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ABSOLUTE EXPONENTIAL STABILITY OF A CLASS OF NEURAL NETWORKS*

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Abstract: This paper investigates the absolute exponential stability (AEST) of a class of neural networks with a general class of partially Lipschitz continuous and monotone increasing activation functions. The main obtained result is that if the interconnection matrix T of the neural system satisfies that $-T$ is an H-matrix with nonnegative diagonal elements, then the neural system is AEST.

Keywords: Absolute exponential stability, activation functions, partial Lipschitz continuity, neural networks.

1. INTRODUCTION

Recently, the analysis of absolute stability (ABST) of neural networks, especially to Hopfield neural networks and cellular neural networks, has received much attention in the literature (Forti, 1994; Kaszkurewicz and Bhaya, 1994, 1995; Arik, 1996 and 1998;.Liang and Wu, 1998; Liang and Wang, 2000) An absolutely stable neural network has the ideal characteristics that , for any neuron activation in a proper class of sigmoid functions and other network parameters, the network has a unique and globally asymptotically stable (GAS) equilibrium point. The ABST property of neural networks is very attractive in their applications for solving optimization problems, such as linear and quadratic programming, because it implies that the optimization neural networks are devoid of the spurious suboptimal responses for any activation functions in the proper class and other network parameters. The ABST neural networks are thus regarded as the most suitable ones for solving optimization problems (Liang and Wang, 2000).

The existing ABST results of neural networks in (Forti, 1994; Kaszkurewicz and Bhaya, 1994, 1995; Arik, 1996 and 1998;.Liang and Wu, 1998) were obtained within the classes of bounded and differentiable activation functions. However, in practical optimization applications, it is not uncommon that the activation functions in

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optimization neural networks are unbounded and/or non-differentiable, as demonstrated in previous work (Forti and Tesi, 1995; Sudharsanan and Sundareshan, 1991; Bouzerman and Pattison, 1993; Liang and Wang, 2000). Moreover, it is desirable that the optimization neural networks are globally exponentially stable (GES) at any prescribed exponential convergence rate (Sudharsanan and Sundareshan, 1991; Bouzerman and Pattison, 1993; Liang and. Wang, 2000). Furthermore, for a GES neural network we can make a quantitative analysis and therefore, know the convergence behaviors of the neural network arrives at a solution with a specified precision. Thus, the analysis of absolute exponential stability (AEST) of neural networks is deemed necessary and rewarding. An absolutely exponentially stable (AEST) neural network means that the network has a unique and GES equilibrium point for any activation functions in the proper class and other network parameters (Liang and Wang, 2000).

The main purpose of this paper is to provide an AEST result, which can be stated as follows: if the interconnection matrix T of the network system satisfies that $-T$ is an H-matrix with nonnegative diagonal elements, then the network system is AEST with respect to (w.r.t.) a general class of partially Lipschitz continuous (p.l.c.) and monotone increasing activation functions. The obtained AEST result of the neural networks in the paper is first proposed in the literature.

2. MODEL AND PRELIMINARIES

Consider the neural network model descibed by the system of differential equations in the form

$$\dot{x} = -Df(x) + Tg(x) + I \tag{N}$$

where $x = (x_1, x_2, \ldots, x_n)^T \in$ R ^{n} , D is an $n \times n$ constant diagonal matrix with diagonal elements $d_i > 0, i = 1,2,\ldots,n,$ $T = (T_{ij})$ is an $n \times n$ constant interconnection matrix, $f(x) = (f_1(x_1), f_2(x_2), \ldots, f_n(x_n))^T \in$ R ^{n} and $f_i(x_i)(i = 1,2,\ldots,n)$ is defined as

$$f_i(x_i) = \begin{cases} m(x_i - 1) + 1, & x_i \geq 1 \\ x_i, & |x_i| < 1 \\ m(x_i + 1) - 1, & x_i \leq -1 \end{cases} \tag{1}$$

where $m \geq 1$. $g(x) = (g_1(x_1), g_2(x_2), \ldots, g_n(x_n))^T$: R ^{n} $\rightarrow$ R^n is a nonlinear vector-valued activation function and $I = (I_1, I_2, \ldots, I_n)^T \in$ R^n is a constant input vector.

