \zeta_b\sqrt{K}$$

Hence, the above two expressions give us a transformation relation from $\left\|\Delta x^i\right\|_\infty \le \zeta, \left|\Delta y^i\right| \le \zeta_b$ into $\|\Delta X\|_2 \le \eta, \|\Delta Y\|_2 \le \eta_b$. That means we can form the RFT model by solving the RLS problem of (2.13) with $\eta = \zeta\sqrt{(n+1)p}$, $\eta_b = \zeta_b\sqrt{K}$. Proof ends.

Theorem 2 (S. Chandrasekaran, G.H.Golub, et.al.1998): Given $A \in R^{m\times n}$,with $m \ge n$ and A full rank, $b \in R^m$, and nonnegative real numbers $(\eta \quad \eta_b)$. The following optimization problem

$$\min_{\hat{x}} \max\left\{\|(A+\Delta A)\hat{x}-(b+\Delta b)\|_2 : \|\Delta A\|_2 \le \eta, \|\Delta b\|_2 \le \eta_b\right\}$$

always has a solution $\hat{x}$. The solution(s) can be constructed as following.

- Introduce the SVD of A, $A = U\begin{bmatrix}\Sigma \\ \mathbf{0}\end{bmatrix}V^T$,

where $U \in R^{m\times m}$ and $V \in R^{n\times n}$ are orthogonal, and $\Sigma = diag(\sigma_1, \cdots, \sigma_n)$ is diagonal, with $\sigma_1 \ge \sigma_2 \ge \cdots \ge \sigma_n > 0$ being the singular values of A.

- Partition the vector $U^T b$ into $\begin{bmatrix}b_1 \\ b_2\end{bmatrix} = U^T b$

where $b_1 \in R^n$, $b_2 \in R^{m-n}$

- Introduce the secular function $\Omega(\alpha) = b_1^T(\Sigma^2 - \eta^2 I)(\Sigma^2 + \alpha I)^{-2} b_1 - \dfrac{\eta^2}{\alpha^2}\|b_2\|_2^2$.
- Define $\tau_1 = \dfrac{\|\Sigma^{-1}b_1\|_2}{\|\Sigma^{-2}b_1\|_2}$ and $\tau_2 = \dfrac{\|A^T b\|_2}{\|b\|_2}$.

First case: b does not belong to the column span of A. If $\eta \ge \tau_2$, then the unique solution is $\hat{x} = 0$. If $\eta < \tau_2$, then the unique solution is $\hat{x} = \left(A^T A + \hat{\alpha} I\right)^{-1} A^T b$, where $\hat{\alpha}$ is the unique positive root of the secular equation $\Omega(\alpha) = 0$.

Second case: b belongs to the column span of A. If $\eta \ge \tau_2$, then the unique solution is $\hat{x} = 0$. If $\tau_1 < \eta < \tau_2$, then the unique solution is $\hat{x} = \left(A^T A + \hat{\alpha} I\right)^{-1} A^T b$, where $\hat{\alpha}$ is the unique positive root of the secular equation $\Omega(\alpha) = 0$. If $\eta \le \tau_1$, then the unique solution is $\hat{x} = V\Sigma^{-1}b_1 = A^+ b$. If $\eta = \tau_1 = \tau_2$, then there are infinitely many solutions that are given by $\hat{x} = \beta V\Sigma^{-1}b_1 = \beta A^+ b$, for $0 \le \beta \le 1$.

Supplements: Theorem 2 gives us a general method to get the RLS solution. In order to apply it in RFT model, two supplements should be analyzed.

First, although $\|\Delta b\|_2 \le \eta_b$ is a given condition, η_b doesn't really affect the value of a RLS solution $\hat{x}$. Correspondingly the condition of $\left|\Delta y^i\right| \le \zeta_b$ in the RFT model with unstructured bounded data uncertainties can be deduced not to affect the value of the robust least squares solution $v_{\tilde{T}}$.

Secondly, according to the definitions of X, ΔX, x^i and Δx^i, we find that some columns of X which correspond to the constant term of x^i are unrelated to input and thus do never smeared by perturbations. We can arrange these precise columns as frontal part and other smeared columns as later part. This new constructed matrix becomes the matrix in a restricted perturbation problem (S. Chandrasekaran, G.H.Golub, et.al.1998). Given $A = [A_1 \quad A_2]$, $A \in R^{m\times n}$ with $\|\Delta A_2\|_2 \le \eta_2$, $\|\Delta b\|_2 \le \eta_b$ and

$$= R \quad = \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \\ 0 & 0 \end{bmatrix}, \qquad {}^T b = \begin{bmatrix} \bar{b}_{1A} \\ \bar{b}_{2A} \\ \bar{b}_2 \end{bmatrix}$$

