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IMPROVED MULTIPLE MODEL ADAPTIVE CONTROL AND ITS STABILITY ANALYSIS [1]

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Abstract: Multiple model adaptive control (MMAC) of discrete time plant is considered in this paper. The plant can be a time invariant system with unknown parameters or a time variant system with jumping parameter. Localization method is combined with the design of discrete time system MMAC. Multiple models of the plant are set up to cover the uncertainties of the plant dynamics. Every sample time, by using localization method, only a few of models which are closer to the "true" model of the plant are selected to form a multiple model controller, so the big computation burden of the multiple model algorithm is greatly reduced. It is proved that the closed-loop system is stable when multi-model controller is used to a linear time-invariant plant with unknown parameters. The simulation results are given to show the usefulness of the proposed method. *Copyright © 2002 IFAC*
Key words: adaptive control, discrete time, models, parameters, closed-loop

1.INTRODUCTION

Conventional adaptive control system based on a fixed or slowly adaptive model can give good control performance. But when the parameters of the system change abruptly from one context to another, the conventional adaptive control will react slowly due to the large errors in the parameters identified, the output of the system will change abruptly and may be out of control at this time. Multi-model adaptive control (MMAC) is one of effective methods to solve this kind of problem. The earliest MMAC appeared in initial 70's(Athans, M, et al, 1977, Lainiotis, D. G., 1971), controller is always produced as the probability-weighted average of elemental controller outputs, and the stability of closed loop system is difficult to prove. From mid 1980's to now, a lot of MMAC algorithms combined with a switching index function have been given successively, and this kind of MMAC can guarantee the stability of closed-loop system.

Generally, MMAC with switching index function can be divided into two categories, indirect switching and direct switching. Indirect switching control can also be called as supervisory control because a supervisory function is used to decide when and which controller should be switched. Narendra, K. S. and Autenrieth, T. Using this method to improve the transient response of systems(Narendra, K. S., and Balakrishnan, J.1994,1997; Narendra, K.S. and Xiang, C.1998, Autenrieth T. and Rogers E.1999). In direct switching control, the choice of when to switch to the next controller in a predetermined sequence is based directly on the output of the plant(Fu M.Y. and Barmish B.R.1986). The results above in switching MMAC are most in continuous time system. The study of MMAC of discrete time system is just at beginning(Narendra, K.S. and Xiang, C,2000; Xiaoli Li, Wei Wang. 1999) some useful results in continuous systems are difficult to be extended to discrete time systems.

Another problem in the switching MMAC is that a lot of models are needed to keep the accuracy of control, but too much models would lead heavy computation and excessive "competition" from the "unnecessary" models may degrade the performance of MMAC, all these would make MMAC impractical

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for actual implementation. Recently, localization method is always used in design procedure of direct MMAC(Zhivoglyadov, P, Middleton, R.H., Fu,M.,1997; Zhivoglyadov P., Middleton R.H. and Fu,M.Y.,2000), In every sample time, only the models that are closer to the "true" model of plant are used, and the multiple model adaptive controller are formed by these selected model. This method can speed up the convergence of switching in discrete time direct multiple model adaptive control.

This paper combine localization method with the discrete time MMAC, and the computation burden of multiple model algorithm is great reduced. It is proved that when the multi-model adaptive controller is used to control a linear time-invariant plant, the stability of the closed-loop system can be guaranteed. And from the simulations it can be seen that this kind of MMAC can improve the control performance greatly to the plant with jumping parameters.

2. PROBLEM FORMULATION AND ADAPTIVE CONTROL

Consider a discrete time system with input-output description

$$y(t+d)=\sum_{i=0}^{n-1}\alpha_i(t)y(t-i)+\sum_{j=0}^{n-1}\beta_j(t)u(t-j) \quad (1)$$

where $y(t)$ and $u(t)$ are the output and input of the system, the $2n$ parameters $\alpha_i(t)$ and $\beta_i(t)(i=0,1,...,n-1)$ are unknown, the delay d through the system is known, and β_0, the coeffcient of $u(t)$, is not equal to zero. The above model may be derived from an equivalent representation of the plant in the form

$$A(q^{-1},t)y(t)=q^{-d}B(q^{-1},t)u(t) \quad (2)$$

where

$$A(q^{-1},t)=1+a_1(t)q^{-1}+a_2(t)q^{-2}+\cdots+a_n(t)q^{-l}$$
$$B(q^{-1},t)=b_0(t)+b_1(t)q^{-1}+b_2(t)q^{-2}+\cdots b_m(t)q^{-m}$$

q^{-1} is the unit shift operator. From the point of view of mathematical tractability, (1) is found to be the most convenient.

