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DESIGN OF ROBUST OUTPUT AFFINE QUADRATIC CONTROLLER

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Abstract: The paper addresses the problem robust output feedback controller design with guaranteed cost and affine quadratic stability for linear continuous time affine systems. The proposed design method leads to a non-iterative LMI based algorithm. A numerical example is given to illustrate the design procedure.

Keywords: Affine Linear Systems, Affine Quadratic Stability, Robust Controller, Output Feedback

1. INTRODUCTION

Robustness has been recognized as a key issue in the analysis and design of control systems for the last two decades. During the last decades numerous papers dealing with the design of static robust output feedback control schemes to stabilize uncertain systems have been published (Benton and Smith,1999; Crusius and Trofino, 1999; Ghaoui and Balakrishnan, 1994; Geromel, De Souza, and Skelton, 1998; Henrion, Tarboriech and Garcia, 1999; Kose and Jabbari, 1999; Li Yu and Jian Chu, 1999; Mehdi, Al Hamid and Perrin, 1996; Pogyeon and et al, 1999; Tuan and et al, 2000; Xu and Darouch, 1998; Yong Yan Cao and You Xian Sun, 1998). Various approaches have been used to study the two aspects of the robust stabilization problem, namely conditions under which the linear system described in state space can be stabilized via output feedback and the respective procedure to obtain a stabilizing or robustly stabilizing control law.

The necessary and sufficient conditions to stabilize the linear continuous time invariant system via static output feedback can be found in Kučera and De Suoza , 1995, Veselý, 2001. In the above and other papers, the authors basically conclude that despite the availability of many approaches and numerical algorithms the static output feedback problem is still open.

Recently, it has been shown that an extremely wide array of robust controller design problems can be reduced to the problem of finding a feasible point under a Biaffine Matrix Inequality (BMI) constraint. The BMI has been introduced in Goh, Safonov and Papavassilopous, 1995. In this paper, the BMI problem of robust controller design with output feedback is reduced to a LMI problem (Boyd, El Ghaoui, Feron and Balakrishnan, 1994).The theory of Linear Matrix Inequalities has been used to design robust output feedback controllers in Benton and Smith,1999; Crusius and Trofino, 1999; El Ghaoui and Balakrishnan, 1994;Henrion, Tarboriech and Garcia, 1999; Li Yu and Jian Chu, 1999; Tuan, Apkarian, Hosoe and Tuy, 2000, Vesely, 2001. Most of the above works present iterative algorithms in which a set of LMI problems are repeated until certain convergence criteria are met. The V-K iteration algorithm proposed in El Ghaoui and Balakrishnan, 1994 is based on an alternative solution of two convex LMI optimization problems obtained by fixing the

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Lyapunov matrix or the gain controller matrix. This algorithm is guaranteed to converge, but not necessarily, to the global optimum of the problem depending on the starting conditions. This paper is concerned with the class of uncertain linear systems that can be described as.

$$\dot{x}(t) = (A_0 + A_1\theta_1 + \ldots + A_k\theta_k)x(t) \quad (1)$$

where $\theta = [\theta_1...\theta_k] \in R^k$ is a vector of uncertain and possibly time varying real parameters (Gahinet, Apkarian and Chilali, 1996).
The system represented by(1) is a polytope of linear affine systems which can be described by a list of its vertices

$$\dot{x}(t) = D_{ci}x(t), \quad i = 1, 2, ..., N \quad (2)$$

where $N = 2^k$.
The system represented by (2) is quadratically stable if and only if there is a Lyapunov matrix $P > 0$ such that

$$D_{ci}^T P + PD_{ci} < 0, \quad i = 1, 2, ..., N \quad (3)$$

A weakness of quadratic stability is that it guards against arbitrary fast parameter variations. As a result, this test tends to be conservative for constant or slow-varying parameters θ. To reduce conservatism when (1) is affine in θ and the parameters of system are time invariant, in Gahinet, Apkarian and Chilali, 1996 the parameter-dependent Lyapunov functions $P(\theta)$ has been used in the form

$$P(\theta) = P_0 + \theta_1 P1 + ... + \theta_k P_k \quad (4)$$

In this paper, new necessary and sufficient conditions to stabilize continuous time systems via static output feedback (Veselý, 2001) have been used to design a robust controller for system (1). For guaranteed cost and system (1)this leads to a non iterative LMI based algorithm. The design procedure guarantees with sufficient conditions the robust affine quadratic stability for closed loop systems.
The paper is organized as follows. In Section 2 the problem formulation and some preliminary results are brought. The main results are given in Section 3. In Section 4 the obtained theoretical results are applied. We have used the standard notation. A real symmetric positive (negative) definite matrix is denoted by $P > 0$ ($P < 0$). Much of the notation and terminology follows the references of Kučera and De Souza, 1995 and Gahinet, Apkarian and Chilali, 1996.

