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ROBUST FLIGHT CONTROLLER DESIGN FOR HELICOPTERS BASED ON GENETIC ALGORITHMS

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Abstract: With handling qualities requirements ADS-33 taken as design criteria, the robust controller for a helicopter in low-speed forward flight is designed by the use of H_∞ mixed sensitivity approach. In the process of designing the controller, genetic algorithms are used to optimize the parameters of weighting functions in order to search for the H_∞ controller which meets design specifications in both time domain and frequency domain. A robust flight controller for UH-60A helicopter is synthesized using the proposed technique. The resulting flight control system not only has robust stability, but also satisfies ADS-33D level 1 requirements. *Copyright ©2002 IFAC*

Keywords: Flight control, Robust control, H-infinity control, Helicopter, Multiobjective optimizations, Genetic algorithms

1. INTRODUCTION

A military helicopter has to satisfy the handling qualities requirements ADS-33 (U. S. Army Aviation and Troop Command, 1994), which put severe constraints on the allowable cross-coupling and the required frequency responses of the helicopter for the specific mission. However, a helicopter is characterized by the inherent instability, the high number of degrees of freedom, and the high degree of coupling between the state variables. Therefore, it is a challenge to develop a helicopter flight control system to meet the stringent ADS-33 level 1 requirements. In the last decade, a lot of research achievements (Ingle and Celi, 1994; Manness and Murray-Smith, 1992; Low and Garrard, 1993; Rozak and Ray, 1997; Takahashi, 1994; Dudgeon, *et al*., 1997) have been made on approaches to designing a flight controller for a rotorcraft system. The literature can be classified into two categories: Eigenstructure assignment and Riccati-based methods such as H_2, H_∞ and μ synthesis. Among these multivariable control methods the H_∞ technique has a broad base of support because of its robustness to uncertainties and reliable design algorithms. However, it is less tailored to design criteria such as those associated with ADS-33 and its success depends on the proper selection of the weighting functions.

The selection of appropriate weighting matrices reflecting the stability and the performance of a system remains a delicate task (Postlethwaite, *et al*., 1990; Yang, *et al*., 1997). There are no generic methods for systematically and efficiently selecting the weighting matrices, which relies on a designer's experience and familiarity with the design approach in most practical applications. The weighting functions chosen to shape the sensitivity functions are obtained through analysis of the uncertainties present in the system as well as from frequency- and /or time-domain requirements. However, H_∞ robust control is a frequency-domain design method and the time-domain specifications are also not easily transferable into the frequency domain. Hence, it is difficult to directly consider the time-domain performance requirements in the choice of the weighting matrices.

When weighting matrices are regarded as variables, H_∞ robust design can be formulated as a multi-objective optimization problem, which needs to simultaneously satisfy design specifications in both time-domain and frequency-domain. This optimization problem, however, is usually very complicated with many constraints (Tang, *et a*l., 1996; Whidborne, *et al*., 1994), and in most cases, the involved multi-objectives are conflicting. Hence, it is difficult to solve the constrained optimization problem by using traditional methods.

Genetic Algorithms (GA's) are search procedures based on the mechanics of natural, which are efficient computational tools (Goldberg, 1989; Zalzala, and Fleming, 1997) to solve complicated nonlinear optimization problems which are intractable traditionally. It can be characterized by group searching strategy, random information exchange among individuals in a population, and a global search algorithm that does not rely on the knowledge of gradient. Owing to the above advantages, it can be used to solve the above multi-objective optimization problem.

In this paper, genetic algorithms are used to search for the optimal weighting matrices in the framework of H_∞ mixed sensitivity design, and thus the satisfactory H_∞ controller meeting the given performance requirements are effectively obtained. A robust flight controller for UH-60A helicopter is synthesized using the proposed technique.

2. MATHEMATICAL MODELS OF THE HELICOPTER

The linear, time-invariant mathematical model (Dai Jiyang, Mao Jianqin, Yang Chao, 2000) used in this study is extracted from a fully nonlinear flight dynamic model of the Sikorsky UH-60A helicopter (Yang Chao, 1995) through numerical linearization about a trimmed flight condition at 30-kn forward flight.