We shall focus our attention on the multi-model adaptive control for a class of plants with invariant parameters or jumping change parameters. So plant (1) should satisfy the following assumptions.

A1) the parameters $\alpha_i(t)$, $\beta_i(t)$ $i\in\{0,...,n-1\}$ are the constant functions of t or piecewise constant functions of t, the parameters vector

$$[\alpha_0(t),...,\alpha_{n-1}(t);\beta_0(t),...,\beta_{n-1}(t)]^T$$

lie in a closed convex region Ω.

A2) the coefficient β_0 is known;

A3) the roots of the polynomial $B(q^{-1})$ in (2) lie inside the unit circle in the complex plane (i.e. the system is minimum phase)

From assumption A1 we know that plant (1) can be a linear time invariant system (LTI) or a linear time variant system (LTV) with jumping parameters, plant (1) can be written in a regression form as following

$$\begin{aligned} y(t+d)&=\phi^T(t)\theta^*(t)\\ \phi(t)&=[y(t),...,y(t-n+1),u(t),...,u(t-n+1)]^T\\ \theta^*(t)&=[\alpha_0(t),...,\alpha_{n-1}(t),\beta_0(t),...,\beta_{n-1}(t)]^T \end{aligned} \quad (3)$$

To obtain an adaptive control algorithm the following recursive least-squares estimator is used

$$\hat\theta(t)=proj\left\{\hat\theta(t-1)+\frac{\phi(t-d)e(t)}{1+\phi(t-d)^T\phi(t-d)}\right\} \quad (4)$$

where

$$\hat\theta(t)=[\hat\alpha_0(t),...,\hat\alpha_{n-1}(t),\hat\beta_0(t),...,\hat\beta_{n-1}(t)]^T$$

$$e(t)=y(t)-\hat y(t)=y(t)-\phi^T(t-d)\hat\theta(t-1) \quad (5)$$

where proj(.) is a projection operator constraining $\hat\theta(t)$ in known convex region Ω.

The objective of adaptive control problem is to determine a bounded control input $u(t)$ such that the output $y(t)$ of the plant asymptotically tracks a specified bounded reference output $y^*(t)$, i.e.

$$\lim_{t\to\infty}\left|y(t)-y^*(t)\right|=0 \quad (6)$$

When $\theta^*(t)$ is known, and $y^*(t+d)$ is known at time t, the input $u(t)$ can be computed from the equation

$$y^*(t+d)=\phi^T(t)\theta^*(t) \quad (7)$$

When $\theta^*(t)$ is unknown, using $\hat\theta$ in place of $\theta^*(t)$, we have

$$y^*(t+d)=\phi^T(t)\hat\theta(t) \quad (8)$$

from which the input $u(t)$ can be computed.

So from (3)-(8), it can be seen that a conventional adaptive controller of system (1) is formed. For LTI plant (1), when the initial value of estimator (4)-(5) is far from the true value, the transient response of adaptive control (8) will be not very well. For the same reason, when conventional adaptive control is applied to the plant (1) with jumping parameters, the control performance will be very bad. One method to solve this kind of problem is to improve control performance by using multiple models to cover the uncertainties of the plant.

