Abstract: This paper addresses the problem of identifying nonlinear models, directly from input-output measured data, for a benchmark water canal system located in Évora, Portugal. The scarcity of fresh water in many regions of the world demands for a careful management and control of such resource. In order to achieve such objective, techniques from nonlinear modeling and control are being applied and tested in such systems. In this paper, a weighted combination of several local linear models given in state-space is proposed to identify nonlinear models for the canal directly from available data. Such identification methodology can provide a viable way to address the modeling of general nonlinear systems, such as the one proposed in the paper, which is shown by comparing the performance of modeling results with an existing theoretical model.

Keywords: Nonlinear systems, System identification, Natural and environmental systems, Management of natural resources, Integration of technology and environment

1. INTRODUCTION

Fresh water, which is a scarce resource in many regions, is vital for life and impacts both on the economic development and quality of living. The demand for fresh water has steeply increased in the last decades due to several factors: human consumption, agriculture, industry and households. The management of current distribution networks is mostly performed by humans and are prone to spill. These distribution networks could benefit from the use of automatic management systems, which are common in systems and control, in order to satisfy human consumption needs, while minimizing spill (Figueiredo and Ayala Botto (2005); Xavier and Fromion (2004); Nabais and Botto (2010)).

Water distribution networks are large complex systems, spatially distributed with nonlinear dynamics that require suitable modeling and control techniques. Recent technological advances in several engineering areas have triggered new developments in systems theory, in order to provide tools to help with the analysis and design of complex system. According to Murray-Smith and Johansen (1997) two main trends have been followed in this context: the development of new sophisticated tools using the the nonlinear framework; the adaptation of existing tools from linear system theory into a new framework, which is normally denominated by multiple model or operating regime approaches.

The multiple model approaches are based on a divide-and-conquer strategy, which is in-line with many traditional problem-solving methodologies, therefore providing simple ways to cope with the global system complexity. These approaches consist of dividing the global problem into a set of subproblems, which are valid for a restricted neighborhood of the system operating range and for which the linearity assumption is valid. The result is a set of multiple models/controllers that can be handled using tools adapted from linear systems theory, which are normally based on a well known theory and have been systematically applied to several real-life systems. Necessarily, the multiple model approaches need a supervisor, or scheduler, that coordinates the local actions of the model/controller to give the global description.

Although the concept of multiple model approaches may seem very appealing, it also presents several research challenges, i.e. the decomposition of the system full range into operating regimes, the selection of the local model/controller structure and the respective system identification, or the determination of the scheduler for the combination of local models into a global model. Additionally, the type of nonlinearities, e.g. whether it is smooth or hard, may not suit into the framework.

Several applications using the multiple model approach have been reported in the literature. A good overview on the multiple model and operating regime approaches covering several topics, which cover both the modeling and the control aspects, is presented by Murray-Smith and Johansen (1997).

This paper is organised as follows: section 2 presents the framework for composite local models, section 3 describe
the water canal system and presents results from identification. Conclusions are drawn in section 4.

2. COMPOSITE LOCAL LINEAR MODELS

Similarly to Verdult et al. (2002), the Composite Local Linear Models (CLLM) are defined in state-space form as:

\[ x_{k+1} = \sum_{i=1}^{s} p_i(\phi_k) (A_i x_k + B_i u_k + O_i) \]  

\[ y_k = C x_k + v_k \]

where \( O_i \) denotes the offsets for each \( i \)-th local model, \( s \) is the number of local models, \( x_k \in \mathbb{R}^n \) is the state vector, \( u_k \in \mathbb{R}^m \) is the input, \( y_k \in \mathbb{R}^r \) is the output, \( v_k \in \mathbb{R}^t \) is a white-noise sequence and \( p_i(\phi_k) \in \mathbb{R}^s \) are the weighting vectors. The weighting vectors \( p_i(\phi_k) \in \mathbb{R}^s \) are parameterized using normalized radial basis functions:

\[ p_i(\phi_k) = \frac{r_i(\phi_k; c_i, w_i)}{\sum_{j=1}^{r} r_j(\phi_k; c_j, w_j)} \]

with,

\[ r_i = \exp\left(- (\phi_k - c_i)^T \text{diag}(w_i)^2 (\phi_k - c_i) \right) \]

where \( \phi_k \in \mathbb{R}^q \) is the scheduling vector, \( c_i \) is the center and \( w_i \) the width of the \( i \)-th radial basis function.