with $R_{11} \in R^{(n-r)\times(n-r)}$, $R_{22} \in R^{r\times r}$, $\bar{b}_{1A} \in R^{n-r}$, $\bar{b}_{2A} \in R^r$, $\bar{b}_2 \in R^{m-n}$, the solution of the restricted perturbation problem is constructed as $\hat{x} = [\hat{x}_1 \quad \hat{x}_2]$, with $\hat{x}_1 = R_{11}^{-1}[\bar{b}_{1A} - R_{12}\hat{x}_2]$ and $\hat{x}_2$ being the solution of

$$\min\max\left\{\left\|\left(\begin{bmatrix} {}_{22} \\ 0 \end{bmatrix} + \Delta\begin{bmatrix} {}_{22} \\ 0 \end{bmatrix}\right)\hat{} - \left(\begin{bmatrix} \bar{b}_2 \\ \bar{b}_2 \end{bmatrix} + \Delta\begin{bmatrix} \bar{b}_2 \\ \bar{b}_2 \end{bmatrix}\right)\right\|_2 : \left\|\Delta\begin{bmatrix} {}_{22} \\ 0 \end{bmatrix}\right\|_2 \le \eta, \left\|\Delta\begin{bmatrix} \bar{b}_2 \\ \bar{b}_2 \end{bmatrix}\right\|_2 \le \eta_b\right\}$$

Hence, based on this method, we can get the robust least squares solution $v_{\tilde{T}}$ from the restricted perturbation matrix X.

2.3 Implementation of the RFT with unstructured data uncertainties

Given $\left\{(x^i, y^i) \mid x^i \in R^{n+1}, y^i \in R, i = 1,2,\cdots K\right\}$ and unstructured bounded data uncertainties $\left\|\Delta x^i\right\|_\infty \le \zeta, \left|\Delta y^i\right| \le \zeta_b$, a RFT model with data uncertainties can be implemented as following. Firstly, form a FT model according to algorithm 1. Secondly, find the robust linear coefficients $v_{\widetilde{T}}$ and form the RFT model according to algorithm 2.

Algorithm 1:

(1) Initialize parameters of the root node—let $\mu_{r(T)}(x)=1$ and estimate $c_{r(T)}$. Initialize α_t ($t \in \widetilde{T}$) and a error bound ER;

(2) Calculate e_t and e_t^c of each leaf node by using (2.7—2.8). Use the condition of $e_t > e_t^c$ to find out all divisional leaf nodes. If they/it exist(s), then continue, else end algorithm1 and go to algorithm 2;

(3) Divide every divisional node. Calculate $\{\theta_t \in \widetilde{T}\}$ and $\mu_t(x)$ of the new leaf nodes by using (2.4—2.6). Finally estimate all c_t ($t \in \widetilde{T}$) from (2.3);

(4) Train all α_t ($t \in \widetilde{T}$) and c_t ($t \in \widetilde{T}$). Calculate (2.3). If E declines and does not achieve ER, then go to (2), else go to algorithm 2.

Algorithm 2:

(1) Initialize $\eta = \zeta\sqrt{(n+1)p}$ and $\eta_b = \zeta_b\sqrt{K}$;

(2) Arrange all columns of X to become matrix A with the frontal p precise columns (suppose there are p leaf nodes);

$$\left.\begin{array}{l} A_{i,h} = X_{i,[(n+1)(h-1)+1]} \\ A_{i,[p+n(h-1)+q]} = X_{i,[(n+1)(h-1)+1+q]} \end{array}\right\} \begin{array}{l} h=1,2,\cdots p \\ i=1,2,\cdots K \\ q=1,2,\cdots n \end{array}$$

(3) Put the output vector Y as b into supplements and theorem 2 to get the solution $\hat{x}$;

(4) Inversely arrange the columns of $\hat{x}$ to get $v_{\widetilde{T}}$.

$$\left.\begin{array}{l} v_{\widetilde{T}(n+1)(h-1)+1} = \hat{x}_h \\ v_{\widetilde{T}(n+1)(h-1)+1+q} = \hat{x}_{p+n(h-1)+q} \end{array}\right\} \begin{array}{l} h=1,2,\cdots p \\ q=1,2,\cdots n \end{array}.$$

In summary, RFT model with unstructured data uncertainties not only possesses features of FT model, but also works on unstructured data uncertainties. Thus RFT model can be viewed as a robust version of FT model. Moreover, this model gives us a new idea to obtain the corresponding robust versions of other TS fuzzy logic models to work with unstructured uncertainties $\|\Delta x^i\|_\infty \le \zeta, |\Delta y^i| \le \zeta_b$.