2. PRELIMINARIES AND PROBLEM FORMULATION

We shall consider the following linear time invariant continuous time uncertain systems

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) \quad (5)$$
$$y(t) = C(\theta)x(t), \quad x(0) = x_0$$

where $x(t) \in R^n$ is the plant state; $u(t) \in R^m$ is the control input; $y(t) \in R^l$ is the output vector of system; $A(\theta), B(\theta), C(\theta)$ are matrices of appropriate dimensions. The following definition and theorem will be exploited in the next development (Gahinet, Apkarian and Chilali, 1996) .
Definition 1. The linear system

$$\dot{x}(t) = A_c(\theta)x(t), \quad x(0) = x_0 \quad (6)$$

is affine quadratically stable if there exist $k + 1$ symmetric matrices $P_0, P_1, ..., P_k$ such that

$$P(\theta) = P_0 + P_1\theta_1 + ... + P_k\theta_k > 0 \quad (7)$$

and

$$\frac{dV(x,\theta}{dt} = x(t)^T (A_c^T(\theta)P(\theta) + \quad (8)$$
$$P(\theta)A_c(\theta) + \frac{dP(\theta)}{dt})x(t) < 0$$

□

Note that quadratic stability corresponds to the case $P_1 = ... = P_k = 0$. Sufficient affine quadratic stability conditions are given by the next theorem.
Theorem 1. Consider the linear systems governed by (6), where $A_c(\theta)$ depends affine on the uncertain parameter vector $\theta = [\theta_1...\theta_k]$ and θ_i satisfies

$$\theta_i \in <\underline{\theta_i}, \overline{\theta_i}>, \quad \dot{\theta}_i \in <\underline{\nu_i}, \overline{\nu_i}> \quad (9)$$

$i = 1, 2, ..., k$
where $\underline{\theta_i}, \overline{\theta_i}, \underline{\nu_i}, \overline{\nu_i}$ are known lower and upper bounds. Let Γ and Λ denote the sets of corners of the parameters box (9) and of the rate of variation box (9), respectively

$$\Gamma = \{(\gamma_1, ..., \gamma_k) : \gamma_i \in <\underline{\theta_i}, \overline{\theta_i}>\} \quad (10)$$
$$\Lambda = \{(\lambda_1, ..., \lambda_k) : \lambda_i \in <\underline{\nu_i}, \overline{\nu_i}>\}$$

and let

$$\theta_m = [\frac{\underline{\theta_1} + \overline{\theta_1}}{2}, ..., \frac{\underline{\theta_k} + \overline{\theta_k}}{2}]$$

denote the average value of the uncertain parameters vector. This system is affine quadratically stable if $A_c(\theta_m)$ is stable and there exist $k+1$ symmetric matrices $P_0, P_1, ..., P_k$ such that $P(\theta) > 0$ satisfies

$$L(\gamma, \lambda) = A_c(\theta)^T P(\theta) + \tag{11}$$

$$P(\theta) A_c(\theta) + P(\lambda) - P_0 < 0$$

for all $(\gamma, \lambda) \in \Gamma \times \Lambda$ and

$$A_{ci}^T P_i + P_i A_{ci} \geq 0 \tag{12}$$

for $i = 1, 2, ..., k$. When (11) and (12) are met, a Lyapunov function for (6) and all trajectories $\theta(t)$ satisfying (9) is given by

$$V(x, \theta) = x^T(t) P(\theta) x(t)$$

□

The following performance index is associated with the system (5)

$$J = \int_0^\infty (x(t)^T Q x(t) + u(t)^T R u(t)) dt \tag{13}$$

where $Q = Q^T \geq 0, R = R^T > 0$ are matrices of compatible dimensions.
The problem studied in this paper can be formulated as follows: For a continuous time system described by (5) design a static output feedback controller with the gain matrix F and control algorithm

$$u(t) = Fy(t) = FC(\theta)x(t) \tag{14}$$

so that the closed loop system

$$\dot{x} = (A(\theta) + B(\theta) F C(\theta)) x(t) \tag{15}$$

is affine quadratically stable with guaranteed cost.
Definition 2. Consider the system (5). If there exists a control law u^* and a positive scalar J^* such that closed loop system (15) is stable and the closed loop value cost function (13) satisfies $J \leq J^*$, then J^* is said to be the guaranteed cost and u^* is said to be the guaranteed cost control law for system (5). □