3. MULTI-MODEL ADAPTIVE CONTROL

A localization method is used to set up the multi-model set. First, decomposing the parameter set Ω to obtain a finite cover $\{\Omega_i\}_1^L$, which satisfies the following condition:

C1: $\Omega_i \subset \Omega, \quad \Omega_i \neq \{\}, \quad i = 1,...,L$;

C2: $\bigcup_{i=1}^{L} \Omega_i = \Omega$

C3: for each $i = 1,...,L$, let θ_i and $r_i > 0$ denote the "center" and "radius" of Ω_i, i.e. $\theta_i \in \Omega_i$, and $\|\theta - \theta_i\| \leq r_i$ for all $\theta \in \Omega_i$

The L fixed models are set up according to C1, C2, C3, every model has parameter θ_i and output

$$\hat{y}_i(t) = \phi(t-d)^T \theta_i \quad ,i = 1,...,L$$

Two adaptive models $\theta_{A_1}, \theta_{A_2}$ just like (4)-(5) are used to assure stability and good transient state performance.

$$\theta_{A_i}(t) = proj\left\{\theta_{A_i}(t-1) + \frac{\phi(t-d)e_{A_i}(t)}{1+\phi(t-d)^T \phi(t-d)}\right\} \quad (9)$$

where

$$\theta_{A_i}(t) = [\hat{\alpha}_1(t),...,\hat{\alpha}_n(t), \hat{\beta}_1(t),...,\hat{\beta}_n(t)]^T$$

$$e_{A_i}(t) = y(t) - \hat{y}_{A_i}(t) = y(t) - \phi^T(t-d)\theta_{A_i}(t-1)$$

$$i \in \{1, \quad 2\} \quad (10)$$

Adaptive model 1 have the model parameter θ_{A_1} just like conventional adaptive model, adaptive model 2 have parameter θ_{A_2} which starting adaptation from the location of the fixed model which is closest to the given plant, adaptive model 2 is used to improve transient response. To obtain a multi-model adaptive controller, a definition should be given below.

Definition1: switching function

$$J_{A_i}(t,t_0) = \sum_{j=t_0}^{t} \frac{e_{A_i}^2(j)}{1+\phi^T(t-d)\phi(t-d)}, \quad i = 1,2$$

$$J_i(t,t_0) = \sum_{j=t_0}^{t} \frac{e_i^2(j)}{1+\phi^T(t-d)\phi(t-d)}, \quad i = 1,2,...L$$

(11)

From definition 1, we can obtain the following multi-model adaptive control based on localization method.

MMAC of LTI plant(LTIMMAC) based on localization method

1) $t = t_0, \quad I(t_0) = I_0 = \{1,2,...,L\}$

2) for $t > t_0$ define

$$\hat{I}(t) = \{j : |\phi(t-d)^T\theta_j - y(t)| \leq r_j \|\phi(t-1)\|, j \in I(t-1)\}$$

let

$$I(t) = \hat{I}(t)$$

$$i(t) = \arg\min_{l \in I(t)} J_l(t,t_0)$$

$$j(t) = \arg\min_{l \in \{1,2\}} J_{A_l}(t,t_0)$$

if $J_{A_{j(t)}}(t,t_0) \leq J_{i(t)}(t,t_0)$, let $\hat{\theta}(t) = \theta_{A_{j(t)}}$, calculate control input as (8), and let $\theta_{A_2} = \theta_{A_{j(t)}}$, $J_{A_2}(t,t_0) = J_{A_{j(t)}}(t,t_0)$, t=t+1, go back to 2).

else, let $\hat{\theta}(t) = \theta_{i(t)}$, calculate control input as (8), and let $\theta_{A_2} = \theta_{i(t)}$, $J_{A_2}(t,t_0) = J_{i(t)}(t,t_0)$, t=t+1 go back to 2).