The basic model consists of 25 states including fuselage rigid body modes, flap degrees of freedom for main rotor and tail rotor, first harmonic dynamic inflow for the main rotor and tail rotor dynamic inflow. The 25-state model is augmented with an 8-state model of the actuator dynamics. The actuators are modeled by four identical, uncoupled, second-order transfer functions. The highest order model used in this study therefore had 33 states, which was used to test the robustness and performance of the designed flight controller for the helicopter.

Based on the basic model, a direct truncation method was used to reduce the order of the helicopter model according to the principal component modules of the helicopter and the degree of coupling between modes. In comparing various reduced-order models, a 17-state model was used as the nominal design model, which includes 8 fuselage rigid body modes, 6 flap modes of the main rotor and tail rotor, 3 dynamic inflow modes for the main rotor and 1 tail rotor dynamic inflow mode.

3. HANDLING QUALITIES REQUIREMENTS

The goal of rotorcraft control system design is to achieve level 1 ADS-33D handling qualities performance. The ADS-33D specification is divided into hover/low speed and forward flight regimes. The UH-60A is a utility aircraft and falls under the "Moderate Maneuvering" MTE category. Satisfaction of the design requirements is assumed to be for a "Attitude Command Attitude Hold (ACAH)" response type performing the "Target Acquisition and Tracking Mission Task Element" in a low-speed flight with a "Useable Cue Environment (UCE)" of 1 and divided attention operations. The design criteria used are based on a representative subset of the low speed requirements of ADS-33D, which are listed below:

Table 1 Representative requirements of ADS-33D

	ADS-33 requirements		Section
1.	Bandwidth——	Pitch Axis	3.3.2.1
		Roll Axis	3.3.2.1
		Yaw Axis	3.3.5.1
2.	Divided Attention Operations （Pole Placement）		3.3.2.2.2 3.3.5.2.2
3.	Inter-axis Coupling		3.3.9
4.	Attitude Quickness		3.3.3, 3.3.6
5.	Collective Climb Rate Response		3.3.10.1

4. H_∞ CONTROLLER DESIGN BASED ON GENETICAL ALGORITHMS

4.1 H¥ mixed sensitivity design

Consider the tracking control problem of a continuous-time, linear time-invariant system shown in Fig. 1.

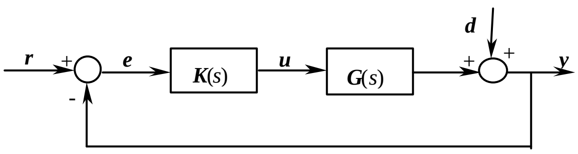

Fig. 1 Tacking control problem

where $\boldsymbol{r}$, $\boldsymbol{e}$, $\boldsymbol{u}$, $\boldsymbol{d}$ and $\boldsymbol{y}$ are the reference input, the tracking error, the control input, the disturbance and the feedback output, respectively. $\boldsymbol{K}(s)$ is the controller, and $\boldsymbol{G}(s)$ the plant.

Using the frequency-dependent weighting functions $\boldsymbol{W}_1(s)$, $\boldsymbol{W}_2(s)$ and $\boldsymbol{W}_3(s)$ to weight the signals $\boldsymbol{e}$, $\boldsymbol{u}$ and $\boldsymbol{y}$ respectively, a generalized system is constructed, which can be furthermore converted into the standard small gain problem. The closed-loop transfer function of the generalized system is defined by

$$T_{zr} = \begin{bmatrix} W_1 S \\ W_2 KS \\ W_3 GKS \end{bmatrix} = \begin{bmatrix} W_1 S \\ W_2 R \\ W_3 T \end{bmatrix} \tag{1}$$

where $S(s)$ is sensitivity function, and $T(s)$ is complementary sensitivity function.