Assume that g belongs to the class PLI of activation functions defined by the property that $g \in$ PLI if for $i = 1,2,\ldots,n, g_i(x_i)$: R $\rightarrow$ R is a partially Lipschitz continuous and monotone increasing function. A function $h(\rho): \mathrm{R} \rightarrow \mathrm{R}$ is said to be partially Lipschitz continuous (p.l.c.) in R if for any $\rho \in \mathrm{R}$ there exists a positive number l_ρ such that

$$|h(\theta) - h(\rho)| \leq l_\rho |\theta - \rho|, \qquad \forall \theta \in \mathrm{R}. \tag{2}$$

It can be seen that a function $g \in$ PLI may be unbounded and/or non-differentiable. Liang and Wang, (2000) have shown that several classes of activation functions used in the literature (Hopfield (1985); Chua (1988); Forti and Tesi (1995)) are special ones of the PLI class.

Lemma 1 (Liang and Wang, 2000) If g belongs to sigmoid functions defined by the property that for $i = 1,2,\ldots,n$, $g_i(x_i) \in C^1$ (R) is a bounded function and has positive derivative everywhere in R, i.e. $g \in$ s , then $g \in$ PLI.

If $g \in$ PLI, then the vector field defined by the right hand of system (N), $-Df(x) + Tg(x) + I,$ satisfies a local Lipschitz condition. By the Theorem of Local Existence and Uniqueness for the solutions of ordinary differential equations (ODE) (see Hale, 1969), for any $x_0 \in$ R ^{n} , there exists a unique solution of the autonomous system (N) denoted by $x(t, x_0)$ for $t \in [0; t^*(x_0))$ satisfying $x(0; x_0) = x_0$, where $t^*(x_0) \in (0, +\infty)$ or $t^*(x_0) = +\infty$ such that $[0, t^*(x_0))$ is the maximal right existence interval of the solution $x(t; x_0)$. It will be found , in section 3, that the solution $x(t; x_0)$ is actually bounded for $t \in [0, t^*(x_0))$ .By the Continuation Theorem for the solutions of ODE (see Hale, 1969), we can conclude that $t^*(x_0) = +\infty$. In the following definitions of stability, we will denote $x(t; x_0)$ for $t \in [0, +\infty)$ as the global solution of system (N) uniquely determined by the initial condition $x(0; x_0) = x_0 \in \mathrm{R}^n$ .Moreover, we will use two equivalent norms of vector x in R ^{n} , i.e., $\|x\| = \left(\sum_{i=1}^n x_i^2\right)^{1/2}$ and $\|x\|_1 = \sum_{i=1}^n |x_i|$.

Definition 1 An equilibrium point $x^* \in$ R ^{n} of system (N) is a constant solution of (N), i.e., it satisfies the algebraic equation $-Df(x^*) + Tg(x^*) + I = 0$. The equilibrium x^* is said to be GES if there exist two positive constants

$\alpha \geq 1$ and $\beta > 0$ such that for any $x_0 \in \mathrm{R}^n$ and $t \in [0,+\infty)$

$$\left\| x(t; x_0) - x^* \right\| \leq \alpha \left\| x_0 - x^* \right\| \exp(-\beta t).$$

Definition 2 (Liang and Wang, 2000) System (N) is said to be AEST with respect to the class PLI if it possesses a GES equilibrium point for every function $g \in$ PLI, every input vector $I \in \mathrm{R}^n$, and any positive diagonal matrix D.

It is obvious that an AEST neural network system (N) is ABST because the GES property implies the GAS one.

For the proof of AEST result of neural network model (N) in Section 3, we require some knowledge in matrix types with their characteristics and some concepts from degree theory. For convenience, we give those of particular relevance to our need.

Definition 3 Let the $n \times n$ matrix A have non-positive off-diagonal elements, then each of the following conditions is equivalent to the statement "A is an M-matrix."

(M_1) All principal minors of A are non-negative, i.e., $A \in \mathrm{P}_0$.

(M_2) $A+K$ is nonsingular for any positive diagonal matrix K.

Definition 4 An $n \times n$ matrix A is said to belong to the class P_0 if A satisfies one of the following equivalent conditions:

(P_1) All principal minors of A are nonnegative.

(P_2) For each $x \in \mathrm{R}^n$, if $x \neq 0$, there exists an index $i \in \{1,2,\ldots,n\}$ such that $x_i \neq 0$ and $x_i(Ax)_i \geq 0$, where $(Ax)_i$ denotes the ith component of the vector Ax.