3. SIMULATION RESULTS

Example 1: Consider a nonlinear function $y = x + \sin(x)$, $x \in [0, 2\pi)$. Data set is $\left\{(x^i, y^i) \mid x^i = \frac{2\pi}{N}(i-1) \in R, y^i = x^i + \sin(x^i), i = 1,2,\cdots N\right\}$ with $N = 126$. Form a RFT model and a FT model with 8 fuzzy rules. Compare the root mean squared errors (RMSE) for perturbed input of the two models in table 1.

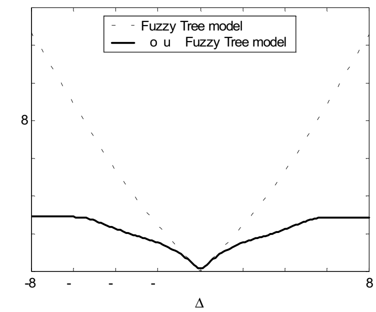

Figure 2. RMSE with $\zeta = 8.0$, $\Delta x \in [-8, 8]$

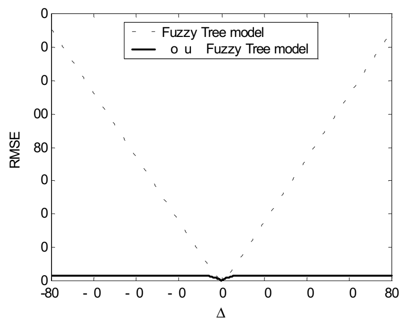

Figure 3. RMSE with $\zeta = 8.0$, $\Delta x \in [-80, 80]$

Figure 2 plots error functions of the two models. In the range of $\|\Delta x\|_\infty \le \zeta$ the error of the FT model is less than the error of the RFT model in a little range of $\|\Delta x\|_\infty \to 0$, but out of which the RFT model has less error. When $\|\Delta x\|_\infty$ increases from 0, the error of the FT model increases quickly while the error of the RFT model increases slowly. Furthermore, figure 3 indicates clearly that even when $\|\Delta x\|_\infty$ increases much larger than ζ, the error of the RFT model does not increase while the error of the FT model increases largely.

Other RFT models considering different $\|\Delta x\|_\infty \le \zeta$ are given in table 1. Data result that all these RFT models decreases drastically the sensitivity of the FT models to bounded uncertainties.

Table 1 Comparison of two models

ζ	Δx range	FT (RMSE)		RFT (RMSE)	
		Max	Min	Max	Min
1.0	[-1,1]	1.207	0.0055	0.981	0.159
4.0	[-4,4]	12.66	0.0055	2.883	0.159
8.0	[-8,8]	27.89	0.0055	2.883	0.159

***Example 2*:** Consider a nonlinear function $f(u, y, z) = (1 + u^{0.5} + y^{-1} + z^{-1.5})$. Denote input vector by $x = [1 \; u \; y \; z]$. Data set is $\left\{(1, {}^{i}, {}^{i}, {}^{i}) \mid {}^{i} = \frac{6\times}{N}, {}^{i} = \frac{6\times}{N}, {}^{i} = \frac{6\times}{N}, = 1,2,\cdots N\right\}$ with $N = 6$. Bounded uncertainty is $\|\Delta x\|_\infty \le \zeta = 3.0$. Form a robust tree model and a FT model with 16 fuzzy rules.

Table 2 Comparison of two models

Δx	FT (RMSE)		RFT (RMSE)	
	Max	Min	Max	Min
[0.82 0.82 0.34]	26.76	1.345	3.240	1.561
[0.31 0.57 -0.70]	9.061	1.345	3.278	1.563
[0.80 0.52 0.17]	26.56	1.345	3.489	1.567
[0.74 -0.01 0.20]	25.03	1.345	4.240	1.567

Random 3-dimensional perturbations in $\|\Delta x\|_\infty \le \zeta$ with uniform distribution are used to check the robustness of the RFT model and the FT model in table 2. Results show that the RFT model keeps smaller errors for different positions and amplitudes of perturbations.

Numerical simulation results show that RFT model is more robust for different positions and amplitudes of perturbations than FT model. In addition, these numerical experiments are obtained in less than one minute and thus show that the RFT model really reduces computational burden drastically than the former RFT model (Zhang J, 1999).

4. CONCLUSION

Based on RLS proposed by S. Chandrasekaran, G.H.Golub and FT model, this paper proposed RFT model with unstructured bounded data uncertainties. The RFT not only keeps the features that FT model can deal with high dimensional problem, has less computation load and has high precise, but also decreases drastically the sensitivity of FT model to uncertainties and thus possesses good robustness. Moreover, the procedure gives us a new idea to obtain corresponding robust versions of other TS fuzzy logic models.

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