The H_∞ mixed sensitivity design problem is to find a proper rational transfer controller that stabilizes the closed-loop system and satisfies

$$\left\| T_{zr} \right\|_\infty \le g \;\; (g > g_0, g_0 = \min \left\| T_{zr} \right\|_\infty) \tag{2}$$

These transfer function matrices are all diagonal to maintain the desired decoupled control response. In order to guarantee the good ability of rejecting the disturbance and tracking, low-pass filters are used on the diagonal of W_1. W_2 and W_3 are both high-passed. The former are usually taken as diagonal constant matrix to avoid increasing the order of the controller, and the latter may be a diagonal matrix with elements of non-proper real rational transfers, but it needs to satisfy the inequality constraint:

$$\underline{s}(W_1^{-1}(jw)) + \overline{s}(W_3^{-1}(jw)) \ge 1 \tag{3}$$

4.2 Weighting matrices optimization problem

In the mixed sensitivity design, the weighting matrices W_1, W_2 and W_3 can be regarded as the design parameters. The H_∞ design of a control system can be expressed as a minimization problem under the inequality constraints.

Given a nominal plant $G(s)$, to find (W_1, W_2, W_3) such that

$$\underset{W_1, W_2, W_3}{\text{Minimize}} \; y\,(W_1, W_2, W_3) \tag{4}$$

subject to

$$\underline{s}(W_1^{-1}(jw)) + \overline{s}(W_3^{-1}(jw)) \ge 1 \tag{5}$$

$$g_0(W_1, W_2, W_3) \le e_0 \tag{6}$$

$$f_i(W_1, W_2, W_3) \le e_i \,, \qquad i = 1, 2, \cdots, n \tag{7}$$

where y is the performance index to be minimized, and f_i's are performance indices representing rise time, overshoot, and bandwidth, etc., and e_0 and e_i are real numbers representing the desired bounds on g_0 and f_i, respectively.

The optimal weighting matrices can be found, and simultaneously the satisfactory controller can be synthesized by solving the optimization problem (4)-(7). Since the constrained optimization is usually a non-convex, non-smooth, and multi-objective problem with several conflicting design aims, it is hard to be solved by traditional optimization approaches. Hence, an efficient numerical search algorithm, namely a multiobjective genetic algorithm, is proposed to find solutions to such optimization problem.

4.3 Chromosome coding

According to the requirements on the robustness and performance of the system, there are a variety of different structures of the weighting matrices to select, e.g. scalar, first-order and second-order transfer functions. However, when performing continuous-time H_∞ synthesis, the weighting functions need to be bi-proper as to insure the row rank of D_{12} is full. Without loss of generality, in this investigation the structures of W_1 and W_3 are both selected as first order bi-proper forms, and the structure of W_2 is selected as scalar form, that is

$$W_1 = diag(w_{11}, w_{12}, \cdots, w_{1m}) \tag{8}$$

$$W_2 = diag(w_{21}, w_{22}, \cdots, w_{2m}) \tag{9}$$

$$W_3 = diag(w_{31}, w_{32}, \cdots, w_{3m}) \tag{10}$$

where

$$w_{1i} = \frac{k_{1i}(\frac{1}{w_{1i}} s + 1)}{(\frac{1}{w_{2i}} s + 1)}, \quad w_{3i} = \frac{k_{3i}(\frac{1}{w_{3i}} s + 1)}{(\frac{1}{w_{4i}} s + 1)},$$

$$w_{2i} = k_{2i}, \quad i = 1, \cdots, \; m; \tag{11}$$

where m is the number of the dimension of the output vector.

Each parameter in (11) is coded by a binary string, and accordingly a chromosome or an individual is generated by joining all the strings in series, which is defined by

$$g_r = \{ w_{11}, w_{12}, \cdots, w_{4m}, k_{11}, k_{12}, \cdots, k_{3m} \} \tag{12}$$

4.4 Objective function and fitness function

The following algorithm is presented to calculate the objective function in the mixed optimization problem.

Algorithm 1:

1) For each individual, generate the corresponding W_1, W_2, W_3 by decoding.