The goal is to determine, from a finite number of measurements of the input \( u_k \) and output \( y_k \), the matrices \( A_i \), \( B_i \), \( O_i \), \( C \), and the centers \( c_i \) and widths \( w_i \) that describe the radial basis functions. Let the vectors \( c \) and \( w \) consist of the centers \( c_i \) and \( w_i \), respectively. Let \( \theta \) parameterize the system matrices \( A_i \), \( B_i \), \( O_i \), \( C \). The estimation of parameters \( \theta \), \( c \) and \( w \), is based on the minimization of the following cost function:

\[ J_N(\theta, c, w) = E_N^T E_N \]

where \( E_N = Y_N - \hat{Y}_N \) denotes the error vector, with \( Y_N \) a vector containing \( N \) samples of the measured outputs and \( \hat{Y}_N \) a vector containing the outputs of the estimated model.

2.1 Separable least squares

Given the state-space system (1)–(2), the estimated output for a given time instant \( k \) is given by:

\[ \hat{y}_k = C \left( \prod_{i=0}^{k-1} A_i \right) x_0 + \sum_{j=1}^{k-1} C \left( \prod_{h=j+1}^{k-1} A_h \right) G(p(\phi_j) \odot u_j) \]

where the symbol \( \odot \) denotes the Kronecker product, \( A_h := \sum_{i=1}^{\ell} A_i p_i(\phi_k) \), \( u_j := [u_j^T 1]^T \), and

\[ p(\phi_k) := [p_1(\phi_k) \ p_2(\phi_k) \ \cdots \ p_s(\phi_k)] \]

\[ G := [B_1 \ O_1 \ B_2 \ O_2 \ \cdots \ B_s \ O_s] \]

Equation (6) can be further simplified by considering \( k \in [1, \ldots, N] \) and assuming the initial state equal to zero, as follows,

\[ \hat{Y}_N = \Phi(\eta) \theta_\ell \]

where

\[ \eta := \begin{bmatrix} \theta_n \\ w \end{bmatrix}, \quad \theta_\ell := \text{vec}(G), \quad \theta_n := \text{vec}(A) \]

with \( A := [A_1 \ A_2 \ \ldots \ A_s] \). Matrix \( \Phi(\eta) \) is defined as,

\[ \Phi(\eta) := \begin{bmatrix} 0 \\ \sum_{j=0}^{N-2} (p(\phi_j) \odot \bar{u}_j)^T \odot C (N-2 \prod_{h=j+1}^{N-2} A_h) \end{bmatrix} \]

From (7) \( E_N \) can be written as,

\[ E_N = Y_N - \Phi(\eta)\theta_\ell \]

For a fixed \( \eta \), minimizing the norm of \( E_N \) with respect to \( \theta_\ell \) yields

\[ \hat{\theta}_\ell(\eta) = \Phi(\eta)^\dagger Y_N \]

with \( \Phi^\dagger = (\Phi^T \Phi)^{-1} \Phi^T \). By evaluating equation (9) for \( \theta_\ell = \hat{\theta}_\ell(\eta) \), it is possible to define a new error vector,

\[ \bar{E}_N(\eta) = E_N(\theta_\ell, \eta)|_{\theta_\ell = \hat{\theta}_\ell(\eta)} = (I - P(\eta))Y_N \]

where \( P(\eta) = \Phi(\eta)\Phi(\eta)^\dagger \). The principle of Separable Least Squares (SLS) from Golub and Pereyra (1973) states that the optimization problem of estimating \( (\hat{\eta}, \hat{\theta}_\ell) = (\hat{\theta}, \hat{\omega}, \hat{c}) \) using the cost function (5) is equivalent to, first, estimate \( \hat{\eta} \) by minimizing

\[ J_N(\eta) := \bar{E}_N^T \bar{E}_N \]

which is independent of \( \theta_\ell \) and, then, estimate \( \hat{\theta}_\ell \) using the linear least squares optimization (10).