(P_3) For each diagonal matrix $K = diag(K_1, K_2, \ldots, K_n)$ with $K_i > 0$, $i = 1,2,\ldots,n$, $\det(A+K) \neq 0$.

Definition 5 An $n \times n$ matrix $A = (a_{ij})$ is said to be an H-matrix if its comparison matrix $\mathrm{M}(A) = (m_{ij})$ defined by for $i,j = 1,2,\ldots,n$,

$$m_{ij} = \begin{cases} \left| a_{ij} \right|, & i = j \\ -\left| a_{ij} \right|, & i \neq j \end{cases}$$

is an M matrix.

Definition 6 Let the $n \times n$ matrix $A = (a_{ij})$ have non-positive off-diagonal elements, then each of the following conditions is equivalent to the statement "A is a nonsingular M-matrix."

(M_1') All principal minors of A are positive.

(M_2') A has all positive diagonal elements and there exists a positive diagonal elements and there exists a positive diagonal matrix $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_n)$ such that $A\Lambda$ is strictly diagonally dominant; that is

$$a_{ii}\lambda_i > \sum_{j \neq i} \left| a_{ij} \right| \lambda_j, \quad i = 1,2,\ldots,n.$$

By Definition 3 and definition 6, the matrix A with non-positive off-diagonal elements is an M-matrix if and only if its transposition A^T is also.

The following two lemmas will be used in this paper.

Lemma 2 (Liang and Wang, 2000) Let the matrix A have non-positive off-diagonal elements, then A is an M-matrix if and only if $A+K$ is a nonsingular M-matrix for any positive diagonal matrix K.

Lemma 3 (Liang and Wang, 2000) If A is an H-matrix with nonnegative diagonal elements, then $A \in \mathrm{P}_0$.

Now, we introduce some concepts from the degree theory. The following facts and their details can be found in Liang and Wang (2000) and its references therein.

Let Ω be a nonempty, bounded and open subset of R^n, and $\overline{\Omega}$ and $\partial\Omega$ be the closure and boundary of Ω, respectively. Let $C(\overline{\Omega})$ be the space of all continuous vector-valued functions or mappings defined on $\overline{\Omega}$ into R^n. Let $f(x) \in C(\overline{\Omega})$ and that $f(x) = 0$ has no solution in $\partial\Omega$. then the degree of $f(x)$ relative to 0 and Ω, denoted by $d(f;0;\Omega)$, can be well defined by an algorithm whose details can be seen in Chua and Wang (1977). Roughly speaking, the degree of $f(x)$ relative to 0 and Ω can be regarded a the algebraic number of solutions of $f(x) = 0$ in Ω. For example, if $0 \in \Omega$, then the identity mapping $id(x) = x (x \in \mathrm{R}^n)$ has the degree $d(id;0,\Omega) = 1$. A particularly useful fact is that if $d(f;0,\Omega) \neq 0$, then there exists at least one solution of $f(x) = 0$ in Ω.

We will employ the homotopy invariance property in the degree theory. A homotopy $h(z;\lambda)$ over $\overline{\Omega}$ is any continuous vector-valued function from $\overline{\Omega} \times [0,1]$ to R^n. Let $h(z;\lambda)$ be a homotopy over $\overline{\Omega}$. If $h(z;\lambda) = 0$ has no solution in $\partial\Omega$ for any $\lambda \in [0,1]$, then $d(h(z;\lambda);0,\Omega)$ is a constant independent of λ. In this case, $h(z;0)$ and $h(z;1)$

are said to be homotopic to each other over $\overline{\Omega}$, and we say that $h(z;\lambda)$ connects $h(z;0)$ and $h(z;1)$ homotopically.

Similar to Liang and Wang (2000), the above presented facts from the degree theory will be used to prove the existence of equilibrium of the network system (N).

3. AEST RESULT AND ITS PROOF

The main AEST result in the paper is the following.

Theorem 1: If $-T$ is an H -matrix with nonnegative diagonal elements, then the neural network system (N) is AEST w.r.t. the class PLI.

Proof. Fix $g \in$ PLI, $I \in R^n$ and the positive diagonal matrix D.

