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# HIGHWAY TRAFFIC DENSITY ESTIMATION WITH AN SDRE FILTER

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Abstract: A single origin and single destination highway traffic network with multiple routes and sections is considered. For dynamic routing of traffic, accurate measurement of traffic densities is needed, which requires the placement of many sensors, one for each section, which can be a costly solution. In this work, a simpler solution is presented based on a single sensor placed at the destination node and the use of a state dependent Riccati equation filter for estimation of traffic densities in each section of every route. Two sets of simulations are provided for illustration. In addition, a comparative robustness study with extended Kalman filter involving performance under different noise level is included. 
Keywords: Observers, Nonlinear filter, State estimation, Sensor, Traffic control.

## 1. INTRODUCTION

Dynamic feedback traffic routing is becoming a more viable solution to the traffic and incident management problems in Intelligent Transportation Systems (ITS). The references Papageorgiou (1990), Tan, et al (1993), and Peta and Mahmassani (1995) contain a small sample of feedback control approaches to highway traffic routing. These developments are the result of advances in sensor, actuator, communication, and control technologies as reported in Hasan and Yaz (1999). In dynamic routing, the effectiveness of a control strategy depends largely on the ability to accurately measure the state variables. This necessitates a very expensive solution to the feedback control implementation problem, namely, one typically needs to place a sensor in each section of every route to measure these traffic densities.

In this work, a nonlinear estimation technique is proposed, which uses only one sensor strategically placed at the destination node. This is a very cost effective alternative to the usual sensor placement scheme. The sensor measures the traffic occupancy defined in Gaber and Hoel (1997) through which the outflowing traffic density can be computed. This computed value is used as the measured output of the system where the sensing and computational errors are included in the measurement model as an additive noise term. A state dependent Riccati equation (SDRE) filter is used to then estimate individual traffic densities in every section of every route. On-line use of state estimators in this context is relatively new. Traditionally, the use of estimators has been confined to off-line computations for origin-destination trip table coefficients as in Okutani (1987) and Yang and Zhou (1998). Iwata, et al (1996) and Bartolini, et al (1996) describe on-line applications of nonlinear estimators. In this work, the performance of SDRE is compared against the extended Kalman filter (EKF) which has been used as the benchmark in nonlinear estimation problems. Simulation results demonstrate the effectiveness of this proposed sensor placement/estimation scheme by comparing the EKF and SDRE results with the true state values. In section 2, the dynamic traffic and the measurement models are given. Section 3 introduces EKF and SDRE filter used. Simulation studies are presented in section 4.

## 2. SYSTEM DESCRIPTION

Figure 1 illustrates a single-origin and single-destination freeway flow system where $u$ is the input traffic flow, $\beta_i$ is the split factor for the $i^{th}$ route, $\rho_{i,j}$ is the traffic density on

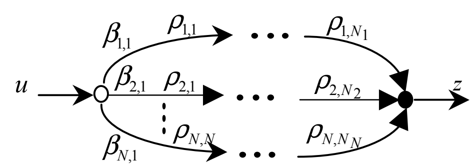


*Fig. 1 Single-origin/single-destination freeway*

the $i^{th}$ route $j^{th}$ section and $z$ is the output flow. There are $N$ alternate routes and each route is subdivided into $N_i$ sections. To facilitate mathematical modeling of this traffic flow system, it is assumed that: i) the traffic density in each section is uniform and ii) there is drivers' full compliance with $\beta_i$ (or if not, then the compliance factor can be made available to the estimator). Using these assumptions in conjunction with Greenshield (1935) model, the freeway flow system can be represented by the following space-discretized equations

$$\dot{\rho}_{i,j} = \frac{1}{\delta_{i,j}}[q_{i,(j-1)} - q_{i,j}] \tag{2.1}$$

$$v_{i,j} = v_{i,j}^f(1-\frac{\rho_{i,j}}{\rho_{i,j}^m}) \quad (2.2)$$

$$q_{i,j} = v_{i,j} \cdot \rho_{i,j} \quad (2.3)$$

where $\delta_{i,j}$ is the length of the $j^{th}$ section on the $i^{th}$ route, $q_{i,j}$ is the traffic flow in that section, $v_{i,j}$ is the equivalent section speed, $v_{i,j}^f$ is the section free-flow speed and $\rho_{i,j}^m$ is the maximum traffic jam density. Combining (2.1), (2.2) and (2.3), for $j$ = 1, and $i$ = 1,2,…,$N$-1, we obtain