Proposition 1 (Convergence of parameter estimates) Let's consider the state-space CLLM denoted by equations (1)–(2), the optimal estimates for \( \hat{\eta}_n \) and \( \hat{\theta}_\ell \), and as well that all local models are strictly stable, observable and controllable. Assume that the input \( u(k) \in \mathbb{R}^m \) is a zero-mean white noise sequence and that the following conditions are satisfied:

1) The input sequence \( u(k) \) is independent of the initial state \( x_0 \in \mathbb{R}^n \), such that,

\[ \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} u(k)x(k_0) = 0 \]

2) The input sequence is independent of the measurement noise \( v(k) \in \mathbb{R}^m \), such that,

\[ \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} u(k)v(k) = 0 \]

3) Matrix,

\[ \lim_{N \to \infty} \frac{1}{N} \Phi(\eta_n)\Phi(\eta_n) \]

has full rank.
Then the following equality holds asymptotically for \( \theta_\ell \),
\[
\lim_{N \to \infty} \frac{1}{N} \hat{\theta}_\ell - \theta_\ell = 0 \tag{13}
\]

**Proof**

Let's start by considering the residue \( E_N = Y_N - \hat{Y}_N \). Upon replacement of the outputs in equation (7), while using also the optimal estimates for \( \eta_n \), results the following expression,
\[
E_N = \Gamma (\eta_n) x(0) + \Phi(\eta_n) \theta \ell + V - \Phi(\eta_n) \bar{\theta}_\ell
\]

Given the condition for convergence that the residues becomes 0 for \( N \) large, results after some straightforward manipulation,
\[
\Phi(\eta_n) \left( \bar{\theta}_\ell - \theta_\ell \right) = \Gamma (\eta_n) x(0) + V,
\]
\[
\Phi^T(\eta_n) \Phi(\eta_n) \left( \bar{\theta}_\ell - \theta_\ell \right) = \Phi^T(\eta_n) \Gamma (\eta_n) x(0) + \Phi^T(\eta_n) V,
\]

therefore,
\[
\lim_{N \to \infty} \frac{1}{N} \Phi^T(\eta_n) \Phi(\eta_n) \left( \bar{\theta}_\ell - \theta_\ell \right) = 0, \tag{14}
\]

iff,
\[
\lim_{N \to \infty} \frac{1}{N} \Phi^T(\eta_n) \Gamma (\eta_n) x(0) = 0, \tag{15}
\]
\[
\lim_{N \to \infty} \frac{1}{N} \Phi^T(\eta_n) V = 0. \tag{16}
\]

Given the definition for matrix \( \Phi^T(\eta_n) \), the statement of equation (16) is immediately verified since the inputs are uncorrelated with the measurement noise through assumption 2) of the theorem. The statement in equation (15) needs some intermediate steps. Consider an alternative procedure to compute matrix \( \Phi(\eta_n) \) based on the simulation of several CLLM, as follows,
\[
\Phi(\eta_n) = \left[ y_{1,1} \; y_{2,1} \; \cdots \; y_{n,1} \right]
\]
where the vectors for a generic pair \( (\alpha, \beta) \) are given by,
\[
y_{\alpha,\beta}^T(0) y_{\alpha,\beta}^T(1) \cdots y_{\alpha,\beta}^T(N - 1) \right)^T,
\]
The outputs result from the simulations of the following CLLM,
\[
x_{\alpha,\beta}(k + 1) = \sum_{i=1}^{s} p_i(\phi(k)) \left( A_i x_{\alpha,\beta}(k) + E_{\alpha,\beta} u(k) \right),
\]
\[
y_{\alpha,\beta}(k) = C x_{\alpha,\beta}(k),
\]
where \( E_{\alpha,\beta} \), with \( \alpha \in \{1, 2, \ldots, n\} \) and \( \beta \in \{1, 2, \ldots, s(m + 1)\} \), is an auxiliary matrix that is equal to zero everywhere, except for the term in position \( (\alpha, \beta) \) that equals one. Given the new definition for matrix \( \Phi(\eta_n) \), a generic term can be computed as follows,
\[
y_{\alpha,\beta}^T \Gamma (\eta_n) x(0) = \sum_{k=0}^{N-1} x_{\alpha,\beta}^T(k) C T \prod_{i=0}^{k-1} A_i x(0)
= \sum_{k=0}^{N-1} \left( x^T(0) \otimes x_{\alpha,\beta}^T(k) \right) \vec{C} T \prod_{i=0}^{k-1} A_i
\]