MMAC of plant with jumping parameters (JPMMAC) based on localization method

1) $t = t_0, \quad I(t_0) = I_0 = \{1,2,...,L\}$

2) for $t > t_0$ define

$$\hat{I}(t) = \{j : |\phi(t-d)^T\theta_j - y(t)| \leq r_j \|\phi(t-1)\|, j \in I(t-1)\}$$

and calculate

$$I(t) = \begin{cases} I(t-1) \cap \hat{I}(t) & , if \quad I(t-1) \cap \hat{I}(t) \neq \{\} \\ I_0 & , else \end{cases}$$

$$i(t) = \arg\min_{l \in I(t)} J_l(t,t_k)$$

$$j(t) = \arg\min_{l \in \{1,2\}} J_{A_l}(t,t_k)$$

where t_k is the biggest positive number which satisfy $t_k < t$ and $I(t_k) = I_0$

if $J_{A_{j(t)}}(t,t_k) \leq J_{i(t)}(t,t_k)$, let $\hat{\theta}(t) = \theta_{A_{j(t)}}$, calculate control input as (8), and let $\theta_{A_2} = \theta_{A_{j(t)}}$, $J_{A_2}(t,t_k) = J_{A_{j(t)}}(t,t_k)$, t=t+1 go back to 2).

else, let $\hat{\theta}(t) = \theta_{i(t)}$, calculate control input as (8), and let $\theta_{A_2} = \theta_{i(t)}$, $J_{A_2}(t,t_k) = J_{i(t)}(t,t_k)$, t=t+1 go back to 2).

4. STABILITY ANALYSIS OF MMAC

In this section, we can show that when LTIMMAC is applied to a LTI plant, the closed loop system is BIBO stable.

Lemma 1(Goodwin, G.C. and Sin, K.S.,1984)**:** for linear time invariant system, the estimation scheme (4)-(7) has the following properties.

(1) $$\lim_{t \to \infty} \frac{e^2(t)}{1+\phi(t-d)^T\phi(t-d)} = 0$$

(2) $\lim_{N\to\infty}\sum_{t=1}^{N}\frac{e(t)^2}{1+\phi(t-d)^T\phi(t-d)}<\infty$

(3) $\lim_{t\to\infty}\|\theta(t)-\theta(t-d)\|=0$

Theorem 1: when conventional adaptive control (3)-(8) is applied to linear time invariant system of (1), then all the signal will remain bounded and

$$|e_c(t)|=|y(t)-y^*(t)|\to 0 \text{ , as } t\to\infty$$

Proof: Let the tracking error be defined as

$$\begin{aligned} e_c(t)&=y(t)-\hat{y}(t)+\hat{y}(t)-y^*(t)\\ &=e(t)+\phi^T(t-d)\left[\hat{\theta}(t-1)-\hat{\theta}(t-d)\right]\end{aligned}$$

then

$$\frac{|e_c(t)|}{[1+\phi^T(t-d)\phi(t-d)]^{1/2}}$$

$$\le\frac{|e(t)|}{1+[\phi^T(t-d)\phi(t-d)]^{1/2}}+\frac{\|\phi(t-d)\|\|\hat{\theta}(t-1)-\hat{\theta}(t-d)\|}{[1+\phi^T(t-d)\phi(t-d)]^{1/2}}$$

by using lemma 1,it immediately follows that both terms in the right-hand side of the inequality tend to zero so that

$$\lim_{t\to\infty}\frac{|e_c(t)|}{1+\phi^T(t-d)\phi(t-d)}=0 \qquad (12)$$

By the minimum phase assumption A3, we have

$$|u(t-d)|\le m_1+m_2\max_{1\le\tau\le t}|y(\tau)|$$

for some $m_1>0, m_2>0$

from the definition of $\phi(t)$, we have

$$|\phi(t-d)|\le m_3+m_4\max_{1\le\tau\le t}|y(\tau)|$$

for some $m_3>0, m_4>0$

since

$$|y(t)|=|e_c(t)+y^*(t)|\le|e_c(t)|+|y^*(t)|$$

$|y^*(t)|$ is bounded, then

$$\|\phi(t-d)\|\le m_5+m_6\max_{1\le\tau\le t}(|e_c(t)|)$$

$$m_5>0, m_6>0 \qquad (13)$$

from (12) the norm of the regression vector $\phi(t-d)$ can be either bounded for all *t*, or grow in an unbounded fashion. In the former case, it directly follows that $|e_c(t)|\to 0$ as $t\to\infty$; if $\|\phi(t-d)\|$ grows in an unbounded fashion, from (13), it can been seen that it can not grow faster than $|e_c(t)|$. However, this leads to a contradiction of (12). Hence, all the signals in the system must be bounded and $\lim_{t\to\infty}|e_c(t)|=0$.