$$\dot{\rho}_{i,1} = -\frac{1}{\delta_{i,1}}[v_{i,1}^f\rho_{i,1}(1-\frac{\rho_{i,1}}{\rho_{i,1}^m}) - \beta_i u] + w_{i,1} \quad (2.4)$$

where $w_{i,1}$ is the (formal) white noise with zero mean and $W_{i,1}$ variance. This formal description is justified in this case because the noise term has a constant coefficient that allows the use of regular calculus not necessitating the use of Ito calculus (Gard, 1988). This noise term physically represents the errors in modeling, especially the computational error made in space-discrimination of original partial differential equations, see Musha and Higuchi (1987). For $i = N$, the traffic density becomes

$$\dot{\rho}_{N,1} = -\frac{1}{\delta_{N,1}}[v_{N,1}^f\rho_{N,1}(1-\frac{\rho_{N,1}}{\rho_{N,1}^m}) - (1-\sum_{i=1}^{N-1}\beta_i)u] + w_{N,1} \quad (2.5)$$

and for $j \neq 1$, it is defined as:

$$\dot{\rho}_{i,j} = -\frac{1}{\delta_{i,j}}[v_{i,j}^f\rho_{i,j}(1-\frac{\rho_{i,j}}{\rho_{i,j}^m}) - v_{i,(j-1)}^f\rho_{i,(j-1)}(1-\frac{\rho_{i,(j-1)}}{\rho_{i,(j-1)}^m})] + w_{i,j} \quad (2.6)$$

It will be assumed that a single sensor is placed at the destination node. In this case, there are several choices for the sensor. In the present work, it is assumed that the occupancy variable is being measured at the destination node. Therefore, the following relationship can be used to compute the sum of the densities (Gaber and Hoel, 1997):

$$\sum_{i=1}^{N}\rho_{i,N_i} = \gamma\frac{5280}{L_{eff}} \quad (2.7)$$

where $\gamma$ is the percent of time that a detector is sensing a vehicle's presence to the total time in some chosen time interval, 5280 is the conversion factor from miles to feet, and $L_{eff}$ is the effective vehicle length in ft. Therefore the measurement equation becomes:

$$y = [0,0,..0,1,0,...,0,1,0,.....0,1]\begin{bmatrix}\rho_{1,1}\\ \vdots \\ \rho_{1,N_1} \\ \vdots \\ \rho_{N_1,N_N}\end{bmatrix} + \zeta \quad (2.8)$$

where the first part denotes the calculated sum of densities, and $\zeta$ denotes the measurement and computational error modeled as white noise with zero mean and $Z > 0$ variance. A compact form of the freeway flow dynamics description can be written as follows:

$$\dot{x} = f(x) + v + w \quad (2.9)$$

$$y = Cx + \zeta \quad (2.10)$$

where

$$x = \begin{bmatrix}\rho_{1,1} & \rho_{1,2} & \cdots & \rho_{2,1} & \cdots & \rho_{N,N_N}\end{bmatrix}^T \quad (2.11)$$

$$v = \begin{bmatrix}\frac{\beta_{1,1}}{\delta_{1,1}}u, & \frac{\beta_{1,2}}{\delta_{1,2}}u, & \cdots & \frac{\beta_{2,1}}{\delta_{2,1}}u, & \cdots & \frac{1-\sum_{i=1}^{N-1}\beta_{N,N_N}}{\delta_{N,N_N}}\end{bmatrix}^T \quad (2.12)$$

and $W$ is the covariance of the process noise in (2.9).

## 3. THE NONLINEAR ESTIMATORS

Given the system and the measurement scheme in (2.9) and (2.10), the nonlinear estimation problem involves constructing estimates of the state vector based on the measurement history. The full knowledge of system parameters including the split factors and the traffic inflow variable is assumed in this work. Subsequent works will involve relaxation of these assumptions.

Under these assumptions, the EKF (see Gelb,1974), which can be considered the industry standard, and SDRE, (Mracek, et al 1996), with promising performance characteristics (see Mracek, et al 1996, Azemi and Yaz, 1999) are used for state estimation. Obviously, there are other possible candidates for this task (see Azemi and Yaz, 1999). Employment of alternative nonlinear estimation techniques and their performance comparison for this system description will be left to future works.