2) If $\underline{s}(W_1^{-1}(jw)) + \overline{s}(W_3^{-1}(jw)) \ge 1$,

a) Synthesize H_∞ controller $K(s)$ using Riccati-based method, and Calculate g_0.

b) If $g_0 < e_0$, calculate the performance indices y and f_i of the closed-loop system, and calculate the objective function by

$$f = y + \sum_{i=1}^{n} r_i \tag{13}$$

where $r_i = \begin{cases} 0, & f_i \le e_i \\ (e_i - f_i)^2, & f_i > e_i \end{cases}$

c) If $g_0 \ge e_0$, the objective function is defined by $f = ng_0$.

3) If $\bar{s}(W_1^{-1}(jw)) + \bar{s}(W_3^{-1}(jw)) < 1$, Take f as a large real number (for instance, 10^6).

In order to search for the optimal solutions by means of GA efficiently, a linear ranking approach is used to convert the objective function f to the fitness value.

4.5 Search for optimal weighting matrices and corresponding H¥ controller

The following algorithm is given to optimize the weighting matrices.

Algorithm 2:

1) For the nominal plant $\boldsymbol{G}(s)$, Determine y and f_i.
2) Determine e_0 and e_i in terms of the requirements on robustness and performance.
3) Determine the structures of the weighting matrices $\boldsymbol{W}_1$, $\boldsymbol{W}_2$ and $\boldsymbol{W}_3$, and determine the search domains of the parameters.
4) Determine the control parameters for GA, such as population size, selection rate, crossover rate, mutation rate, etc.
5) Randomly generate the first population.
6) Calculate the objective value and assign the fitness value to each chromosome.
7) Start the process of GA search:
   a) Perform selection operation using elitist and tournament selection strategies.
   b) Perform crossover and mutation operations and generate new individuals.
   c) Calculate the objective values of the new individuals.
8) Terminate if the condition (for example, the number of generation exceeds the given maximum D); otherwise, repeat 7).

Since the selection of the weighting matrices is a multi-objective optimization problem, only satisfactory feasible solutions are found by the above algorithm. Meanwhile, the H∞ controller satisfying the design specifications can be obtained.

5. HELICOPTER FLIGHT CONTROLLER DESIGN

5.1 Helicopter ACAH system

The helicopter flight control system is designed to be an ACAH system. The structure of the helicopter H∞ control system is adopted as shown in Fig. 1. The reference input vector was defined to be the pilot commands in heave, pitch, roll, and yaw:

$$\boldsymbol{r}(t) = [d_c \; d_e \; d_a \; d_p]^T \tag{14}$$

The input vector to the design plant is comprised of the swashplate tilt and main and tail rotor collective:

$$\boldsymbol{u}(t) = [q_0 \; q_{1s} \; q_{1c} \; q_{0T}]^T \tag{15}$$

The pitch rate and roll rates q, p are included in the ACAH design because the plant can not be stabilized without rate feedback. Thus the feedback output vector is defined by

$$\boldsymbol{y}(t) = [\dot{H} \;\; q + k_q q \;\; f + k_p p \;\; r]^T \tag{16}$$

where $\dot{H}$ is height rate, and k_q and k_p are constants whose values are taken as k_q=k_p=0.1.

5.2 Determination of objective functions

The objective functions can be determined according to level 1 ADS-33D as follows:

(1) Attitude quickness can be described by error functions produced by on-axis step responses of the helicopter. The helicopter flight control system is a 4-input and 4-output system. Let $y_{ji}(t)$ (i=1, 2, 3, 4; j=1, 2, 3, 4) denote the outputs of the closed-loop system to unit step inputs $r_i(t)$=1(t) (i=1, 2, 3, 4), the error functions are defined by

$$e_i = \int_0^{T_0} [r_i(t) - y_{ii}(t)]^2 \, dt \tag{17}$$

$$e_{ji} = \int_0^{T_0} [0 - y_{ji}(t)]^2 dt \quad (j \neq i) \tag{18}$$

where $y_{ii}(t)$ and $y_{ji}(t)$ ($j \neq i$) are the on-axis and off-axis responses respectively, and T_0 is a time constant which is greater than the transient time of the system and is set to be 5s.

Hence, the performance index to be minimized is defined by

$$y = k_e \max_i (e_i) \tag{19}$$

where k_e is an adjusting factor and k_e=0.5.

