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RATE-BASED CONGESTION CONTROLLERS FOR HIGH-SPEED COMPUTER NETWORKS

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Abstract: The present paper proposes a control-theoretic approach to design rate-based controllers in order to flow-regulate the best-effort traffic through high-speed computer communication networks. Classical control theory and Schur-Cohn stability test are exploited to design the traffic controllers for high-speed networks. The stability of closed-loop congestion controlled systems is analysed by utilizing Schur-Cohn stability criterion, which leads to certain necessary and sufficient stability condition under which the controlled network is asymptotically stable in terms of buffer occupancy. The proposed stability condition is then shown to be a key tool in designing a wide scope of adaptive controllers. Simulations are performed that show good performance of such controlled networks. *Copyright © 2002 IFAC*

Keywords: computer communication networks, computer simulation, control system analysis, linear control systems, stability analysis.

1. INTRODUCTION

Congestion in a network is a state when the system performance degrades due to the saturation of network resources such as link capacities, processor cycles, and data buffers. Lacking *congestion control* gives rise to adverse effects including the long delay of packet or cell delivery, low throughput, and even possible network collapse. Congestion control has thus become a challenge in designing and managing a high-speed network due to ever-growing intensive network applications.

In high-speed networks, e.g., Asynchronous Transfer Mode (ATM) switching networks, two basic classes of service are currently under investigation, namely reserved traffic with *guaranteed service*, and *best-effort service* traffic, e.g. ABR (available-bit-rate) (Iliadis, 1995) service. Correspondingly, two classes of traffic control approaches have been proposed for high-speed networks, they are *open-loop control* and *closed-loop control* (Kung *et al*., 1994) respectively. Open-loop control approach has some advantages in supporting real-time and delay-sensitive communication service such as CBR (constant-bit-rate) service and VBR (variable-bit-rate) service. However, open-loop control is not suitable for dealing with best-effort traffic due to the fact that bandwidth requirements for this kind of traffic can be unpredictable and variable over time. For the best-effort traffic, it is shown (Kung *et al*., 1994; Yang and Reddy, 1995) that the closed-loop feedback control provides the relatively more effective solution to bandwidth sharing among all competing users. Two types of control mechanisms have been used in closed-loop control, i.e., *window control* and *rate control*. In the case of window control, a maximum number of cells (or packets) is specified that a source can transmit, the window size thus limits the maximum number of cells, and hence the source throughput, that can be transmitted in a round-trip interval; while the rate-based control, for example, *forward explicit congestion notification* (FECN) and *backward explicit congestion notification* (BECN) (Yang and Reddy, 1995), regulates the source rate based on feedback information on the *buffer occupancy* in the switching node. These kinds of control strategies are very effective in conventional packet-switched networks and attracting increasing research interests (Kalarov and Ramamurthy, 1997; Zhang *et al*., 2000).

Stability of closed-loop system is critical in any congestion control scheme due to the fact that, *propagation delay* encountered in high-speed networks may cause the controllers and the whole network to operate at an unstable point. This yields the notorious *oscillation problem* that greatly degrades the network performance. Concerning this issue, some control-theoretic concepts were proposed in (Benmonhamed and Meekov, 1993) for ATM networks, and were further applied in Benmonhamed and Meekov (1994) and Kalarov and Ramamurthy (1997). These approaches, however, usually require

