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GAME CONTROL OF MOVING OBJECTS

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Abstract: The paper describes the application of elements of the game theory for the purpose of optimal control of some dynamic continuous processes. Using as an example the process of safe ship's control, the paper presents the problem of applying a positional non-cooperative game and multi-step matrix game of *j* objects for the description of the process considered as well as for the synthesis of optimal strategies. A mathematical model of dynamic game is formulated and its approximation in the form of dual linear programming problem is used for the synthesis of safe ship's trajectory as a multistage process decision. The considerations have been illustrated an example of a computer simulation the POSGAME and RISKGAME algorithms to determine the safe ship's trajectory in situation of passing a big number of the objects encountered, recorded on the radar screen in real navigational situation at sea. *Copyright © 2002 IFAC*

Keywords: game theory, safety analysis, ship control, computer simulation.

1. INTRODUCTION

The process of the own control object using the dynamic game model with *j* participants can describe on example of safe ship control in collision situations at sea. In practice the ship control as dynamic object depends both on the accuracy of details concerning the current navigational situation obtained from the ARPA (Automatic Radar Plotting Aids) anti-collision system and the form of the models of the process used for algorithms control synthesis.
The ARPA system ensures automatic monitoring of at least 20 *j* encountered objects, determining their movement parameters (speed V_j and course ψ_j) and elements of approaching to own ship ($D^j_{min} = DCPA_j$ Distance of the Closest Point of Approach and $T^j_{min} = TCPA_j$ Time to the Closest Point of Approach) and also assess the risk of collision r_j.
In order to ensure the safety of navigation ships are obliged to comply with Rules of the Convention on the International Regulations for Preventing Collisions at Sea (COLREG) (Fig.1).

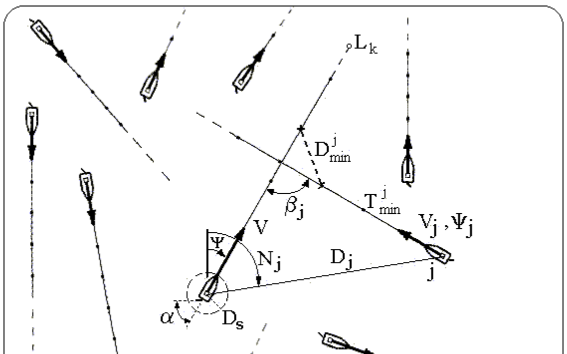

Fig.1. The process of the own ship passing *j* encountered objects.

However, these Rules refer to two ships under the conditions of good visibility. In case of a limited visibility the Rules only specify recommendations of a general nature. Consequently, the actual process of ship's passing very often occurs under the conditions of uncertainty and conflict together with unprecise co-operation of the ships with reference to the COLREG Rules. It is, therefore, reasonable to investigate the methods of ship's safe control using a theory of dynamic games and computational intelligence methods (Lisowski, 1986).

2. DYNAMIC GAME OF *J* MOVING OBJECTS

The most general description of the own control object passing the *j* number of other encountered moving objects is the model of a dynamic game of *j* participants:

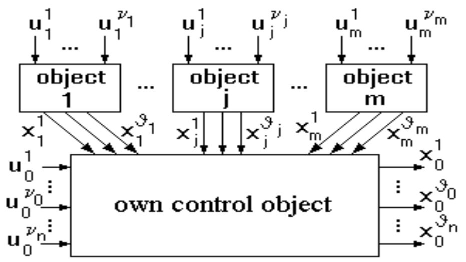

Fig. 2. A block diagram of a dynamic game model.

General dynamic features of the process are described by state equations (Lisowski, 1978, 1985; Lisowski *et al.*, 2000; Miloh and Sharma, 1977):

$$\dot{\mathbf{x}}_{\mathbf{i}} = \mathbf{f_i}[(\mathbf{x}_0^{\vartheta_0}, \mathbf{x}_1^{\vartheta_1}, ..., \mathbf{x}_{\mathbf{j}}^{\vartheta_{\mathbf{j}}}), (\mathbf{u}_0^{\mathbf{v}_0}, \mathbf{u}_1^{\mathbf{v}_1}, ..., \mathbf{u}_{\mathbf{j}}^{\mathbf{v}_{\mathbf{j}}}), \mathbf{t}] \quad (1)$$