3. THE MAIN RESULTS

In this paragraph we present a new procedure to design a static output feedback controller for affine continuous time linear systems (5) which ensure the guaranteed cost and affine quadratic stability of closed loop system. The following theorem is one of the main results.
Theorem 2. For system (5) and Lyapunov function $V(\theta) = x(t)^T P(\theta) x(t) > 0$ the following statements are equivalent:

- System (5) is static output feedback affine quadratic stabilizable (AQS) with guaranteed cost

$$\int_0^\infty x(t)^T (Q + C(\theta)^T F^T R F C(\theta)) x(t) dt$$

$$\leq x_0^T P(\theta) x_0 \tag{16}$$

- There exist $k+1$ symmetric matrices $P_0, P_1, ...,$ P_k that $P(\theta) > 0$, positive definite matrices Q and R, and matrix F such that the following inequality holds

$$(A(\theta) + B(\theta) F C(\theta))^T P(\theta) + \tag{17}$$

$$P(\theta)(A(\theta) + B(\theta) F C(\theta)) + \dot{P}(\theta) +$$

$$Q + C(\theta)^T F^T R F C(\theta) < 0$$

- There exist $k+1$ symmetric matrices $P_0, P_1, ...,$ P_k that (7) holds, positive definite matrices Q and R, and matrix F such that the following inequality holds

$$A(\theta)^T P(\theta) + P(\theta) A(\theta) - P(\theta) B(\theta) R^{-1}$$

$$B(\theta)^T P(\theta) + \dot{P}(\theta) + Q + \tag{18}$$

$$G(\theta)^T R^{-1} G(\theta) < 0$$

where

$$G(\theta) = B(\theta)^T P(\theta) + R F C(\theta)$$

- There exist $k+1$ symmetric matrices $P_0, P_1, ...,$ P_k that (7) holds, positive definite matrices Q and R, and matrix F such that the following inequality holds

$$A(\theta)^T P(\theta) + P(\theta) A(\theta) - \tag{19}$$

$$P(\theta) B(\theta) R^{-1} B(\theta)^T P(\theta) + \dot{P}(\theta) + Q < 0$$

$$G(\theta) \phi(\theta)^{-1} G(\theta)^T - R < 0 \tag{20}$$

where

$$\phi(\theta) = A(\theta)^T P(\theta) + P(\theta) A(\theta) -$$

$$P(\theta) B(\theta) R^{-1} B(\theta)^T P(\theta) + \dot{P}(\theta) + Q$$

Proof of this theorem is omitted. □
Because of *Theorem 1*, inequalities (17), (18) and (19), (20) are negative in the box (10) if they take negative values at the corners of (10); that is if they are negative for all γ in the vertex set Γ given by (10) and inequality (12) holds for all $i = 1, 2, ..., k$. In the vertex set (10) define the polytopic system in the form

$$\{(D_1, E_1, H_1), ..., (D_N, E_N, H_N)\} \tag{21}$$

where

$$D_1 = A_0 + \underline{\theta_1} A_1 + \underline{\theta_2} A_2 + ...$$
$$E_1 = B_0 + \underline{\theta_1} B_1 + \underline{\theta_2} B_2 + ... \quad (22)$$
$$H_1 = C_0 + \underline{\theta_1} C_1 + \underline{\theta_2} C_2 + ...$$

In (21) each vertex is calculated for different permutation of the k variables $\theta_i, i = 1, 2, ..., k$ alternatively taken at maximum and minimum values. Let us introduce the inverse Lyapunov matrix $S(\theta)$ as (Gahinet, Nemirovski, Laub and Chilali, 1995)

$$S(\theta) = P(\theta)^{-1} = S_0 + \theta_1 S_1 + ... + \theta_k S_k \quad (23)$$