### 3.1 Observer Design Using EKF

Consider the freeway flow dynamics description as given by (2.9) and (2.10). We define the Jacobian:

$$F(t) = \frac{\partial f(x)}{\partial x}\Big|_{x=\hat{x}(t)} \quad (3.1)$$

where $\hat{x}(t)$ is the state estimate obtained by the EKF for the given dynamical description which is as follows:

$$\dot{\hat{x}} = f(\hat{x}) + v + K(t)(y - C\hat{x}) \quad (3.2)$$

with the initial condition $\hat{x}(0)$ is set to the expected value of $\hat{x}(0) = E\{x(0)\}$, where the Kalman gain satisfies

$$K(t) = P(t)\,C^T Z^{-1} \quad (3.3)$$

and $P(t)$ is the solution to the matrix Riccati differential equation

$$\dot{P}(t) = F(t)P(t) + P(t)F^T(t) - P(t)C^T Z^{-1} CP(t) + W \quad (3.4)$$

with initial condition $P(0)$ equal to the covariance of the initial state if it is available.

### 3.2 Observer Design Using SDRE

Application of SDRE technique has become more popular in recent years. SDRE design methods have been used in nonlinear filter development and control designs for some nonlinear benchmark problems (Mracek, et al 1996). In this approach, an algebraic Riccati equation is solved at each time by updating its parameters via substitution of the present state estimate that evolves over time.

Let us consider the same system dynamics given by (2.9) and (2.10), and also assume that the nonlinear dynamics are representable by the following linear structure having state-dependent coefficient form:

$$\begin{aligned} \dot{x} &= A(x)x \\ y &= C(x)x \end{aligned} \quad (3.5)$$

The SDRE filter is given by

$$\dot{\hat{x}} = A(\hat{x})\hat{x} + K(t)[y - C(\hat{x})\hat{x}], \qquad \hat{x}(0) = \hat{x}_0 \quad (3.6)$$

where the filter gain is computed as

$$K(t) = P(t)C^T(\hat{x})Z^{-1} \quad (3.7)$$

and $P(t)$ is the solution to the algebraic Riccati equation

$$\begin{aligned} &A(\hat{x})P(t) + P(t)A^T(\hat{x}) - P(t)C^T(\hat{x})Z^{-1} \\ &\times C(\hat{x})P(t) + W = 0 \end{aligned} \quad (3.8)$$

that needs to be solved at each different $t \geq 0$.

## 4. SIMULATION STUDIES

In this section, two examples will be given to demonstrate the application of the SDRE in comparison with EKF to estimation of traffic densities of all sections based on processing of the measurement made at the destination node.

Example 1: Let us assume a traffic network with two routes having two sections each. For cost-effectiveness, we use one sensor placed at destination node as indicated in Fig. 2.

The following values of the model parameters are used in the simulation: The maximal vehicle density in each section: $\rho_{1,1}^m = \rho_{1,2}^m = \rho_{2,1}^m = \rho_{2,2}^m = 150$. The initial vehicle density in each section: $\rho_{1,1}(0) = 11.5$, $\rho_{1,2}(0) = 10.5$, $\rho_{2,1}(0) = 12.5$, $\rho_{2,2}(0) = 14.5$. The free speeds in each section: $v_{1,1}^f = 55$, $v_{1,2}^f = 65$, $v_{2,1}^f = 40$, $v_{2,2}^f = 55$. The length of each section: $\delta_{1,1} = 10$, $\delta_{1,2} = 9$, $\delta_{2,1} = 8$, $\delta_{2,2} = 7$. The initial values for EKF and SDRE: $\hat{\rho}_{1,1}(0) = 12.765$, $\hat{\rho}_{1,2}(0) = 8.925$, $\hat{\rho}_{2,1}(0) = 15.625$, $\hat{\rho}_{2,2}(0) = 10.875$. The diversion factor for the first route: $\beta = 0.4$. The inflow of traffic: $u(t) = 2400 \cdot |\sin(t)|$. The sampling interval: $T = 10^{-4}$. The $P_0$ matrix in EKF: $P_0 = 10I_4$. The modeling and computational error $w_{i,j}(i,j=1,2) \sim N(0,.01)$ or Gaussian white noise with zero mean and 0.01 variance. The sensor and computational error $\zeta \sim N(0, 0.01)$.