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an online turning of control parameters to ensure stability and good performance under different network conditions, which definitely bring inconvenience to actual network implementation. Mostly recently in Mascolo (2000) *Smith's principle* was applied in designing a control law for ABR input rates in ATM networks. Zhang *et al*. (2000) proposed a hop-by-hop congestion controller designing method, in which system stability was met to the occasionally chosen controller. Such specially chosen controller guaranteed the requirement of system stability in this special occasion, but may not be able to meet other performance requirements of actual network in other circumstances, for example, to limit the duration of response time and maximize the throughput (Schwartz, 1996). In Izmailov (1995), two *linear feedback control* algorithms have been proposed for the case of a single connection with a constant service rate. Pan *et al.* (1996) considered a single-controlled traffic source, sharing a bottleneck node with other sources, whereas H^{∞} control approach was used for designing the controller. Although the issue of stability in modelling and analysing network congestion control system have been concerned in one way or another in the aforementioned publications, no explicit general stability condition has been obtained so far. Such general statement of stability is considered (Schwartz, 1996) to be important in actual network congestion controller designing, for only on the basis of it can one choose a wide scope of controllers to meet a wide range of performance requirements.

Concerning the rate-based congestion control schemes which are widely applied in high-speed switching networks, the present paper exploits classical control theory and Schur-Cohn stability criterion to design the traffic controllers for them. Specifically, the stability and transient response of closed-loop congestion-controlled systems are analysed by using Schur-Cohn stability criterion, which leads to certain sufficient and necessary stability condition under which the controlled switching network is asymptotically stable in terms of buffer occupancy. The stability condition is then shown to be a key tool in designing a wide scope of adaptive controllers. Simulations are performed that show good performance of such controlled networks.

2. NETWORK SYSTEM MODELING AND BASIC ANALYSES

A data communication network generally consists of a number of source/destination nodes which are geographically distributed. Cells or packets generated at a source node are delivered to their destination through a series of intermediate nodes. In modelling the traffic through these nodes, one has to know the number of source/destination pairs and the rates at which these sources introduce cells or packets into the network. Figure 1 represents the *continuous time system* model which we are going to consider in the sequel. For the sake of simplicity, only one single switching node is considered, which is connected by two virtual connections (VCs). One VC carries the uncontrolled traffic (e.g., guaranteed traffic) which is not throttled at the source node as long as it confirms to the specifications. The other one carries the controlled traffic (e.g., best-effort traffic) which can only be transmitted when there does not exist congestion in the network. The switching node has a limited *buffer size* K to store the incoming cells or packets and an output link with the capacity of $\overline{\mu}(t)$. There are two kinds of delays for the controlled VC: τ_1' is the input delay from the source node to the switch node and τ_2' is the feedback delay from the switch node to the source node. Let $w(t)$ denote the total uncontrolled traffic rate, and $q(t)$ denotes the controlled traffic rate. Based on the buffer occupancy $x(t)$ which is measured and sent back to the source node every T seconds, $q(t)$ will be adjusted.

Under the above notations and assumptions, the dynamics of a switching node in a network can be described by the following non-linear time-delayed equation (Benmonhamed and Meekov, 1993)

$$\dot{x}(t) = Sat_K\{q(t-\tau_1') + w(t) - \overline{\mu}(t)\} \tag{1}$$

where the saturation function $Sat_K\{x\}$ is given by

$$Sat_K\{x\} = \begin{cases} K, & x > K \\ x, & 0 \le x \le K \\ 0, & x < 0 \end{cases}$$

Without loss of generality we only consider the case where the input delay τ_1' and the feedback delay τ_2' are exactly integral multiples of T, i.e. , $\tau_1' = \tau_1 T$, $\tau_2' = \tau_2 T$, τ_1 and τ_2 are integers, for otherwise one can always add small delays to the input delay and the feedback delay to achieve this. (1) can be further discredited into

$$\begin{aligned} x((n+1)T) &= x(nT) + \\ &\quad + T\, Sat_K\{q((n-\tau_1)T) + w(nT) - \overline{\mu}(nT)\} \end{aligned} \tag{2}$$