Suppose that $-T$ is an H -matrix with nonnegative diagonal elements. Then, its comparison matrix $M(-T)$ is an M -matrix which diagonal elements are $-T_{ii}$ $(i=1,2,\ldots,n)$. By Lemma 2, for any positive diagonal matrix $K=$ diag $(K_1, K_2, \ldots, K_n)$ the matrix $M(-T)+K$ is a nonsingular M -matrix. Thus, its transposition $(M(-T)+K)^T$ is also. From condition $(M_2^{'})$ in Definition 6, it follows that there exists a positive diagonal matrix $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$ such that

$$\lambda_j T_{jj} + \sum_{i \neq j} \lambda_i |T_{ij}| < \lambda_j K_j, \quad j=1,2,\ldots,n. \quad (3)$$

Step 1: Let $H(x)=Df(x)-Tg(x)-I\,(x \in R^n)$, then $x^* \in R^n)$ is an equilibrium of the network system the form $H(x)=Df(x)-TG(x)+V$, where the function $G(x)=(G_1(x_1),G_2(x_2),\ldots,G_n(x_n))^T=g(x)-g(0) \in$ PLI satisfying $G(0)=0$, and the vector $V=(V_1,V_2,\ldots,V_n)^T=-Tg(0)-I \in R^n$.

Since $g \in$PLI, by (1) there exist positive constants $l_i > 0 (i=1,2,\ldots,n)$ such that

$$|G_i(x_i)| = |g_i(x_i) - g_i(0)| \le l_i |x_i|$$

for $x_i \in R$ and $i=1,2,\ldots,n.$ We can select the positive diagonal matrix K as $K_j = d_j / 2l_j > 0 (j=1,2,\ldots,n)$ for which the inequality (2) holds for some positive diagonal matrix $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$.

Construct the nonempty, bounded and open subset $\Omega_r = \{x \in R^n \, \| x \|_1 < r\} \supseteq \{0\}$ for some $r>0$ and the homotopy $h(x;\lambda)=[h_1(x;\lambda),h_2(x;\lambda),\ldots,h_n(x;\lambda)]^T \in R^n$ defined as

$$h(x;\lambda) = \lambda f(x) + (1-\lambda) H(x), \qquad x \in \overline{\Omega}_r, \lambda \in [0,1]$$

where $\overline{\Omega}_r = \{x \in R^n \, \|x\|_1 \le r\}$. In the following, we will prove that for sufficiently large $r>0$, $h(x;\lambda) \neq 0$ for $x \in \partial\Omega_r = \{x \in R^n \, \|x\|_1 = r\}$ and $\lambda \in [0,1]$.

Let the signum function $\mathrm{sgn}(\rho) (\rho \in R)$ be defined as 1 if $\rho > 0$; 0 if $\rho = 0$; and -1 if $\rho < 0$. Then, we have

$$\sum_{i=1}^{n} \lambda_i \,\mathrm{sgn}(x_i) h_i(x;\lambda) = \sum_{i=1}^{n} \lambda_i \,\mathrm{sgn}(x_i)[\lambda + (1-\lambda) d_i] f(x_i)$$
$$+ \sum_{i=1}^{n} \lambda_i \,\mathrm{sgn}(x_i) \left[(\lambda - 1) \sum_{j=1}^{n} T_{ij} G_j(x_j) + (1-\lambda) V_i \right]$$
$$\ge \sum_{j=1}^{n} \lambda_j [\lambda + (1-\lambda) d_j] |x_j| - (1-\lambda) \sum_{j=1}^{n} \lambda_j T_{jj} |G_j(x_j)|$$
$$- (1-\lambda) \sum_{j=1}^{n} \sum_{i \neq j} \lambda_i |T_{ij}| |G_j(x_j)| - \theta$$
$$\ge \sum_{j=1}^{n} \lambda_j [\lambda + (1-\lambda) d_j] |x_j| - (1-\lambda) \sum_{j=1}^{n} \lambda_j K_j |G_j(x_j)| - \theta$$
$$\ge \sum_{j=1}^{n} \lambda_j \left[\lambda + \frac{1}{2} (1-\lambda) d_j \right] |x_j| - \theta$$
$$\ge \omega \|x\|_1 - \theta,$$

where $\theta = (1-\lambda) \sum_{j=1}^{n} \lambda_j |V_j| \ge 0$ and $\omega = \min_{j=1,2,\ldots,n} (\min(\lambda_j / 2, \lambda_j d_j / 4)) > 0.$

Thus, if $r > \theta / \omega$, from the inequality (3), then we can get that for $x \in \partial\Omega_r$ and $\lambda \in [0,1]$, $\sum_{i=1}^{n} \lambda_i \,\mathrm{sgn}(x_i) h_i(x;\lambda) > 0$, which implies that $h(x;\lambda) \neq 0$.