For the new variable $S(\theta)$, equations (11), (12) and (19) read as follows

$$L(\gamma, \lambda) = S(\gamma)A(\gamma)^T + A(\gamma)S(\gamma) - \quad (24)$$
$$(\dot{S}(\lambda) - S_0) < 0$$
$$S_i A_i^T + A_i S_i \geq 0 \quad i = 1, 2, ..., k \quad (25)$$

and

$$S(\gamma)A(\gamma)^T + A(\gamma)S(\gamma) - B(\gamma)R^{-1}B(\gamma)^T -$$
$$(\dot{S}(\lambda) - S_0) + S(\gamma)QS(\gamma) < 0 \quad (26)$$

For reducing the conservatism of the AQS test (Gahinet, Apkarian, and Chilali, 1996) nonnegative matrices $M_i \geq 0, i = 1, 2, ..., k$ are added to (26) and (25) as follows

$$S(\gamma)A(\gamma)^T + A(\gamma)S(\gamma) - B(\gamma)R^{-1}B(\gamma)^T -$$
$$(\dot{S}(\lambda) - S_0) + S(\gamma)QS(\gamma) + \quad (27)$$
$$\sum_{i=1}^{k} \theta_i^2 M_i < 0$$

and

$$S_i A_i^T + A_i S_i + M_i \geq 0 \quad i = 1, 2, ..., k \quad (28)$$

The resulting test is generally less conservative for (27) and (28). However, this improvement is at the expense of higher computational needs since the number of optimization variables is increased in the new LMI problem (27), (28). Combining the results of (27), (28) and (20) the following algorithm for computation of a robust output feedback controller with guaranteed affine quadratic stability has been proposed.

Algorithm

1. Find the solution of (27) at all vertex $(\gamma, \lambda) \in \Gamma \times \Lambda$ with respect to the variables $S_0, S_1, ..., S_k$, $M_1, M_2, ..., M_k$ from the following LMI inequalities

$$\begin{bmatrix} N_i(\gamma, \lambda) & S(\gamma)Q \\ QS(\gamma) & -Q \end{bmatrix} \leq 0 \quad (29)$$

where

$$N_i(\gamma, \lambda) = S(\gamma)D_i^T + D_i S(\gamma) - E_i R^{-1} E_i^T -$$
$$(\dot{S}(\lambda) - S_0) + \sum_{j=1}^{k} \theta_j^2 M_j$$

$i = 1, 2, ..., N$

$$K_j = S_j A_j^T + A_j S_j + M_j \geq 0 \quad (30)$$
$$M_j \geq \rho_1 I, S(\gamma) \geq \rho_2 I, S_j \leq \rho_3 I, j = 1, 2, ..., k$$

where I is identity matrix with corresponding dimensions and ρ_1, ρ_2, ρ_3 are some nonnegative constants.

2. Calculate the value of the inverse Lyapunov matrix S_{ni} and P_{ni}, $i = 1, 2, ..., N$ at all vertex of $\gamma \in \Gamma$.

3. Compute the value of Riccati equation at all vertex of $(\gamma, \lambda) \in \Gamma \times \Lambda$.

$$\phi_i(\lambda) = D_i^T P_{ni} + P_{ni} D_i - P_{ni} E_i R^{-1} E_i^T P_{ni} + \quad (31)$$
$$\dot{P}(\lambda) + Q$$

4. Compute the gain matrix F from the following LMI inequalities

$$\begin{bmatrix} -R & E_i^T P_{ni} + RFH_i \\ (E_i^T P_{ni} + RFH_i)^T & \phi_i(\lambda) \end{bmatrix} \leq 0 \quad (32)$$

$i = 1, 2, ..., N$ and $\lambda \in \Lambda$

$$K_j + S_j C_j^T F^T B_j^T + B_j F C_j S_j \geq 0 \quad (33)$$

$j = 1, 2, ..., k$ □

Note that for example $S(\gamma)$ in (29) reads for $i = 1$ as follows

$$S(\gamma) = S_0 + \underline{\theta_1} S_1 + \underline{\theta_2} S_2 + ...$$

If the LMI problems (29)-(33) are feasible, the resulting gain matrix F guarantees the affine quadratic stability and simultaneously ensures the guaranteed cost (16) for the closed loop system (15).