where $\vec{x}_0^{\vartheta_0}(t)$ is ϑ_0 dimensional vector of the process state of the own control object, $\vec{x}_j^{\vartheta_j}(t)$ is ϑ_j dimensional vector of the process state for the *j*th object, $\vec{u}_o^{v_0}(t)$ is v_0 dimensional control vector of the own control object and $\vec{u}_j^{v_j}(t)$ is v_j dimensional control vector of the *j*th object (Isaacs, 1965).
The constraints of the control and the state of the process are connected with the basic condition for the safe passing of the objects at a safe distance D_s in compliance with COLREG Rules:

$$\mathbf{g_j}(\mathbf{x}_{\mathbf{j}}^{\vartheta_{\mathbf{j}}}, \mathbf{u}_{\mathbf{j}}^{\mathbf{v_j}}) \le 0 \quad (2)$$

The synthesis of the decision making pattern of the object's control leads to the determination of the optimal strategies of the playersw. For the class of non-coalition games, often used in the control techniques, the most beneficial conduct of the own control object as a player with *j*th object is the minimization of her goal function in the form of the payments – the integral payment and the final one:

$$\mathbf{I}_0^{\mathbf{j}} = \int_{\mathbf{t}_0}^{\mathbf{t_k}} [\mathbf{x}_0^{\vartheta_0}(\mathbf{t})]^2 \mathbf{dt} + \mathbf{r_j}(\mathbf{t_k}) + \mathbf{d}(\mathbf{t_k}) \rightarrow \min \quad (3)$$

The integral payment determines the loss of way of the own control object to reach a safe passing of the encountered objects and the final one determines the risk of collision and final game trajectory deflection from reference trajectory.
The state equations (1) for the basic game model of safe ship control have the form:

$$\begin{aligned}
\dot{x}_0^1 &= x_0^2 \\
\dot{x}_0^2 &= a_1 x_0^2 x_0^3 + a_2 x_0^3 \left|x_0^3\right| + b_1 x_0^3 \left|x_0^3\right| u_0^1 \\
\dot{x}_0^3 &= a_4 x_0^3 \left|x_0^3\right| \left|x_0^4\right| x_0^4 \left(1 + x_0^4\right) + a_5 x_0^2 x_0^3 x_0^4 \left|x_0^4\right| + \\
&\quad a_6 x_0^2 x_0^3 x_0^4 + a_7 x_0^3 \left|x_0^3\right| + a_8 x_0^5 \left|x_0^5\right| x_0^6 + b_2 x_0^3 x_0^4 \left|x_0^3 u_0^1\right| \\
\dot{x}_0^4 &= a_3 x_0^3 x_0^4 + a_4 x_0^3 x_0^4 \left|x_0^4\right| + a_5 x_0^2 x_0^4 + a_9 x_0^2 + \\
&\quad b_3 x_0^3 u_0^1 \\
\dot{x}_0^5 &= a_{10} x_0^5 + b_4 u_0^2 \\
\dot{x}_0^6 &= a_{11} x_0^6 + b_5 u_0^3 \\
\dot{x}_j^1 &= -x_0^3 + x_j^2 x_0^2 + x_j^3 \cos x_j^3 \\
\dot{x}_j^2 &= -x_0^2 x_j^1 + x_j^3 \sin x_j^3 \\
\dot{x}_j^3 &= -x_0^2 + b_{j+5} x_j^3 u_j^1 \\
\dot{x}_j^4 &= a_{j+11} x_j^4 \left|x_j^4\right| + b_{j+6} u_j^2
\end{aligned} \quad (4)$$

The state variables $x_0^{\vartheta_0}$ are represented by the values: course, angular turning speed, speed, drift angle, rotational speed of the screw propeller and pitch of the adjustable propeller-of the own ship and $x_j^{\vartheta_j}$ by the values: distance, bearing, course and speed-of the *j*th object. The control values $u_0^{v_0}$ are represented by: reference rudder angle, reference rotational speed screw propeller and reference pitch of the adjustable propeller-of the own ship and $u_j^{v_j}$ by the values: course and speed of the *j*th object.
Generally two types of the control are taken into consideration, programmed $u_0(t)$ and positional $u_0[x_0(t)]$. The basis for the decision making steering are the decision making patterns of the positional control processes, the patterns with the feedback arrangement representing the dynamic games (Olsder and Walter, 1977; Basar and Olsder, 1982).