with \( \alpha \in \{1, 2, \ldots, n\} \) and \( \beta \in \{1, 2, \ldots, s(m + 1)\} \). Since the state \( x_{\alpha,\beta}(k) \) depends only on the inputs, and these are uncorrelated with the initial state \( x(0) \) according to assumption 1), then,
\[
\lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N-1} \left( x^T(0) \otimes x_{\alpha,\beta}^T(k) \right) \vec{C} T \prod_{i=0}^{k-1} A_i = 0,
\]
therefore, the statement in equation (15) is asymptotically verified for \( \alpha \in \{1, 2, \ldots, n\} \) and \( \beta \in \{1, 2, \ldots, s(m + 1)\} \). Since both the statements of equations (15)–(16) are satisfied, then also the statement of equation (14) is satisfied. Finally, using assumption 3) the statement in equation (13) is also verified, therefore concluding the proof.

### 2.2 Projected gradient search

Full parameterized \( A \) and \( C \) matrices are used along the optimization process. A major drawback of using fully parameterized matrices for the identification of composite local linear state-space models, is the redundancy that is introduced in the optimization procedure. This redundancy results from the nonuniqueness of the cost function values (Borges et al., 2004).

**Proposition 2 (Nonuniqueness of Cost Function)**

Consider the following definitions for generic \( A_T \) and \( C_T \) matrices,
\[
A_T := \left[ T A_1 T^{-1} \; T A_2 T^{-1} \; \cdots \; T A_s T^{-1} \right],
\]
\[
C_T := C T^{-1},
\]
where \( T \in \mathbb{R}^{n \times p} \) is a nonsingular matrix. Then the cost function denoted by equation (12) verifies the following relation,
\[
J_{\eta}^R (\eta(T)) = J_{\eta}^R (\eta_n), \tag{17}
\]
where \( \eta_n(T) \) is the parameter vector built up using a different realization defined with matrices \( A_T \) and \( C_T \), rather than matrices \( A \) and \( C \).

**Proof** The \( k \)-th block-row of matrix \( \Phi(\eta_n(T)) \) is given by:
\[
[\Phi(\eta_n(T))]_k = \sum_{j=0}^{k-2} u(k) \otimes (C T^{-1}) \prod_{h=j+1}^{k-2} \left( \sum_{i=1}^{s} T A_i T^{-1} p_i(\phi(k)) \right),
\]
\[
= \sum_{j=0}^{k-2} u(k) \otimes C \prod_{h=j+1}^{k-2} T \left( \sum_{i=1}^{s} A_i p_i(\phi(k)) \right) T^{-1},
\]
\[
= \left[ \Phi(\eta_n) \right]_k T^{-1}.
\]
The latter relation can be applied to all block-rows of matrix \( \Phi(\eta_n(T)) \) and the following relation holds for the complete matrix,
\[
\begin{align*}
\Phi(\eta_n(T)) &= \Phi(\eta_n) (I \otimes T^{-1}), \\
&= \Phi(\eta_n) T_n.
\end{align*}
\]
where matrix \( I \) has dimensions \( ns(m + 1) \times ns(m + 1) \). The latter relation can be replaced into the residue of cost function denoted by equation (12), as follows,
\[ E_N^\eta(\eta_n(T)) = (I - \Phi (\Phi^T \Phi)^{-1} \Phi^T) Y_N, \]
\[ = (I - \Phi T_n (\Phi T_n)^T \Phi T_n)^{-1} (\Phi T_n)^T Y_N, \]
\[ = E_N^\eta(\eta_n). \]

where \( \Phi = \Phi(\eta_n(T)). \) Therefore, the statement of the proposition is verified, i.e., the cost function in equation (12) is invariant to changes in the parameters due to similarity transformations.