By the homotopy invariance property, we have that $d(h(z;0);0,\Omega_r) = d(h(z;1);0,\Omega_r)$, i.e., that $d(H;0,\Omega_r) = d(id;0,\Omega_r) = 1 \neq 0$. Thus, $H(x)=0$ has at least one solution in $\Omega_r \subseteq R^n$.

Now, we show that there is at most one solution of $H(x)=0$ in R^n by the contradiction method. Assume that $x^{(1)} = (x_1^{(1)}, x_2^{(1)}, \ldots, x_n^{(1)})^T \in R^n$ and $x^{(2)} = (x_1^{(2)}, x_2^{(2)}, \ldots, x_n^{(2)})^T \in R^n$ be two different solutions of $H(x)=0$. This means

$$Df(x^{(1)}) - Tg(x^{(1)}) - I = Df(x^{(2)}) - Tg(x^{(2)}) - I = 0$$

and hence,

$$(-T)\left[g\left(x^{(2)}\right)-g\left(x^{(1)}\right)\right]=D[f(x^{(1)})-f(x^{(2)})]\neq 0,$$

Let $\tilde{x}=(\tilde{x}_1,\tilde{x}_2,\dots,\tilde{x}_n)^T=g(x^{(2)})-g(x^{(1)})\in R^n$, then $(-T)\tilde{x}\neq 0$ and hence $\tilde{x}\neq 0$. Since $-T\in P_0$ from Lemma 3, by condition (P_2) in definition 3, there exists an index $i\in\{1,2,\dots,n\}$ such that $\tilde{x}_i=g_i(x_i^{(2)})-g_i(x_i^{(1)})\neq 0$ and $\tilde{x}_i(-T\tilde{x})_i=d_i\tilde{x}_i[f(x_i^{(1)})-f(x_i^{(2)})]\geq 0$. The last inequality is equivalent to $\tilde{x}_i[x_i^{(1)}-x_i^{(2)}]\geq 0$ because of $d_i>0$ and $[f(x_i^{(1)})-f(x_i^{(2)})][x_i^{(1)}-x_i^{(2)}]\geq 0$. Moreover, noting the inequality $x_i^{(1)}-x_i^{(2)}\neq 0$ from $\tilde{x}_i\neq 0$, we know that $\tilde{x}_i[x_i^{(1)}-x_i^{(2)}]\neq 0$. Therefore, we should have $\tilde{x}_i[x_i^{(1)}-x_i^{(2)}]>0$, i.e., that $[g_i(x_i^{(2)})-g_i(x_i^{(1)})][x_i^{(1)}-x_i^{(2)}]>0$. This is in contradiction with the monotone increasing property of $g_i(x_i)$.

At this point, we have shown that the network system (N) has a unique equilibrium point which can be denoted by $x^*=(x_1^*,x_2^*,\dots,x_n^*)^T\in R^n$.

Step 2: For any $x_0\in R^n$, let $x(t;x_0)$ for $t\in[0,t^*(x_0))$ be the unique solution of the autonomous system (N) satisfying the unique solution of the autonomous system (N) satisfying the initial condition $x(0;x_0)=x_0$, where $t^*(x_0)\in(0,+\infty)$ or $t^*(x_0)=+\infty$ such that $[0,t^*(x_0))$ is the maximal right existence interval of the solution $x(t;x_0)$. Let $z(t)=(z_1(t),z_2(t),\dots,z_n(t))^T=x(t;x_0)-x^*\in R^n$ for $t\in[0,t^*(x_0))$. Then, $z(t)$ satisfies the following ODE of the form

$$dz(t)/dt=-D\bar{f}(z(t))+T\bar{g}(z(t)),\forall t\in[0,t^*(x_0)) \qquad (\tilde{N})$$

with the initial condition $z(0)=x_0-x^*$, where the vector-valued function $\bar{f}(z)=(\bar{f}_1(z_1),\bar{f}_2(z_2),\dots,\bar{f}_n(z_n))^T\in R^n$ is defined by $\bar{f}(z)=\bar{f}(z+x^*)-\bar{f}(x^*)$ and $\bar{g}(z)=(\bar{g}_1(z_1),\bar{g}_2(z_2),\dots,\bar{g}_n(z_n))^T\in R^n$ is defined by $\bar{g}(z)=\bar{g}(z+x^*)-\bar{g}(x^*)$ for $z=(z_1,z_2,\dots,z_n)^T\in R^n$ satisfying $\bar{f}(0)=0$ and $\bar{g}(0)=0$ respectively.