(2) The degree of inter-axis coupling can be reflected by off-axis response errors e_{ji} ($j \neq i$). The smaller e_{ji} is, the smaller the couplings of each axis will be. Set

$$f_1 = \max_{j \neq i}(e_{ji}), \quad e_1 = 0.05 \tag{20}$$

(3) Collective climb rate (to a step input) should resemble a delayed first-order response. Set

$$f_2 = \max_t (y_{11}(t)), \quad e_2 = 1 \tag{21}$$

(4) Let w_{bq}、 w_{bf} and w_{by} be the bandwidths of pitch, roll and raw axes respectively, and let t_{pq} , t_{pf} and t_{py} be delay times of three axes. Set

$$f_3 = -w_{bq}, \quad e_3 = -2 \tag{22}$$

$$f_4 = t_{pq} - 0.1 w_{bq}, \quad e_4 = -0.05 \tag{23}$$

$$f_5 = -w_{bf}, \quad e_5 = -2.5 \tag{24}$$

$$f_6 = t_{pf}, \quad e_6 = 0.16 \tag{25}$$

$$f_7 = -w_{by}, \quad e_7 = -3.5 \tag{26}$$

$$f_8 = t_{py} - 0.11 w_{bq}, \quad e_8 = -0.225 \tag{27}$$

(5) For the requirements on pole placement, assume that there be q pairs of complex poles (x_i, w_{di}) (i=1,$\cdots$,q) satisfying $w_{di} \geq 0.5$ in the

closed poles， then the damp ratios of these poles should satisfy $x_i \geq 0.35$. Hence, set

$$f_9 = -\min_i(x_i)\,, \quad e_i = -0.35 \tag{28}$$

5.3 Choice of parameters and optimization results

Some GA parameters are chosen as the following.

$\boldsymbol{R}_1 = [10^2, 10^3]$, $\boldsymbol{R}_2 = [10^{-3}, 10^{-2}]$, $\boldsymbol{R}_3 = [10^{-3}, 10^{-2}]$,

$\boldsymbol{R}_4 = [10^2, 10^3]$, $\quad \boldsymbol{R}_6 = [10^{-2}, 10]$

$k_{11} = k_{12} = k_{13} = k_{14}$, $\quad \boldsymbol{R}_5 = [50, 10^3]$

$k_{31} = k_{32} = k_{33} = k_{34} = 1/k_{11}$, $\boldsymbol{R}_7 = [0.01, 10]$

and

population size N=31， crossover rate p_c=0.85， mutation rate p_m=0.001， terminal generation D=40 and tournament size L=2.

Using Algorithms 1 and 2, the three satisfactory weighting matrices were effectively found, and simultaneously a 25-order H∞ controller with g_0 being 0.5 was obtained.

6. PERFORMANCE EVALUATION OF HELICOPTER CONTROL SYSTEM

6.1 Stability robustness analysis

For the output multiplicative uncertainty structure of the helicopter plant, we have

$$\boldsymbol{G}_{33}(s) = [\boldsymbol{I} + \boldsymbol{\Delta}(s)]\boldsymbol{G}_{17}(s) \tag{29}$$

where $D(s)$ is a stable unstructured uncertainty. According to the small gain theorem, the system will remain stable if

$$\overline{s}[\boldsymbol{\Delta}(jw)] < \underline{s}[\boldsymbol{T}^{-1}(jw)] \tag{30}$$

where $\overline{s}(\bullet)$ and $\underline{s}(\bullet)$ denote the largest and smallest singular values.

Fig. 2 gives robustness test, which is satisfied in the whole range of frequency. Therefore, the designed flight control system possessed stability robustness.