Theorem 2: when LTIMMAC is applied to a LTI plant, there exists a time T that after $t>T$, the close-loop satisfies:

S1) $\{u(t)\}$ is bounded

S2) $\{y(t)\}$ is bounded

S3) $\lim_{t\to\infty}|e_c(t)|=|y(t)-y^*(t)|=0$

Proof: From lemma1 and LTIMMAC, it follows that the identification error of the adaptive model satisfies the condition

$$\lim_{t\to\infty}J_{A_i}(t,t_0)=\lim_{t\to\infty}\sum_{j=t_0}^{t}\frac{e_{A_i}^2(j)}{1+\phi^T(t-d)\phi(t-d)}<\infty,\quad i=1,2 \qquad (14)$$

For the fixed models, $\lim_{t\to\infty}J_i(t),\ i\in\{1,2,...L\}$ is either bounded or ∞.

If it tends to ∞, from LTIMMAC, then there exists a finite time T, such that

$$J_{A_k}(t,t_0)<J_i(t,t_0)\ \ (k=1,2; i\in I(t)) \qquad (15)$$

for all $t\ge T$, which implies that the switching algorithm will choose the adaptive model and that switching will stop after a finite time.

If $\lim_{t\to\infty}J_i(t),\ i\in\{1,2,...L\}$ of the fixed model *i* is bounded, then we can have that the fix model *i*, $i\in\{1,.,.,L\}$ satisfy that

$$\lim_{t\to\infty}\frac{e_i^2(t)}{1+\phi^T(t-d)\phi(t-d)}=0,\ i\in\{1,2,...L\} \qquad (16)$$

At time *t*, from LTIMMAC, let the *i*th model be chosen, it can be one of the adaptive models $\{A_1,A_2\}$, or fixed model of set $I(t)$. The control input $u(t)$ is computed from

$$y^*(t+d)=\phi^T(t)\theta_i(t) \qquad (17)$$

The control error at time (*t*+*d*) is then given by

$$\begin{aligned} e_c(t+d)&=y(t+d)-\hat{y}_i(t+d)+\hat{y}_i(t+d)-y^*(t+d)\\ &=e_i(t+d)+\phi^T(t-d)[\theta_i(t+d-1)-\theta_i(t)]\end{aligned} \qquad (18)$$

from lemma1 for adaptive model, and evident for fixed model, we have

$$\|\theta_i(t+d-1)-\theta_i(t)\|\to 0 \qquad (19)$$

for all $i\in\{I(t),A_1,A_2\}$ .Hence both term on the r.h.s. of (18), when normalized, i.e.

$$(e_i(t+d))/([1+\phi^T(t)\phi(t)]^{1/2}),$$
$$(\phi^T(t)[\theta_i(t+d-1)-\theta_i(t)])/([1+\phi^T(t)\phi(t)]^{1/2})$$

tend to zero as $t\to\infty$. So as the proof of one adaptive model control in Theorem 1, we can have all the signals in the system are bounded, and $\lim_{t\to\infty}|e_c(t)| = 0$.

Remark 1. It can be seen from the theorem 1 that the LTIMMAC can guarantee the BIBO stability of LTI plant. But the transient response has been greatly improved because of the existence of multiple models.

Remark 2. A LTV discrete time system with jumping parameters can be seen as a LTI system between the interval of the two adjacent parameter jumping. From theorem 2 when the interval is sufficient long, for the same reason as remark 1, JPMMAC can improve the control performance of LTV discrete time system with jumping parameters, and this can be seen from the following simulation example.