Similarly, from the assumption of $g\in$ PLI, by (2) there exist positive constants $\mu_i>0 (i=1,2,\dots,n)$ such that

$$\left|\bar{g}_i(z_i)\right|=\left|g_i(z_i+x_i^*)-g_i(x_i^*)\right|\leq\mu_i|z_i|$$

for $z_i\in R$ and $i=1,2,\dots,n.$ In what follows, we will take the positive diagonal matrix K as $K_j=d_j/(2\mu_j)>0$ $(j=1,2,\dots,n)$ for which there exists a positive diagonal matrix $\Lambda=\mathrm{diag}(\lambda_1,\lambda_2,\dots,\lambda_n)$ satisfying the inequality (2).

Construct the Lyapunov function $V(z)=\sum_{i=1}^n\lambda_i|z_i|$ for $z\in R^n$. Define the right and upper Dini derivative of $V(z)$ along the solution $z(t)$ by

$$\frac{d^+V(z(t))}{dt}=\lim_{h\to 0^+}\sup[V(z(t+h))-V(z(t))]/h.$$

Computing the Dini derivative of $V(z)$ along the solution $z(t)$ for $t\in[0,t^*(x_0))$, we have

$$\frac{d^+V(z(t))}{dt}=\sum_{i=1}^n\lambda_i\,\mathrm{sgn}(z_i(t))\left[-d_i\bar{f}(z_i(t))+\sum_{j=1}^nT_{ij}\bar{g}_j(z_j(t))\right]$$

$$\leq-\sum_{i=1}^n d_j\lambda_j\left|\bar{f}(z_j(t))\right|+\sum_{j=1}^n\lambda_jT_{jj}\left|\bar{g}_j(z_j(t))\right|+\sum_{j=1}^n\left(\sum_{i\neq j}\lambda_i\left|T_{ij}\right|\right)\left|\bar{g}_j(z_j(t))\right|$$

$$\leq-\sum_{j=1}^n d_j\lambda_j\left|z_j(t)\right|+\sum_{j=1}^n\lambda_jK_j\left|\bar{g}_j(z_j(t))\right|$$

$$\leq-\frac{1}{2}\sum_{j=1}^n d_j\lambda_j\left|z_j(t)\right|$$

$$\leq-(d_{\min}/2)V(z(t))$$

$$\leq 0. \qquad t\in[0,t^*(x_0)).$$

where $d_{\min}=\min_{j=1,2,\dots,n}d_j>0.$

By the comparison principal, from the above differential inequality we have

$$V(z(t))\leq V(z(0))\exp(-d_{\min}t/2),\ t\in[0,t^*(x_0)). \qquad (4)$$

Let the two constants $\lambda_{\max}=\max_{j=1,2,\dots n}\lambda_j>0$ and $\lambda_{\min}=\min_{j=1,2,\dots,n}\lambda_j>0,$ then we get $\lambda_{\min}\|z\|_1\leq V(z)\leq\lambda_{\max}\|z\|_1$ for $z\in R^n$ .Thus, it can be inferred from (4) that for $t\in[0,t^*(x_0))$

$$\left\|x(t;x_0)-x^*\right\|_1\leq\frac{\lambda_{\max}}{\lambda_{\min}}\left\|x_0-x^*\right\|_1\exp(-d_{\min}t/2). \qquad (5)$$

The above inequality implies that the solution $x(t;x_0)$ is bounded for $t\in[0,t^*(x_0))$. By the Continuation Theorem for the solutions of ODE, we can conclude that $t^*(x_0)=+\infty$ and the inequality (5) still holds for $t\in[0,\infty)$. In view of the equivalence of the norms $\|x\|_1$ and $\|x\|$, by Definition 1 and (5), x^* is the GES equilibrium of the system (N).

Integrating the above obtained results, we have completed the proof of AEST of the network system (N).

Remark 1 The inequality (5) implies that the exponential convergence rate of any network trajectory has a lower bound of $d_{\min}/2$. On the other hand, putting T equal to the zero matrix in the network model (N), we can see easily that the possible lower bound for the exponential convergence rate of the network trajectory cannot be greater than $d_{\min}$. When the network model (N) is used for solving optimization problem and the larger exponential convergence rate of network trajectories is desired, we can use the following modified network model

$$\tau dx/dt = -Df(x) + Tg(x) + I$$

where $\tau > 0$ is a time constant. It is obvious that the exponential convergence rate of any trajectory for the above modified network model has a lower bound of $d_{\min}/(2\tau)$. Thus, the exponential convergence rate of the network trajectory can be made arbitrarily large by tuning downward the time constant $\tau > 0$.