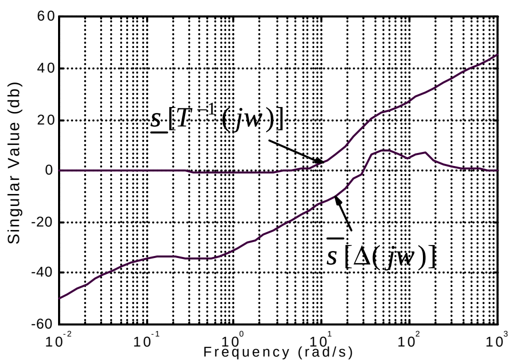

Fig. 2 Multiplicative robustness test

6.2 Time response analysis

The responses of the helicopter to the four pilot commands were simulated using the non-design model with 33rd order, which are shown in Figs. 3-6. In these figures, y1 denotes height rate $\dot{H}$, y2 pitch angle q, y3 roll angle f, and y4 yaw rate r. These simulations showed that the dynamic responses to pilot commands behaved well and were significantly decoupled.

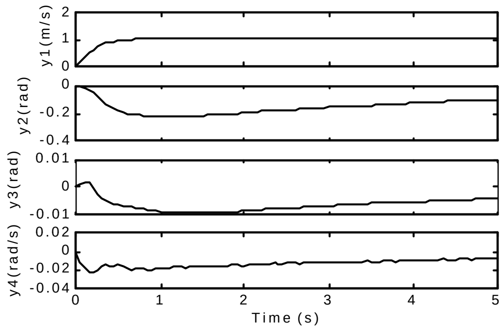

Fig. 3 Height response

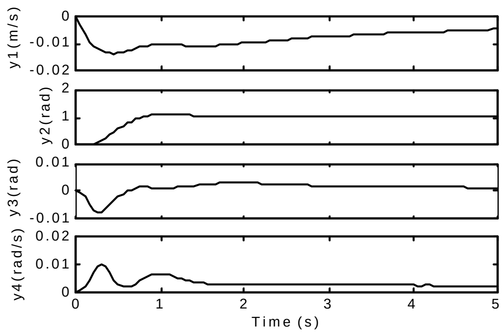

Fig. 4 Pitch response

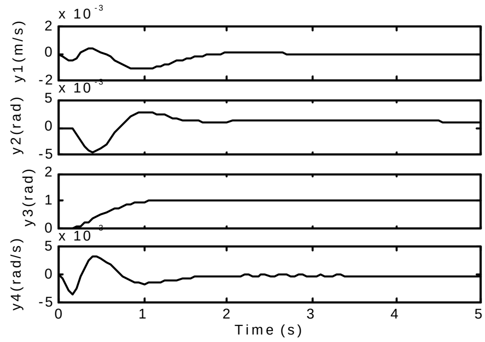

Fig. 5 Roll response

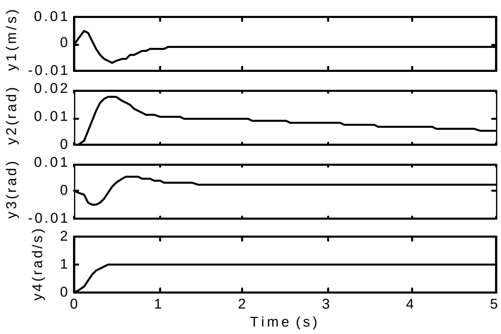

Fig. 6 Yaw response

6.3 ADS-33 handling qualities assessment

According to ADS-33D handling quality requirements on helicopters at hover/ low-speed flight, all required performance indexes are also tested, which meet level 1 ADS-33D handling qualities requirements. Table 2 gives the bandwidth and phase delay parameters. From the test results, the flight control system also had good performance robustness.

Table 2 Bandwidth and phase delay parameters

Off-design model
(33-state helicopter linearized model)

	ω_{BW} (rad/s)	τ_p (s)
Pitch	4.74	0.123
Roll	4.66	0.061
Yaw	4.35	0.054

7. CONCLUSIONS

Robust flight control problem of UH-60A helicopter has been investigated using GA-based H_∞ controller design method. The design criteria used in this study are based on a representative subset of the low-speed requirements of the Military Handling Qualities Specification ADS-33D. The design objectives are achieved by using genetic algorithms to optimize the parameters of the three weighting function matrices in loop shaping H_∞ design method. The resulting flight control system not only has stability robustness, but also meets level 1 ADS-33D handling qualities requirements when unstructured uncertainty is considered.

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