Based on the EKF and SDRE design procedures described in the previous section, the relevant coefficient matrices for the EKF and SDRE are found and indicated in Fig.3 and Fig. 4, respectively with $C = [0 \quad 1 \quad 0 \quad 1]$. Following the procedures mentioned above, we discretized the continuous-time system with time interval $10^{-4}$, and then, carried out the simulation via a MATLAB code. Simulation results are shown in Figs. 5-12. Both EKF and SDRE can estimate the state-variables (traffic densities in our example) well. In other words, each estimation error gradually converges to zero. However, it is found that the convergence rate is highly related to the choice of initial values. Since this is a nonlinear estimation problem where the convergence can only be local, good initial guess may be beneficial for the fast convergence.

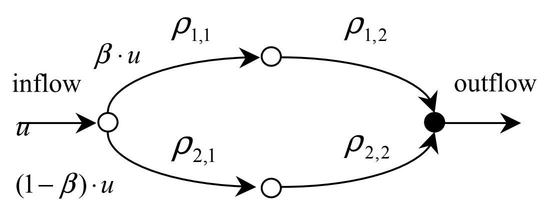


*Fig. 2 Two routes and two sections of each*

*●: Sensor location*

$$F = \begin{bmatrix} -\frac{1}{\delta_{1,1}}[v_{1,1}^f(1-\frac{2\hat{\rho}_{1,1}}{\rho_{1,1}^m})] & 0 & 0 & 0 \\ \frac{1}{\delta_{1,2}}[v_{1,1}^f(1-\frac{2\hat{\rho}_{1,1}}{\rho_{1,1}^m})] & -\frac{1}{\delta_{1,2}}[v_{1,2}^f(1-\frac{2\hat{\rho}_{1,2}}{\rho_{1,2}^m})] & 0 & 0 \\ 0 & 0 & -\frac{1}{\delta_{2,1}}[v_{2,1}^f(1-\frac{2\hat{\rho}_{2,1}}{\rho_{2,1}^m})] & 0 \\ 0 & 0 & \frac{1}{\delta_{2,2}}[v_{2,1}^f(1-\frac{2\hat{\rho}_{2,1}}{\rho_{2,1}^m})] & -\frac{1}{\delta_{2,2}}[v_{2,2}^f(1-\frac{2\hat{\rho}_{2,2}}{\rho_{2,2}^m})] \end{bmatrix}$$

*Fig.3 The EKF system matrix in example 1*

$$A=\begin{bmatrix} -\frac{1}{\delta_{1,1}}[v_{1,1}^{f}(1-\frac{\hat{\rho}_{1,1}}{\rho_{1,1}^{m}})] & 0 & 0 & 0 \\ \frac{1}{\delta_{1,2}}[v_{1,1}^{f}(1-\frac{\hat{\rho}_{1,1}}{\rho_{1,1}^{m}})] & -\frac{1}{\delta_{1,2}}[v_{1,2}^{f}(1-\frac{\hat{\rho}_{1,2}}{\rho_{1,2}^{m}})] & 0 & 0 \\ 0 & 0 & -\frac{1}{\delta_{2,1}}[v_{2,1}^{f}(1-\frac{\hat{\rho}_{2,1}}{\rho_{2,1}^{m}})] & 0 \\ 0 & 0 & \frac{1}{\delta_{2,2}}[v_{2,1}^{f}(1-\frac{\hat{\rho}_{2,1}}{\rho_{2,1}^{m}})] & -\frac{1}{\delta_{2,2}}[v_{2,2}^{f}(1-\frac{\hat{\rho}_{2,2}}{\rho_{2,2}^{m}})] \end{bmatrix}$$