Due to the fact that, $0 \le x(nT) \le K$, (2) is subsequently written as

$$\begin{aligned} &x((n+1)T) = \\ &Sat_{K'}\{x(nT) + \lambda((n-\tau_1)T) + d(nT) - \mu(nT)\}, \end{aligned} \tag{3}$$

where $K' = (T+1)K$, $\lambda(nT) = Tq(nT)$ and $d(nT) = Tw(nT)$ denotes the packets (or cells) flowing into the network from the VC and flowing into the switch from the uncontrolled traffic respectively during the n^{th} interval of T , and $\mu(nT) = T\overline{\mu}(nT)$ denotes the number of packets (or cells) emitted from the switch node during the n^{th} interval of T . One further denotes $\tau = \tau_1 + \tau_2$, and

$$u(nT) = \lambda((n+\tau_2)T) - \mu_1(nT), \tag{4}$$

where $\mu_1(nT)$ is the maximum number of packets allowed for the source node to transmit the data into the network in the n^{th} interval of T, and let $\hat{\mu}(nT)=\mu(nT)-\mu_1((n-\tau)T),$ one then has

$$x((n+1)T)=Sat_{\kappa'}\begin{Bmatrix} x(nT)+u((n-\tau)T) \\ +d(nT)-\hat{\mu}(nT)\end{Bmatrix}. \quad (5)$$

3. STABILITY ANALYSIS AND DESIGN OF CONGESTION CONTROLLERS

In this section, a control-theoretic approach to design a class of end-to-end rate-based congestion controllers will be presented, which is mainly based on the classical control theory and Schur-Cohn stability criterion. To investigate the stability of the network described by (5) with a proportional-like congestion controller being implemented, it has been shown in Benmonhamed and Meekov (1993) that it is suffice to study the locally asymptotical stability of the system by removing the saturation restriction which is posed on the network system (5). There are two-fold reasons to do so, one is that a mathematically rigorous proof of stability of the system using Liapunov's approach may be possible, but this seems not to give any significant insight into system structures; the other one is that this saturation non-linearity will not generally be activated for a network switch node with a reasonably large buffer size (Yang and Reddy, 1995). We therefore focus our attention on the network model (*state equation*) given by

$$\begin{aligned} x((n+1)T)=x(nT)+u((n-\tau)T)+ \\ +d(nT)-\hat{\mu}(nT). \end{aligned} \quad (6)$$

Let the system *output equation* be described by

$$y(nT)=x(nT)+\sum_{i=0}^{n}u(iT), \quad (7)$$

where $u(iT)$ satisfies (4), and where the sum of the previous control inputs acts as the *direct feed-through* to the system. Note that this model is actually a modification to the proposed ones in the literatures (Benmonhamed and Meekov, 1993; 1994). The modifications made therein are (a) rather than assume the source emitting rate $\bar{\mu}$ to be a constant, we let it be a function of time, and (b) we have added a direct feed-through item to shape the system structure. As will seen later, such modified descriptions characterize the system structures in a more precisely manner. Thus we have

$$\begin{aligned} y((n+1)T) &= x((n+1)T)+\sum_{i=0}^{n+1}u(iT) \\ &= x(nT)+u((n-\tau)T)+d(nT) \\ &\quad -\hat{\mu}(nT)+u((n+1)T)+\sum_{i=0}^{n}u(iT) \\ &= y(nT)+u((n-\tau)T)+u((n+1)T) \\ &\quad +d(nT)-\hat{\mu}(nT). \end{aligned} \quad (8)$$

If one chooses the *output feedback*

$$u(nT)=\alpha y(nT), \quad (9)$$

where the parameter α is to be determined such that the closed-loop system is stable, then

$$\begin{aligned} y((n+1)T)=y(nT)+\alpha y((n-\tau)T)+ \\ +\alpha y((n+1)T)+d(nT)-\hat{\mu}(nT). \end{aligned} \quad (10)$$