A way to cope with this problem is to use projected gradient search. Consider the following matrix,
\[ M_{(A,C)} := \sum_{i=1}^n \Pi_i^T \left[ \frac{\Theta_i^T}{\Theta_i} \right] - A \otimes \left[ I_n \right]_{0 \times n} \]
where \( \Pi_i := [0_{n \times (i-1)n} I_n 0_{n \times (n-i)n}]^T. \) Assuming \((A, C)\) is observable, with \( i \in \{1, \ldots, n\}, \) then the matrix \( M_{(A,C)} \) has full column rank and its left null-space \( Q_2 \) is given as
\[ M_{(A,C)} = \left[ Q_1 \; Q_2 \right] \left[ \begin{array}{l} R_1 \\ 0 \end{array} \right] \]
The matrix \( Q_2 \) can be used in a projected gradient search using a Levenberg-Marquardt type of update equation \( \eta^{(i+1)} = \eta^{(i)} + d^{(i)}. \) Taking into account the projection of the gradient, the parameter update is given by:
\[ d^{(i)} = -Q_2 \left( Q_2^T \Psi_N Q_2 + \lambda I \right)^{-1} Q_2^T \Psi_N E_N \]
with,
\[ \Psi_N := \frac{\partial E_N}{\partial \eta_n}, \quad Q_2 := \left[ \begin{array}{l} Q_2^T \\ 0 \end{array} \right] \]
Note that \( Q_2 \) only operates on the \( \theta_n \) part of \( \eta. \)

2.3 Definition of a similarity map

Consider the following definition of a similarity map \( S_{(A,C)}(T) : T \in \mathbb{R}^{n \times n}, \) where the similarity transformation verifies \( \det(T) \neq 0 \rightarrow \mathbb{R}^{(n+t) \times n}, \)
\[ S_{(A,C)}(T) := \left[ \begin{array}{c} T A (I_s \otimes T^{-1}) \\ C (I_s \otimes T^{-1}) \end{array} \right]. \]
This similarity map can be used to characterize all pairs \((A, \tilde{C})\) that yield the same value of the cost function in equation (12),
\[ \mathcal{I}_{(A,C)} := \{(A, C) \mid (A, C) = S_{(A,C)}(T), \det(T) \neq 0 \}. \]

The vectorization of the linearized similarity map is given by,
\[ \text{vec}(S_{(A,C)}(T)) \approx \mathcal{M}_{(A,C)} \text{vec}(T), \]
where,
\[ \mathcal{M}_{(A,C)} := \sum_{i=1}^n \Pi_i^T \left[ \frac{\Theta_i^T}{\Theta_i} \right] - A \otimes \left[ I_n \right]_{0 \times n}, \]
and the property \( \text{vec}(XYZ) = (Z^T \otimes X)\text{vec}(Y) \) has been used.