4. EXAMPLE

In this example we consider the linear model of two cooperating DC motors. The problem is to design two PI controllers for a laboratory MIMO system which will guarantee affine quadratic stability of a closed loop uncertain system. The system model is given by (5) with a time invariant matrix affine type uncertain structure, where

$$A_0 = \begin{bmatrix} 0 & -.2148 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1.014 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.2605 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -.9107 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.1639 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -.8137 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.2279 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -.8251 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$A_{01} = \begin{bmatrix} 0 & -.025 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.1395 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.0938 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.2911 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .0188 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .0208 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.0333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.1173 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{02} = \begin{bmatrix} 0 & .0125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .0594 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .0116 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .0308 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.0188 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.0156 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & .0208 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.0333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} .3148 & 0 \\ .0478 & 0 \\ 0 & -.1028 \\ 0 & -.0091 \\ -.0841 & 0 \\ -.0287 & 0 \\ 0 & .3676 \\ 0 & .2448 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_{01} = \begin{bmatrix} .0625 & 0 \\ -.0798 & 0 \\ 0 & -.0462 \\ 0 & -.0449 \\ .0016 & 0 \\ .0072 & 0 \\ 0 & .077 \\ 0 & -.005 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_{02} = \begin{bmatrix} -.0094 & 0 \\ .0151 & 0 \\ 0 & .0019 \\ 0 & -.003 \\ -.0121 & 0 \\ -.03 & 0 \\ 0 & -.064 \\ 0 & .0189 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad C^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The number of polytope systems are equal to 4 and the polytope vertices are computed for different permutations of the two variables θ_1, θ_2 alternatively taken at their maximum $\overline{\theta_i}$ and minimum $\underline{\theta_i}, i = 1, 2$. The decentralized control structure for the two PI controllers can be obtained by the choice of the static output feedback gain matrix F structure. It is given as follows

$$F = \begin{bmatrix} f11 & 0 & f13 & 0 \\ 0 & f22 & 0 & f24 \end{bmatrix}$$

The results of calculation of a static output feedback gain matrix F for quadratically and affine quadratically stable system for different $Q = qI, R = rI, |\theta_1| = |\theta_2| = 1$ and $\rho_i, i = 1, 2, 3$ are summarized in the following table.

N	q	r	ρ_1	ρ_2	ρ_3	$quad$	$aff.quad$
1	1.5	1	1.5	1	.166	−.1342	−.0954
2	5	1	1.5	1	.166	+.307	−.1277*
3	10	1	1.5	1	.166	−.081	−.1922**
4	20	1	1.5	1	.166	+1.11	−.1148
5	10^{-4}	1	1.5	1	.166	−.0164	+.0011
6	10^{-4}	1	10^{-4}	1	.166	−.0164	−.0136
7	.1	1	1.5	1	.166	−.1383	−.0386
8	.1	1	10^{-4}	1	.166	−.1383	−.1133
9	.1	1	0	1	.166	−.1383	−.1449
10	.1	1	0	1	0	−.1383	−.1448
11	10^{-6}	1	0	1	.166	−.0015	−.0015
12	10^{-6}	.1	0	1	.166	−.0134	−.0178

where
quad and *aff.guad* denote the max(real(eigenvalue)) of the closed loop system for quadratic or affine quadratic stability, respectively. The solutions are feasible for 11 and 12 cases. For other cases the closed loop system is quadratically or affine quadratically stable but the cost is not guaranteed. The static output feedback gain matrix for cases * and ** are given as follow

$$F^* = \begin{bmatrix} -.3582 & 0 & -.376 & 0 \\ 0 & -.7927 & 0 & -.7535 \end{bmatrix}$$

$$F^{**} = \begin{bmatrix} -1.0708 & 0 & -.6317 & 0 \\ 0 & -2.6952 & 0 & -1.671 \end{bmatrix}$$

5. CONCLUSIONS

In this paper, we have proposed a new procedure for robust output feedback controller design for linear systems with affine and possible time varying parameter uncertainty. The feasible solution of the output feedback controller with sufficient conditions guarantee the affine quadratic stability and guaranteed cost. The design procedure is based on new necessary and sufficient conditions for output feedback stabilizability of linear systems and a non-iterative LMI based algorithm. A valuable feature of the robust controller design procedure is that quantitative information about the rate of parameter variation is readily incorporated to reduce conservatism in the time varying case.

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