The z-transformation of (10) is

$$z^T Y(z)=Y(z)+\alpha z^{-\tau T}Y(z)+\alpha z^T Y(z)+D(z)-\Phi(z),$$

which is actually

$$\Delta(z)Y(z)=D(z)-\Phi(z), \quad (11)$$

where we have denoted $Y(z)=\sum_{n=0}^{\infty}y(nT)z^{-n}$,

$D(z)=\sum_{n=0}^{\infty}d(nT)z^{-n},\ \Phi(z)=\sum_{n=0}^{\infty}\hat{\mu}(nT)z^{-n},$ and

$$\Delta(z)=z^T-1-\alpha z^{-\tau T}-\alpha z^T. \quad (12)$$

3.1 Stability of the congestion controlled systems

Considering in (8), $\Delta(z)$ is seen to be the *characteristic polynomial* (**CP**) of the closed-loop system. Stability of closed-loop system is critical in congestion controller designing due to the fact that, *propagation delay* encountered in high-speed networks may cause *oscillation* to the network that greatly degrades the network performance, while CP (12) is closely related to this issue. From a control theory point of view, in order to achieve asymptotic stability of the system it is necessary to choose the appropriate parameter α such that all the zeros of CP (12) lie inside the unit circle. Such choices are obviously abound. As will be shown later, different choices of the parameter α lead to specific congestion controlling schemes. Every kind of control scheme may have its own advantages in some aspect of performance, may in the meantime inherit some disadvantages in other aspects of network performance (see, for example, Yang and Reddy, 1995)). From a rich resource of controlling schemes, one is able to choose the appropriate one to meet specific performance requirements for some particular networking purpose. It is, therefore, quite desirable to have a basic understanding to the CP (12). Concerning this, the following condition can be proposed on the basis of *Schur-Cohn stability criterion* (Rosenbrock, 1970).

Theorem 1: The closed-loop congestion controlled system (6), (9) is stable if and only if $\alpha<\frac{1}{\tau+2}$.

Proof : It is noted that, for the closed-loop congestion controlled system (6), (9) to be stable if and only if all the roots of its CP

$$\begin{aligned} \Delta(z) &= z^T-1-\alpha z^{-\tau T}-\alpha z^T \\ &= z^T(1-\alpha)[1-\frac{1}{1-\alpha}z^{-T}-\frac{\alpha}{1-\alpha}z^{-(\tau+1)T}],(\alpha\neq 1). \end{aligned}$$

lie inside the unit circle (Rosenbrock, 1970). By letting $\Delta_1(z)=1-\frac{1}{1-\alpha}z^{-T}-\frac{\alpha}{1-\alpha}z^{-(\tau+1)T}$, this is seen to be equivalent to the condition that all the roots of $\Delta_1(z)$ lie inside the unit circle for $\Delta(z)$ has all the roots of $\Delta_1(z)$ together with a root $z=0$, which is obvious inside the unit disc.

There are several computational procedures that aid us in determining if any of the roots of $\Delta_1(z)$ lie outside the unit circle. These procedures are called *stability criteria*. We use Schur-Cohn stability test to prove our statement. For this purpose, we construct a polynomial from $\Delta_1(z)$ by $A_{(\tau+1)T}(z)=\Delta_1(z)$, from which we find the *reflection coefficient*

$$K_{(\tau+1)T}=-\frac{\alpha}{1-\alpha},$$

and the *reciprocal or reverse polynomial* of $A_{(\tau+1)T}(z)$ is defined by

$$B_{(\tau+1)T}(z)=z^{-(\tau+1)T}A_{(\tau+1)T}(z^{-1})$$

$$=z^{-(\tau+1)T}-\frac{1}{1-\alpha}z^{-\tau T}-\frac{\alpha}{1-\alpha},$$

thus we compute the lower-degree polynomials $A_{mT}(z), m=\tau,\tau\text{-}1,\cdots,1$, according to the recursive equation $A_{(m-1)T}(z)=\dfrac{A_{mT}(z)-K_{mT}B_{mT}(z)}{1-K_{mT}^2}$, and get the reflection coefficients K_{mT}, m=τ,τ-1,⋯,1. The *Schur-Cohn stability test* states that the polynomial $\Delta_1(z)$ has all its roots inside the unit circle if and only if the coefficients K_{mT} satisfy the condition $|K_{mT}|<1$ for all $\mathrm{m}=\tau,\tau\text{-}1,\cdots,1$. From $|K_{(\tau+1)T}|<1$, it follows