2.4 Computation of projected gradient

Based on the observability assumption for the system with state-space equations (1)–(2), matrix \( M_{(A,C)} \) has full column rank and its left null-space can be used in a projected gradient update. The left null-space results from a QR-decomposition of matrix \( M_{(A,C)} \) with zeros terms discarded, i.e.,
\[ \Xi M_{(A,C)} := [Q_1 (\theta_n) \; Q_2 (\theta_n)] \left[ R_1 (\theta_n) \; 0 \right]. \]
Using both terms from matrix \( Q \) it is possible to decompose the vector of parameters \( \theta_n \) according to,
\[ \theta_n = Q_1 (\theta_n) Q_1^T (\theta_n) \xi (\theta_n) + Q_2 (\theta_n) Q_2^T (\theta_n) \xi (\theta_n). \]
The first part of (22) contains the directions that correspond to similarity transformations and the second part contains the directions that change the value of the cost in (12). The parameter update at each step of the optimization is modified into,
\[ \eta^{new} = \eta^{old} - Q_2 (\eta_n) \xi (\eta_n) (\eta_n)^{-1} Q_2 (\eta_n)^T \Psi_N (\eta_n)^T E_N (\eta_n), \]
where,
\[ \xi (\eta_n) := Q_2 (\eta_n)^T \Psi_N (\eta_n)^T \Psi_N (\eta_n) Q_2 (\eta_n) + \lambda I, \]
\[ Q_2 (\eta_n) := \left[ \begin{array}{c} Q_2 (\eta_n) \\ 0 \end{array} \right]^T. \]
The identity block introduced in \( Q_2 (\eta_n) \) is used here to cope with vectors \( c \) and \( w \) from the radial basis functions, which do not suffer from similarity transformation problems. In this way the Levenberg-Marquardt method can still be applied for the SLS estimation of the CLLM parameters.

2.5 Initialization of local models

The initial values used for the parameter in the Output Error (OE) optimization can significantly influence the performance for the minimization method and, therefore, largely determines the quality for the final parameter estimates for the CLLM model. An ideal methodology to find this initial point could be to use subspace identification techniques for CLLM, although such approach was not developed yet.

An alternative approach is to estimate a Linear Time Invariant (LTI) state-space model using the full data set, and then initialize all local models using this linear model. The reasoning is that the CLLM will describe the dynamic system in a better way then the LTI model. The subspace identification for LTI systems was proposed by Verhaegen (1994) in case of the Multivariable Output Error State Space (MOESP) class of algorithms.
A more sophisticated approach for the initialization of state-space parameters may result from first performing several identification experiments where the local models are estimated as independent LTI models in the range of the inputs. The set of input signals used to excite the system are distributed uniformly according to the range of the scheduling variable. Since these models are estimated independently, than the procedure for computing an embedding state space could be used to bring all local models to the same state basis. Although this is a more elaborated approach, it allows to provide better initialization parameters to the OE estimation method.

The initial values for the scheduling functions, i.e. the radial basis functions, are distributed uniformly over the operating range of the scheduling variable (Verdult et al., 2002).

2.6 Computation of the SLS-cost function value

Using a QR-decomposition of $\Phi$, the computation of the SLS-cost function is quite straightforward. The error vector can be computed as follows,

$$
E_N^n = Y_N - Q_1 Q_1^T Y_N,
$$

where the products are performed from right-to-left. The cost is then computed by replacing the result of (23) into equation (12).

2.7 Computation of gradients

A main step in a gradient based optimization is the computation of gradients for the cost function:

$$
\frac{\partial J}{\partial \eta_j} = E_N^n \frac{\partial E_N^n}{\partial \eta_j},
$$

where $\eta_j \in \eta_n$. The gradients of $E_N^n$ can be compute directly from the gradient of $E_N^n$.

The gradient of $E_N^n$ with respect to a generic $\eta_j \in \eta_n$ is given by:

$$
\frac{\partial E_N^n}{\partial \eta_j} = -(I-P) \frac{\partial \Phi^T}{\partial \eta_j} (\Phi^T Y_N - (\Phi^T) (I-P) Y_N).
$$

A more efficient procedure to compute the gradient of $E_N^n$ might result from using,

$$
\frac{\partial E_N^n}{\partial \eta_j} = -(I-Q_1 Q_1^T) \frac{\partial \Phi^T}{\partial \eta_j} R_1^{-1} Q_1^T Y_N + (R_1^{-1} Q_1^T)^T \frac{\partial \Phi^T}{\partial \eta_j} (Q_1 Q_1^T - I) Y_N,
$$

where $\Phi^T$ denotes the pseudo-inverse and $P := Q_1 Q_1^T$. The products are calculated from right-to-left.