*Fig.4 The SDRE system matrix in example 1*

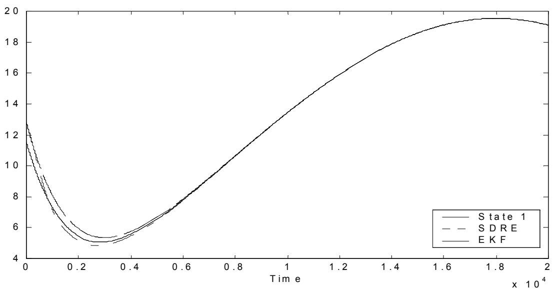


***Fig.5 Comparison of state 1 with its estimates***

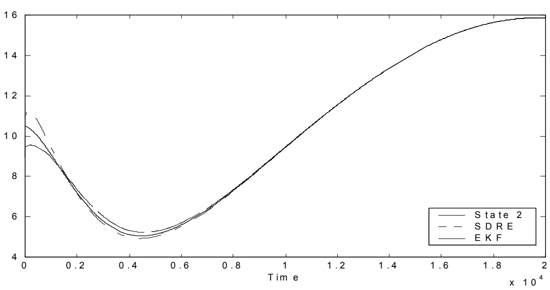


***Fig.6 Comparison of state 2 with its estimates***

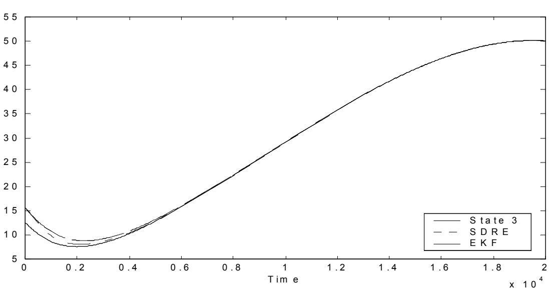


*Fig.7 Comparison of state 3 with its estimates*

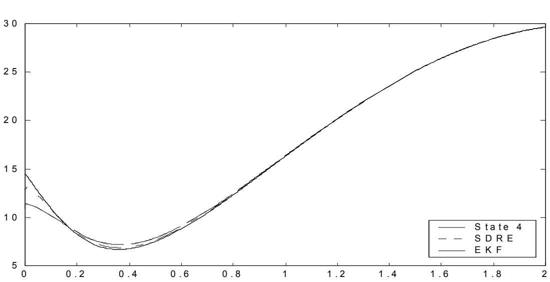


***Fig.8 Comparison of state 4 with its estimates***

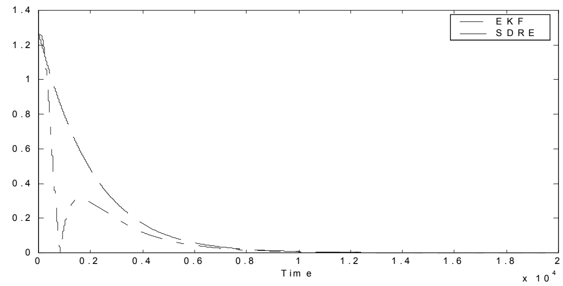


***Fig.9 Comparison of state 1 absolute error in estimation.***

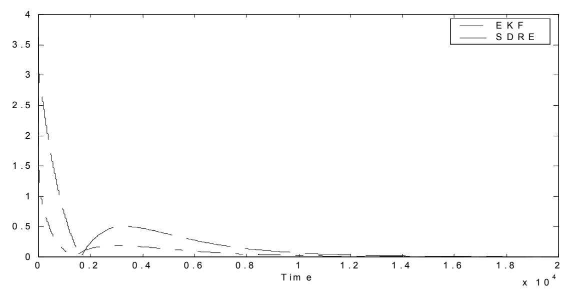


***Fig.10 Comparison of state 2 absolute error in estimation***

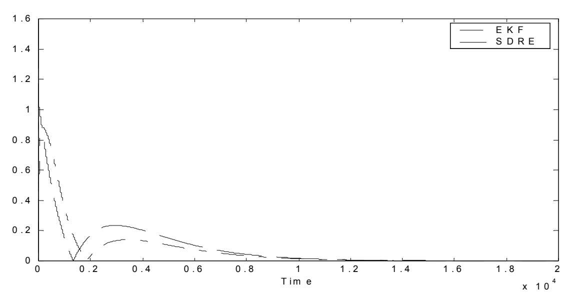


***Fig.11 Comparison of state 3 absolute error in estimation***

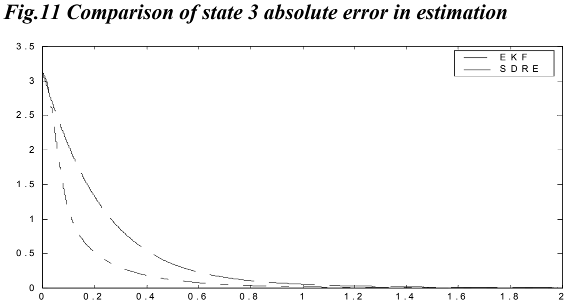


***Fig.12 Comparison of state 4 absolute error in estimation***

Example 2: This time, we design EKF and SDRE observers for a segment of a freeway with three routes having one section each as shown in Fig. 13, assuming the following simulation parameters: The maximal vehicle density in each section: $\rho_1^m = \rho_2^m = \rho_3^m = 150$. The initial vehicle density in each section: $\rho_1(0) = 115$, $\rho_2(0) = 145$, $\rho_3(0) = 100$. The free speeds in each section: $v_1^f = v_2^f = v_3^f = 55$. The length of each section: $\delta_1 = 10$, $\delta_2 = 