$$\alpha<\frac{1}{2}\ . \tag{13}$$

From

$$A_{\tau T}(z)=\frac{A_{(\tau+1)T}(z)-K_{(\tau+1)T}B_{(\tau+1)T}(z)}{1-K_{(\tau+1)T}^2}$$

$$=\frac{1}{1-2\alpha}[-\alpha z^{-\tau T}-(1-\alpha)z^{-T}+(1-2\alpha)],$$

one follows

$$K_{\tau T}=-\frac{\alpha}{1-2\alpha},\ \ |K_{\tau T}|<1,\ \Rightarrow\ \alpha<\frac{1}{3}. \tag{14}$$

Thus

$$B_{\tau T}(z)=z^{-\tau T}A_{\tau T}(z^{-1})$$

$$=\frac{1}{1-2\alpha}[-\alpha-(1-\alpha)z^{-(\tau-1)T}+(1-2\alpha)z^{-\tau T}],$$

subsequently

$$A_{(\tau-1)T}(z)=\frac{A_{\tau T}(z)-K_{\tau T}B_{\tau T}(z)}{1-K_{\tau T}^2}$$

$$=\frac{1}{(1-2\alpha)^2-\alpha^2}[-\alpha(1-\alpha)z^{-(\tau-1)T}-$$

$$\text{-}(1-\alpha)(1-2\alpha)z^{-T}+(1-2\alpha)^2-\alpha^2]$$

from which one knows

$$K_{(\tau-1)T}=-\frac{\alpha}{1-3\alpha},\ \ |K_{(\tau-1)T}|<1,\ \ \Rightarrow\ \alpha<\frac{1}{4}\ . \tag{15}$$

With the above recursive process continuing, one finally arrives

$$K_T=\frac{\alpha}{1-(\tau+1)\alpha},\quad |K_T|<1,\ \Rightarrow\ \alpha<\frac{1}{\tau+2}. \tag{16}$$

Considering in (13-16) and by using Schur-Cohn stability test one concludes the proof. #

Remark 1: The above result displays a general condition on the parameter α in order for the closed-loop congestion-controlled systems to be stable. This then leads to a class of congestion controllers as will be described in the next subsection. From them one is able to choose the specific control algorithms to meet relevant performance requirements in networking engineering applications. The importance of this approach will become evident from the simulation results presented in the sequel.

3.2 Design of rate-based congestion controllers

We next study the designing of end-to-end rate-based congestion controller. The aim of congestion control is to regulate the source rate based on feedback information on the *buffer occupancy* in the switching node. Based on the observation made in Theorem 1, it is established that the following control-theoretic algorithms can flow-regulate the best-effort service and guaranteed service traffic through high-speed networks.

Theorem 2: For any $\alpha<\dfrac{1}{\tau+2}$, for the network described by (6), the source rate is regulated by the following algorithm

$$\begin{aligned}\lambda(nT)&=\alpha x((n-\tau_2)T)+\alpha\sum_{i=0}^{n-\tau_2}\lambda((i+\tau_2)T)\\&\quad+\alpha(n-\tau_2+1)\mu_1(nT)-\mu_1(nT).\end{aligned} \tag{17}$$

Proof: Theorem 1 states that the congestion-controlled network system is stable if $\alpha<\dfrac{1}{\tau+2}$. Considering in (4), (7) and (9), one has

$$\begin{aligned}&\lambda(nT)\\&=u((n-\tau_2)T)-\mu_1(nT) && \text{(from (4))}\\&=\alpha y((n-\tau_2)T)-\mu_1(nT) && \text{(using (9))}\\&=\alpha[x((n-\tau_2)T)+\sum_{i=0}^{n-\tau_2}u(iT)]-\mu_1(nT) && \text{(using (7))}\\&=\alpha\left\{x((n-\tau_2)T)+\sum_{i=0}^{n-\tau_2}[\lambda((i+\tau_2)T)-\mu_1(iT)]\right\}-\mu_1(nT)\ . && \\& && \text{(using (4))}\end{aligned}$$