Remark 2 Let $g(x)$ is piecewise linear activation functions, then the network system (N) is a VLSI-oriented continuous-time cellular neural networks proposed by Espejo et al. (1996). So, the GES result of the VLSI-oriented continuous-time CNNs is obtained simultaneously.

Remark 3 Let $m = 1$ in (1) and $g(x) \in$ PLI, then it is obviously shown that the AEST result of Liang and Wang (2000) is special case of this paper.

4. CONCLUSION

In this paper, we have obtained a new AEST result of the neural networks (N) with globally Lipschitz activation functions with the existing ABST results of neural networks with special classes of sigmoid activation functions of neural networks and demonstrates that the network system has the stronger global exponential stability. Thus, the obtained AEST result allows neural networks (N), which is very suitful for for hardware implementation (see Espejo et al., 1996), to have a wider ranger of applications.

REFERENCES

Arik S. and Tavasanoglu V. (1996). Absolute stability of nonsymmetric neural network, In Proc.1996 IEEE Int. Symp. Circuit Syst., vol.Ⅲ, May, 1996, 441-444.

Arik S. and Tavasanoglu V. (1998). A comment on "Comments on necessary and sufficient condition for absolute stability of neural networks". IEEE Trans. Circuit Syst.I, vol.45, 595-596.

Bouzerman A. and Pattison T.R. (1993) . Neural network for quadratic optimization with bound constraints. IEEE Trans. Neural Networks, vol.4. 293-303.

Chua L.O. and Wang N.N. (1977). On the application of degree theory to the analysis of resistive nonlinear networks. Int. J. Circuit Theory Applicants., vol.5, 45-68.

Chua L. O., Yang L. C. (1988). Cellular neural network: Theory. IEEE Trans. Circuits Syst., 1988, 35(10): 1257-1272.

Chua L.O., Yang L. C. (1988). Cellular neural network: Application. IEEE Trans. Circuits Syst, 1988, 35 (10): 1273-1290.

Espejo S., Carmona R.,et. (1996). A VLSI-oriented continuous-time CNN model. Int. J. Circuit Theory and applications. vol.24, 341-356.

Forti, M.,Manetti S.,and Marini M. (1994). Necessary and sufficient condition for absolute stability of neural networks. IEEE Trans. Circuits Syst. I, vol. 41, 491-494.

Forti M.,A.Liberatore,S.Manetti,and M.Marini(1994). On absolute stability of neural networks. In Proc. 1994 IEEE Int. Symp. Circuit Syst.vol.6, May, 1994, 241-244.

Forti M..(1994). On global asymptotic stability of a class of nonlinear system arising in neural network theory. J. Differential Equations, Vol.113, 246-266.

Forti M. and Tesi A. (1995). New conditions for global stability of neural networks with application to linear and quadratic programming. Problems. IEEE Trans. Circuits Syst.I, vol.42, 354-366.

Hale J. K. (1969). Ordinary Differential Equations. New York: Wiley Intersci..

Hopfield J. J. and Tank D. W. (1985). Neural Computation of decisions in optimization problems. Biol. Cybern. , vol.52, 141-152.

Kaszkurewicz E. and Bhaya A.(1994). On a class of globally stable neural circuits. IEEE Trans.Circuits Syst.I,vol.41, 171-174.

Kaszkurewicz E. and Bhaya A.(1995). Comments on 'Necessary and sufficient condition for absolute stability of neural network'. IEEE Trans. Circuits Syst.I, vol.42, 497-499.

Liang X. B. and Wu L .D. (1998) . New sufficient conditions for absolute stability of neural networks. IEEE Trans. Circuits Syst.. I, vol. 45, 584-586.

Liang X.B. and Wang J. (2000). Absolute exponential stability of neural networks with a general class of activation functions. IEEE Trans. Circuits Syst. I. vol 47.No.8,August 2000, 1258-1263.

Sudharsanan S. L. and Sundareshan M. K. (1991). Exponential stability and a systematic synthesis of a neural network for quadratic minimization. Neural networks, vol.4, No. 5, 599-613.