3. POSITIONAL GAME OF *J* SHIPS

3.1 Model of process.

The dynamic game application to process of safe ships control is reduced to a positional multistage game of a *j*th number of participants who do not co-operate among them. The ship dynamics is included in the form of time manoeuvring t_m (Lisowski, 2001):

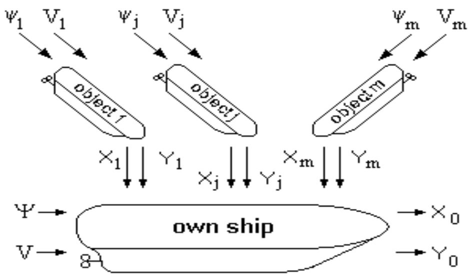

Fig. 3. Block diagram of a positional model of the game.

The essence of the positional game is to make the strategies of the own ship dependent on current positions $p(t_k)$ of the objects. The current state of the process is determined by the co-ordinates:

$$x_0 = (X_0, Y_0), (X_j, Y_j) \tag{5}$$

The system generates its steering at the moment t_k on the basis of the data which are obtained from the ARPA anti-collision system concerning the current positions of the ships:

$$p(t_k) = \begin{bmatrix} x_0(t_k) \\ x_j(t_k) \end{bmatrix} \qquad k = 1, 2, ..., K \tag{6}$$

It is assumed, according to the general concept of the multistage positional game, that at each discrete moment of the time t_k the position of the objects $p(t_k)$ is known on the own ship. The constraints of the state co-ordinates:

$$\{ x_0(t), x_j(t) \} \in P \tag{7}$$

constitute the navigational constraints, while the steering constraints:

$$u_0 \in U_0, \; u_j \in U_j \quad j = 1, 2, ..., m \tag{8}$$

take into consideration the kinematics of the ships movement, the recommendations of the COLREG Rules and the condition to maintain the safe passing distance D_s:

$$D^j_{min} = \min D_j(t) \geq D_s \tag{9}$$

The closed sets U_0 and U_j defined as the sets of the acceptable strategies of the players:

$$U_0[p(t)], U_j[p(t)] \tag{10}$$

are dependent, which means that the choice of the steering u_j by the jth object alter the sets of the acceptable strategies of the other objects. Let refer to the set of the acceptable strategies of the own ship while passing the jth encountered object at a distance D_s. The area, when maintaining stability in time of the course and speed of the own ship and the ship encountered is static and is comprised within the semicircle of a radius equal to the set speed of the own ship V_r within the arrangement of the co-ordinates 0'X'Y' with the axis X' directed to the direction of the set course (Kudriaszov and Lisowski, 1979):

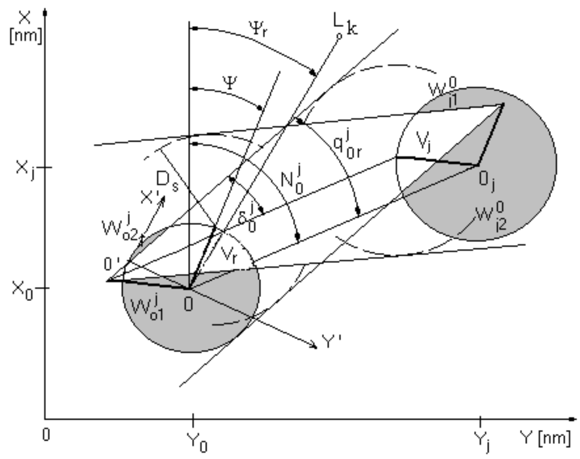

Fig. 4. Determination of the acceptable strategies of the own ship $U^j_0 = W^j_{01} \cup W^j_{02}$ and the jth object encountered $U^0_j = W^0_{j1} \cup W^0_{j2}$.

The area U^j_0 is determined with the inequalities:

$$a^j_0 \, u^x_0 + b^j_0 \, u^y_0 \leq c^j_0 \tag{11}$$

$$\left(u^x_0\right)^2 + \left(u^y_0\right)^2 \leq V^2_r \tag{12}$$

where:

$$\begin{aligned} \vec{V}_r &= \vec{V}_0 = \vec{u}^{\,2}_0(u^x_0, u^y_0) \\ a^j_0 &= -\chi^j_0 \cos(q^j_{0r} + \chi^j_0 \delta^j_0) \\ b^j_0 &= \chi^j_0 \sin(q^j_{0r} + \chi^j_0 \delta^j_0) \\ c^j_0 &= -\chi^j_0 \begin{bmatrix} V_j \sin(q^0_j + \chi^j_0 \delta^j_0) + \\ V_r \cos(q^j_0 + \chi^j_0 \delta^j_0) \end{bmatrix} \\ \chi^j_0 &= \begin{cases} 1 & \text{for} \quad W^j_{01} \quad (\text{starboard}) \\ -1 & \text{for} \quad W^j_{02} \quad (\text{port}) \end{cases} \end{aligned} \tag{13}$$