One thus concludes this proof. #

Remark 2: Compared to the known approaches in Iliadis (1995), Benmonhamed and Meekov, (1994) and Kalarov and Ramamurthy (1997), the fundamental contribution of the above theorem lies in the fact that, it has actually proposed a *class* of congestion controllers to regulate the source rate in the network. All these congestion controllers meet the prerequisite i.e., the stability of the closed-loop systems. This class of controllers is specific to the parameter α , which can be further specified by evaluating simulation results of network performance. Besides the stability of closed-loop network system, any efficient congestion control algorithm should also address the network performance issues such as the greediness (throughput), the response duration of buffer occupancy and the power. From this rich resource of control schemes, one is thus able to choose the appropriate one to meet specific performance requirements for some particular networking engineering purpose. This mechanism is clearly seen from the following simulation and performance evaluation.

4. SIMULATION STUDIES

Since the task of congestion control is to regulate the source rate over time, we are particularly interested in analyzing the transient behaviors of the network. The transient behaviors of the network include the *fluctuations* of buffer occupancy and the *greediness* of the switch node, which are the main concern in the performance analysis. From a control theoretic point of view, even if the closed-loop systems are stable, fluctuations may still arise. Congestion-controlled systems are generally feedback systems which can sometimes sustain a limited range of self-excited fluctuations. However, excessive fluctuations may cause problems to actual network design. For example, large fluctuations of buffer occupancy call for a large buffer size. Implementing large buffers has several disadvantages. Besides increasing the cost, large buffers also increase the queuing delay in the bottleneck switching nodes, which may severely degrade the performance of delay-sensitive applications (see, Schwartz, 1996). Furthermore, to keep the controllers have a high utilization, and it is desirable to have the source rate as nearly as possible to the allocated rate so as to make the source greedy. To evaluate and characterize the greediness of source node, the following mathematical measure is suggested to use, i.e.,

$$\Delta(v) = \overline{\mu}(nT) - \lambda(nT) - d(nT). \tag{18}$$

This is the balance between the out-flowing rate and the total (controllable and uncontrollable) in-flowing rates.

Simulation has been performed on the model given by (5), where the bottleneck switch has a constant capacity of $\overline{\mu}(nT) = 300 Kbps$, and the sampling time $T = 1\sec$. We assume that the maximum allocated rate $\mu_1(nT)$ equal to the switch capacity $\mu(nT)$. The input delay $\tau_1 = 5\,m\sec$ and the feedback delay $\tau_2 = 3\,m\sec$. The total round-trip delay is thus $\tau = 8\,m\sec$. Theorem 1 tells us any parameter $\alpha < 0.1$ is sufficient for the closed-loop systems to be stable. Figure 2, 3 and 4 show the transient behaviors of the network fed by a congestion-controlled source based on different values of $\alpha = -1.2$, $\alpha = -2.0$ and $\alpha = -10$ in control algorithms (17) respectively. In Figure 2, 3, and 4, figure (a) shows the uncontrolled traffic rate $d(nT)$, figure (b) shows the controlled traffic rate $\lambda(nT)$, figure (c) shows the greediness of the source node, and (d) shows the dynamic of the buffer occupancy under the above three different controlling schemes. Attention should be paid to figure 2(c), 3(c) and 4(c). They show that those balances between the out-flowing rate and the (controllable and uncontrollable) in-flowing rates are zero except at some points where there are some glitches. These glitches are produced due to the delay in the response of the controlled input $\lambda(nT)$ to the uncontrolled input $d(nT)$. At these glitches, the source node is not greedy. Obviously, the narrower the glitch is, the better the control scheme is. Observation in Fig. 2(c), 3(c), 4(c), one can see that, as α decreases from -1.2 to -10, the glitches become bigger and wider. Observation in Fig. 2(d), 3(d), 4(d), one can also see that, as α decreases from -1.2 to -10 , the oscillations arising therein become heavier. Considering the simulation results in overall, one can draw a conclusion that, among the above three control schemes, the case corresponding to $\alpha = -1.2$ is the best efficient one due to the fact that, the oscillations arising in buffer occupancy and controllers are mild and $\Delta(\upsilon)$ is mostly approach to zero thus the source node is mostly greedy in this case.