9$, $\delta_3 = 8$. The initial values for the EKF and SDRE observer: $\hat{\rho}_1(0) = 0$, $\hat{\rho}_2(0) = 0$, $\hat{\rho}_3(0) = 0$. The diversion factor for the first route: $\beta_1 = 0.6$. The diversion factor for the second route: $\beta_2 = 0.1$. The inflow of traffic: $u(t) = 2400 \cdot |\sin(t)|$. The sampling interval: $T = 10^{-2}$. The $P_0$ matrix in EKF: $P_0$ = $5I_3$. The modeling and computational error $w_i (i = 1,2,3) \sim N(0,.2)$. The sensor and computational error $\zeta \sim N(0,.1)$.

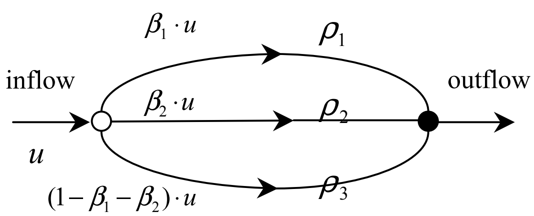


***Fig. 13 Three routes and one section of each***
**●*: Sensor location***

Now, the EKF coefficient matrix becomes

$$F = \begin{bmatrix} -\frac{1}{\delta_1}[v_1^f(1-\frac{2\hat{\rho}_1}{\rho_1^m})] & 0 & 0 \\ 0 & -\frac{1}{\delta_2}[v_2^f(1-\frac{2\hat{\rho}_2}{\rho_2^m})] & 0 \\ 0 & 0 & -\frac{1}{\delta_3}[v_3^f(1-\frac{2\hat{\rho}_3}{\rho_3^m})] \end{bmatrix}$$

and the SDRE coefficient matrix becomes

$$A = \begin{bmatrix} -\frac{1}{\delta_1}[v_1^f(1-\frac{\hat{\rho}_1}{\rho_1^m})] & 0 & 0 \\ 0 & -\frac{1}{\delta_2}[v_2^f(1-\frac{\hat{\rho}_2}{\rho_2^m})] & 0 \\ 0 & 0 & -\frac{1}{\delta_3}[v_3^f(1-\frac{\hat{\rho}_3}{\rho_3^m})] \end{bmatrix}$$

with $C = [1 \quad 1 \quad 1]$. The simulation results are shown in Figs. 14-19. It can be seen that the observers estimate each state-variable well.