The symbol χ^j_0 is determined with the use of an appropriate logical function Z_j which characterises particular COLREG Rules. The form of the function Z_j depends on the interpretation of these recommendations in order to consider it in the control algorithm, however:

$$Z_j = \begin{cases} 1 & \text{then} \quad \chi^j_0 = 1 \\ 0 & \text{then} \quad \chi^j_0 = -1 \end{cases} \tag{14}$$

The resultant area of the acceptable manoeuvres determined by inequalities (11) and (12) is:

$$U_0 = \bigcap_{j=1}^{m} U^j_0 \qquad j = 1, 2, ..., m \tag{15}$$

On the other hand, however, the set of the acceptable strategies of the *j*th object in relation to the own ship U_j^0 is determined by the following inequalities:

$$a_j^0 u_j^x + b_j^0 u_j^y \le c_j^0 \qquad (16)$$

$$\left(u_j^x\right)^2 + \left(u_j^y\right)^2 \le V_j^2 \qquad (17)$$

where:

$$\begin{aligned} \vec{V}_j &= \vec{u}_j(u_j^x, u_j^y) \\ a_j^0 &= -\chi_j^0 \cos(q_j^0 + \chi_j^0 \delta_j^0) \\ b_j^0 &= \chi_j^0 \sin(q_j^0 + \chi_j^0 \delta_j^0) \\ c_j^0 &= -\chi_j^0 V_0 \sin(q_0^j + \chi_j^0 \delta_j^0) \end{aligned} \qquad (18)$$

The value χ_j^0 is determined by analogy to the determination of χ_0^j with the use of the logical function Z_j.
Consideration of the navigational constraints (shallow waters, coastline) generate additional constraints to the set of acceptable strategies:

$$a_0^{l,l-1} u_0^x + b_0^{l,l-1} u_0^y \le c_0^{l,l-1} \qquad (19)$$

where: l – is the nearest point of intersection of the straight lines approximating the coastline.

3.2 POSGAME control algorithm.

The optimal steering of the own ship $u_0^*(t)$, equivalent for the current position p(t) to the optimal positional steering $u_0^*(p)$, is determined:

- from the relationship (16) and (17) for the measured position $p(t_k)$ of the steering status at the moment t_k sets of the acceptable strategies $U_j^0[p(t_k)]$ are determined for the encountered objects in relation to the own ship, and from the relationship (11) and (12) the output sets $U_0^{jw}[p(t_k)]$ of the acceptable strategies of the own ship in relation to each one of the encountered objects,
- a pair of vectors u_j^m and u_0^j, is determined in relation to each *j*th object and then the optimal positional strategy of the own ship $u_0^*(p)$ from the condition:

$$I^* = \min_{u_0 \in U_0 = \bigcap_{j=1}^{m} U_0^j} \left\{ \max_{u_j^m \in U_j} \min_{u_0^j \in U_0^j(u_j)} S_0[x_0(t_k), L_k] \right\} = S_0^*(x_0, L_k) \qquad (20)$$

$$S_0[x_0(t), L_k] = \int_{t_0}^{t_{Lk}} u_0(t)\,dt \qquad (21)$$

S_0 refers to the continuous function of the manoeuvring goal of the own ship, characterising the distance of the ship to the nearest turning point L_k on the assumed route of the voyage.
Using the function of *lp – linear programming* from the Optimization Toolbox contained in the Matlab program, the positional multistage game manoeuvring POSGAME program has been designed for the determination of the safe ships trajectories in a collision situation.

3.3 Computer simulation of multi-stage positional game control.

Simulation tests of the POSGAME algorithm have been carried out with reference to real situation at sea, on board research-trainng ship HORYZONT II, at the RAYTHEON ARPA radar screen (Fig.5).

Fig. 5. The place of navigational situation recording in Kattegat Straits.

Fig.6 shows the computer simulation, performed on the POSGAME algorithm, for the own ship and encountered objects trajectories in a recorded situation.