5. CONCLUSIONS

Classical control theory and Schur-Cohn criterion have been used as key tools for designing effective congestion controllers for high-speed networks. A class of end-to-end rate-based congestion controllers has actually been proposed that have satisfied the relevant stability condition. Further, ideas have been presented on how to choose the mostly desirable controller from this class to meet more specific performance requirements. Mathematical analysis and simulation results show the validity of this approach. Future research would focus on the issue of guaranteeing the required QoS in real time communications and investigations into meeting more performance requirements such as the power and the throughput of the congestion-controlled networks.

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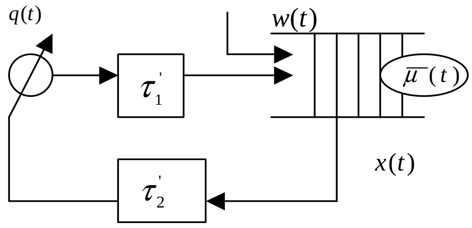

Figure 1: A source nodel's congestion control model

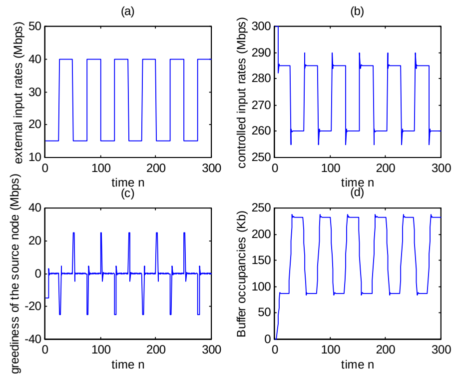

Figure 2: Performance of a single source node implemented by a congestion controller ($\alpha = -1.2$)

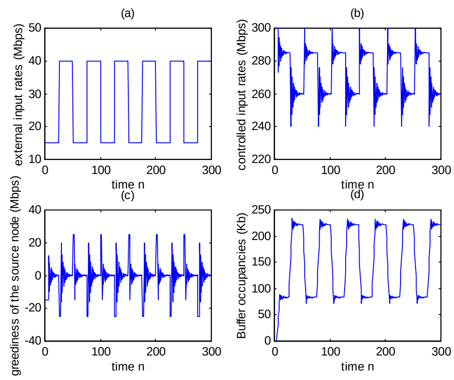

Figure 3: Performance of a single source node implemented by a congestion controller ($\alpha = -2.0$)

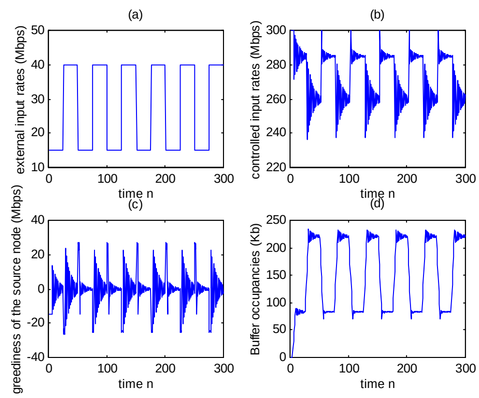

Figure 4: Performance of a single source node implemented by a congestion controller ($\alpha = -10.0$)