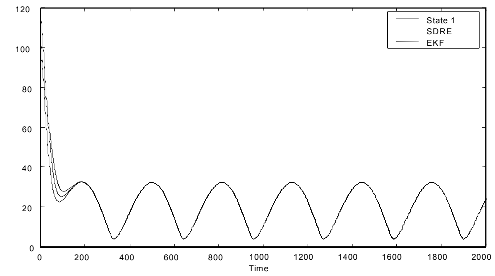


***Fig.14 Comparison of state 1 with its estimates***

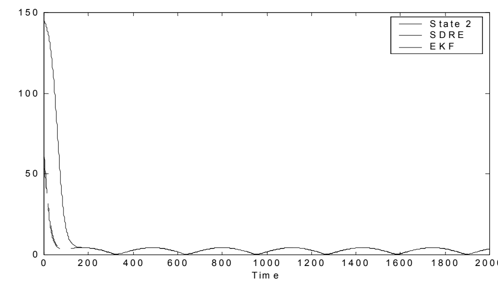


***Fig.15 Comparison of state 2 with its estimates***

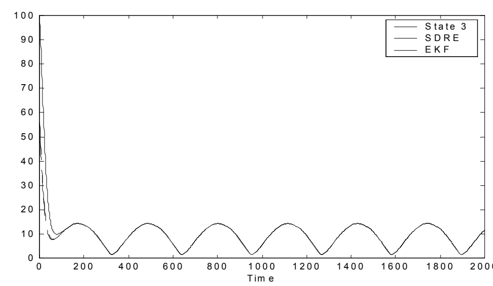


***Fig.16 Comparison of state 3 with its estimates***

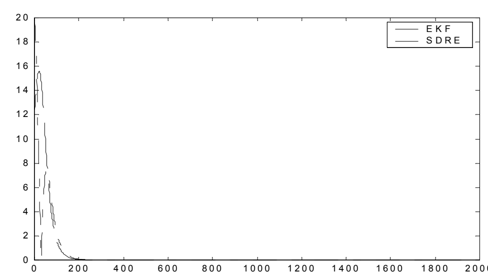


***Fig.17 Comparison of state 1 absolute error in estimation***

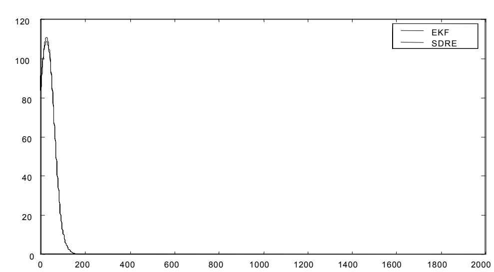


***Fig.18 Comparison of state 2 absolute error in estimation***

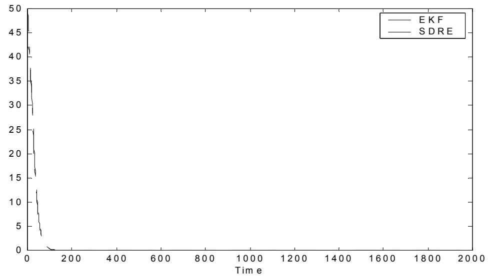


***Fig.19 Comparison of state 3 absolute error in estimation***

The robustness of the above observers in the presence of modeling, sensor and computational errors are investigated. This is done by increasing the process and measurement variances W and Z. This is an important study since in reality, the proposed schemes have to work in noisy environments. The squared errors (SE), which is defined as

$$SE = \int_0^T \sqrt{(x_1 - \hat{x}_1)^2 + (x_2 - \hat{x}_2)^2 + (x_3 - \hat{x}_3)^2}\, dt$$

where $T$ is the final time is calculated for both observers for various process and measurement noise variances. As shown in Table 1 and 2, EKF has better performance as variance is increased.

| ***W*** | **EKF** | **SDRE** |
|---|---|---|
| **0.1** | 7828 | 7863 |
| **1** | 7830 | 7870 |
| **5** | 7926 | 7970 |
| **10** | 8035 | 8111 |
| **15** | 8056 | 8098 |
| **20** | 8091 | 8198 |

*Table 1. SE performances of observers as a function of process noise variance*

| ***W*** | **EKF** | **SDRE** |
|---|---|---|
| **0.1** | 8312 | 8921 |
| **1** | 8334 | 8983 |
| **5** | 8349 | 8956 |
| **10** | 9160 | 9335 |
| **15** | 9171 | 9527 |
| **20** | 9183 | 9601 |

***Table 2. SE performances of observers as a function of measurement noise variance***

## 5. CONCLUSION

A cost-effective alternative to sensor placement and density estimation is presented for possible use in dynamic routing of traffic for networks with multiple routes having multiple sections. Simulation results are promising in that the convergence of the state variable estimates by SDRE is almost the same as that achieved with the EKF approach. However, the convergence rate is observed to be closely dependent on the choice of initial conditions. Further work will involve comparative studies of more types of nonlinear estimators, generalization of this sensor placement and estimation scheme to the network-wide estimation with multiple nodes, and relaxation of the assumption regarding the knowledge of our system parameters.

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