The computations for the gradients of $\Phi(\eta_n)$ result from,

$$
\frac{\partial \Phi}{\partial \eta_j} = \begin{bmatrix}
Y_{\eta_1} \ Y_{\eta_2} \ Y_{\eta_3} \ Y_{\eta_4} \ \cdots \ \ Y_{\eta_n(m+1)s}
\end{bmatrix}.
$$

Each column of (25) is the result of simulating the following state-space system:

$$
X_{\alpha,\beta}(k+1) = \sum_{i=1}^{s} p_i(\phi(k)) (A_i x_{\alpha,\beta}(k) + E_{\alpha,\beta} \tilde{u}(k)) + p_i(\phi(k)) A_i X_{\alpha,\beta}^{\eta_i}(k) + p_i(\phi(k)) \frac{\partial A_i}{\partial \eta_j} x_{\alpha,\beta}(k),
$$

$$
Y_{\alpha,\beta}(k) = \frac{\partial C}{\partial \eta_j} x_{\alpha,\beta}(k) + C X_{\alpha,\beta}(k),
$$

where the following notation is used,

$$
X_{\alpha,\beta}(k) \quad \text{and} \quad Y_{\alpha,\beta}(k)
$$

The same reasoning applies to the computation of gradients with respect to the remaining variables of the parameter vector $\eta$.

The input data is used as a scheduling vector in equations (1)–(2), i.e. $\phi(k) = u(k)$, then the computations of the gradients for $p_i(u(k))$ are simplified (Verdult et al., 2002). For instance, if $\eta_j$ is an element of $A$ then the computations in (26)–(27) simplify to,

$$
X_{\alpha,\beta}(k+1) = \sum_{i=1}^{s} p_i(\phi(k)) A_i X_{\alpha,\beta}^{\eta_i}(k) + p_i(\phi(k)) \frac{\partial A_i}{\partial \eta_j} x_{\alpha,\beta}(k),
$$

$$
Y_{\alpha,\beta}(k) = C X_{\alpha,\beta}(k).
$$

This subject was further developed by Borges et al. (2005)

3. MODELING OF WATER CANAL SYSTEM

The experimental water canal is located in Évora, Portugal, has 4 pools with a trapezoidal cross section of 900 mm height, 150 mm base width and a mean side slope...
of \( m = 1.5 \times 10^{-3} \). The distinct geometric characteristics for each pool are described by Lemos et al. (2010). The 4 pools are divided by three sluice gates. All these sluice gates are electro-actuated and instrumented with position sensors. A rectangular overshot gate is located at the end of the canal with 700 mm width. The off-take valves, equipped with an electromagnetic flow-meter and motorized butterfly valve for flow control, are immediately located upstream of each sluice gate. Counterweight-float level sensors are distributed along the canal, as shown in Figure 1.

At the head of the canal an electro-valve controls the canal inflow. This flow is extracted from a reservoir as depicted in figure 1. The maximum flow capacity is 0.030 \( \text{m}^3/\text{s} \). The water canal nominal capacity provides a flow of 0.030 \( \text{m}^3/\text{s} \) for a uniform water depth of 0.600 m. All electro-actuators and sensors in the canal are connected to local PLCs (Programmable Logic Controllers) responsible for the sensor data acquisition and for the control actions sent to the actuators. All local PLCs are connected through a MODBUS network (RS 485).

Figures 2 and 3 present the modeling results using a composition of 3 local models, each of 4th order versus the measured signal. The quality criteria Variance Accounted For (VAF) and Mean Squared Error (MSE) that were used to assess the performance of the identified model are presented in Table 1.

Table 1. Performance results for the CLLM.

<table>
<thead>
<tr>
<th></th>
<th>VAF [%]</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training data</td>
<td>92.4</td>
<td>0.269 \times 10^{-4}</td>
</tr>
<tr>
<td>Validation data</td>
<td>85.3</td>
<td>0.57 \times 10^{-4}</td>
</tr>
</tbody>
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4. CONCLUSIONS

Composite local linear models provide a suitable identification methodology to model the benchmark water canal system located in Évora, Portugal. The success of identification process is demonstrated through the performance criteria in Table 1. The paper also presents a detailed discussion on the system identification method using this type of state-space models.

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