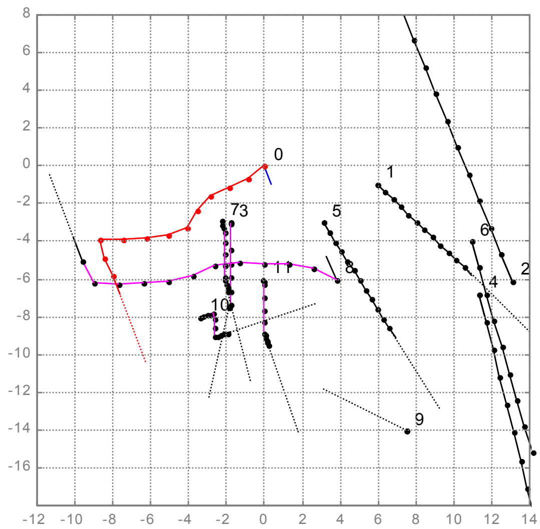

Fig. 6. The computer simulation of positional game trajectories in situation of passing 11 objects at the Kattegat Straits: D_s=*1.5 nm* (nautical miles), t_k=*6 min*, t_m=*3 min*, $r_j(t_k)$=*0*, $d(t_k)$=*9.5 nm*.

4. MATRIX GAME MODEL OF *J* SHIPS

4.1 Model of process.

The dynamic game application to process of safe ships control is reduced to a matrix game of a *j*th number of participants who do not co-operate among them (Lisowski, 2000):

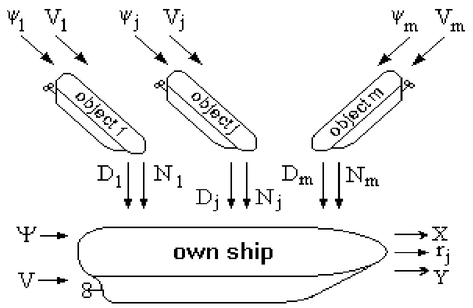

Fig. 7. Block diagram of a matrix model of game.

The ships dynamics is included in the form of time manoeuvring t_m.

The matrix game includes the values determined previously on the basis of data taken from an anticollision system ARPA the value a collision risk r_j with regard to the determined strategies of the own ship and those of the *j*th encountered objects. The form of such a game is represented by the risk matrix $\boldsymbol{R}[r_j(\nu_0,\nu_j)]$ containing the same number of columns as the number of participant I (own ship) strategies. She has; e.g. a constant course and speed, alteration of the course 20^o to starboard, 20^o to port etc., and contains a number of lines which correspond to a joint number of participant II (*j*th objects) strategies.

The constraints affecting the choice of strategies (ν_0,ν_j) are a result of the recommendations of the way priority at sea. Player I (own ship) may use ν_0 of various pure strategies in a matrix game and player II (encountered objects) has ν_j of various pure strategies. As the game, most frequently, does not have saddle point the state of balance is not guaranteed – there is a lack of pure strategies for both players in the game.

The problem of determining an optimal strategy may be reduced to the task of solving dual linear programming problem. Mixed strategy components express the probability distribution of using pure strategies by the players. As a result of using the following form for the steering criterion:

$$\left(I_0^j\right)^* = \min_{\nu_0} \max_{\nu_j} r_j \tag{22}$$

the probability matrix $\boldsymbol{P}[p_j(\nu_0,\nu_j)]$ of using particular pure strategies may be obtained.

The solution for the steering goal is the strategy of the highest probability.

This will also be the optimal value approximated to the pure strategy:

$$\left(u_0^{\nu 0}\right)^* = u_o^{\nu 0}\left\{\left[p_j\left(\nu_j,\nu_o\right)\right]_{max}\right\} \tag{23}$$

4.2 RISKGAME control algorithm.

By applying dual linear programming in order to solve the matrix game you obtain the optimal values of the own course and that of the *j*th object at the smallest deviation from their initial values. If, at a given step, no solution can be found at a speed of the own ship *V*, the calculations are repeated at the speed reduced by 25% until the game has been solved. The calculations are repeated step by step until the moment when all elements of the matrix ***R*** become equal to zero and the own ship, after having passed the encountered objects, returns to her initial course and speed. In this manner optimal safe trajectory of the ship is obtained in a collision situation.

The value of the risk of the collision r_j is defined as the reference of the current situation of the approach described by the parameters D^j_{min} and T^j_{min}, to the assumed assessment of the situation defined as safe and determined by the safe distance of approach D_s and the safe time T_s – which are necessary to execute a manoeuvre avoiding a collision with consideration actual distance D_j between own ship and encountered *j*th ship:

$$r_j = \left[a_1\left(\frac{D^j_{min}}{D_s}\right)^2 + a_2\left(\frac{T^j_{min}}{T_s}\right)^2 + a_3\left(\frac{D_j}{D_s}\right)^2\right]^{-\frac{1}{2}} \tag{24}$$

where the weight coefficients a_1, a_2 and a_3 are depended on the state visibility at sea, dynamic length and dynamic beam of the ship, kind of water region (Fig.8).