5. SIMULATION

Consider the following discrete time system

$$(1 + a_1(t)q^{-1} + a_2(t)q^{-2})y(t) = q^{-1}(b_1(t) + b_2(t)q^{-1})u(t)$$

Case 1: LTI system

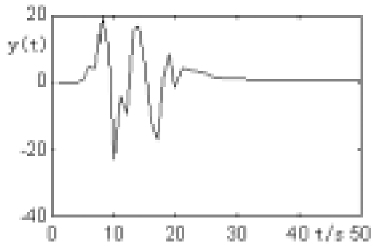
$$a_1(t) = -5.5;\ a_2(t) = 0.5;\ b_1(t) = 2.5;\ b_2(t) = 2;\quad 0 < t < \infty$$

Case 1: LTV system with jumping parameter

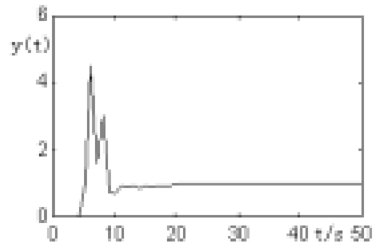
$$a_1(t) = \begin{cases} -1 & 0 \le t \le 49 \\ -2 & 50 \le t \le 99 \\ -3 & 100 \le t \le 149 \\ -5.5 & 150 \le t < \infty \end{cases};$$

$$a_2(t) = 0.5;\ b_1(t) = 2.5;\ b_2(t) = 2;\ 0 < t < \infty$$

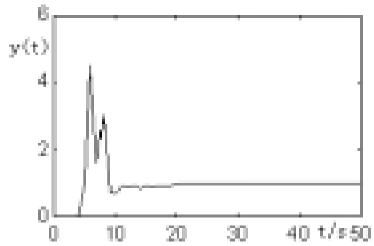
For case 1, LTIMMAC, initial parameter value $\theta_{A_1}(0) = [-2, 0.5, 2.5, 2]^T$, $\theta_{A_2}(0) = [-2, 0.5, 2.5, 2]^T$ for MMAC controller, 186 fixed model are set up; $\hat{\theta}(0) = [-2, 0.5, 2.5, 2]^T$ for single adaptive model control; $y^*(t) = 1,\ 0 < t < \infty$. The simulation result is in Fig 1.



(a)



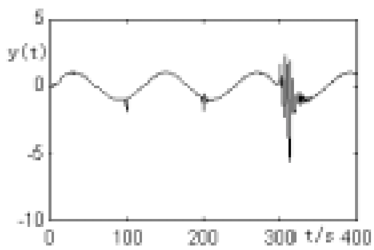
(b)



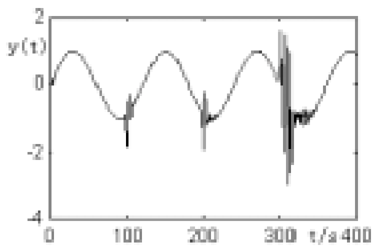
(c)

Fig1. Multiple model adaptive control of linear time invariant system, (a) Single adaptive model adaptive control, simulation time 0.06s; (b) MMAC without using localization method, simulation time 0.77s; (c) MMAC using localization method, simulation time 0.22s

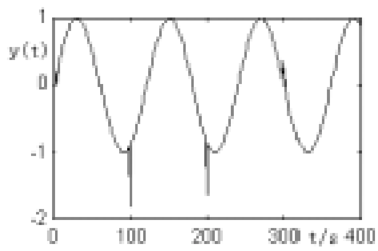
For case 2, JPMMAC, the initial parameter value is same as case 1, $y^*(t) = \sin(\pi t / 60),\quad 0 < t < \infty$, the simulation result is in Fig 2.



(a)



(b)



(c)

Fig2.Multiple model adaptive control of linear time invariant system, (a) Single adaptive model adaptive control, simulation time 0.16s; (b) MMAC without using localization method, simulation time 6.37s; (c) MMAC using localization method, simulation time 0.82s

From the simulation, it can been seen that the multiple model adaptive control with localization can improve transient response greatly, and compared with MMAC without localization, the time of computation is reduced and accuracy is not lost.

6. CONCLUSION

In this paper, the localization method is combined with the design procedure of discrete time system multiple model adaptive controller, and a new type discrete time system multiple model adaptive controller is proposed. It has solved the problem of MMAC that excessive "competition" from the "unnecessary" models in model set with too many models always degrade the control performance, and the accuracy is not lost.

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