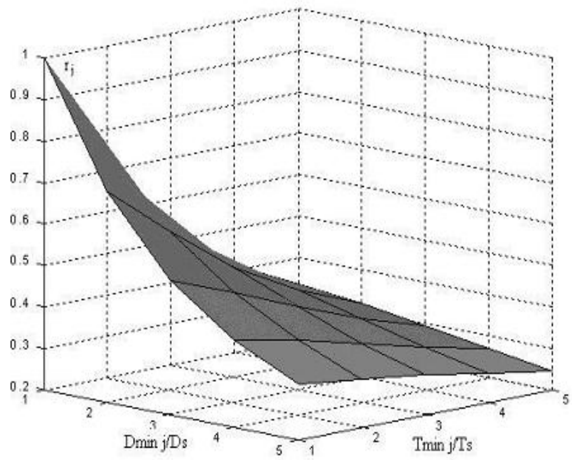

Fig. 8. The surface of the collision risk value r_j in dependence on relative values distance and time of *j*th object approach.

Using the function of *lp – linear programming* from the Optimization Toolbox contained in the Matlab program the RISKGAME algorithm has been designed for the determination of the safe ships trajectories in a collision situation.

4.3 Computer simulation of multi-step risk game control.

Simulation tests of the RISKGAME algorithm have been carried out with reference to real situation at sea. For the basic version of the RISKGAME algorithm the following values for the strategies have been adopted:

$\nu_0 = 13 \rightarrow \left|0^o \div 60^o\right|$ for each of the 5^o

$\nu_j = 25 \rightarrow \left(-60^o \div +60^o\right)$ for each of the 5^o

Fig.9 shows the computer simulation, performed on the RISKGAME program, for the safe own ship's trajectory in a situation recorded at the Kattegat Straits.

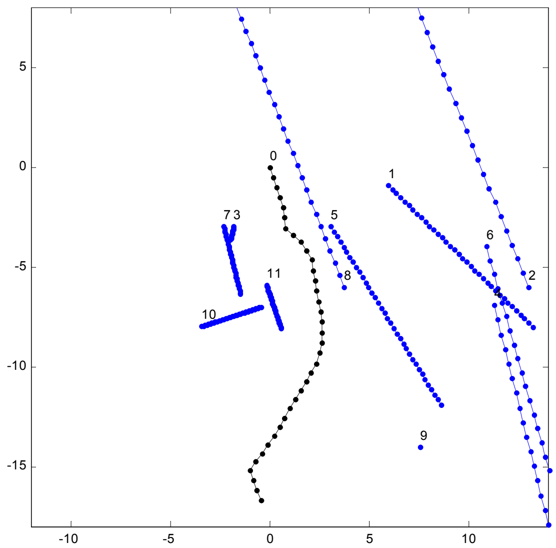

Fig. 9. The computer simulation of the own ship multi-step matrix game trajectory in situation of passing 11 objects at the Kattegat Straits for: D_s=1.5 nm, T_s=18 min, t_k=6 min, t_m=3 min, $d(t_k)$=6.31 nm.

5. CONCLUSION

The application of the approximated dynamic game models for the synthesis of an optimal manoeuvring makes it possible to determine the safe game trajectory of the own ship in situations when she passes a greater number of the encountered objects. The trajectory has been described as\ a certain sequence of manoeuvres with the course and speed. The POSGAME and RISKGAME computer algorithms designed in the Matlab program also takes into consideration the following: Rules of the Convention on the International Regulations for Preventing Collisions at Sea COLREG, advance time for a manoeuvre calculated with regard to the ship's dynamic features and the assessment of the final deviation between the real trajectory and its assumed values.
It results from the performed simulation testing those algorithms able to determine the correct game trajectory when the ship is not in a situation when she approaches too large number of the observed objects or the said objects are found at long distances among them. In the case of the high traffic congestion the program is not able to determine the safe game manoeuvre. This sometimes results in the backing of the own object which is continued until the time when a hazardous situation improves.
The presented model and method of steering a ship may be used in practice both to improve the shipboard ARPA anti-collision system and to construct appropriate training simulator at officer training center.

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