Maneuvering Target Tracking Method based on Unknown but Bounded Uncertainties

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Abstract: In this paper a new maneuvering target tracking method based on a model with unknown but bounded uncertainties is proposed. Firstly, a model with unknown but bounded uncertainties is proposed. Then, an algorithm for tracking the target in both maneuvering and non-maneuvering cases is generated. The proposed method can be used for a large diversity of input types. Also, this method does not require maneuver detection and covariance resetting that are necessary in previous maneuvering target tracking methods. Therefore, it doesn’t consume any time for maneuver detection and covariance resetting. Also simulation results are provided to confirm the theoretical development.

Keywords: target tracking, unknown but bounded uncertainties, input estimation, Kalman filtering

1. INTRODUCTION

Target tracking is estimation of states of the target that include position and velocity of contemporary and mostly its future from the contemporary noisy observations. Tracking of the maneuvering targets is one of the most important problems in air trafficking, navigation and guidance systems. Since the radar cannot measure the target accelerations (maneuver terms) directly, it become more complicated (Mcintyre et al, 1998).

One of the most important classes of maneuvering target tracking (MTT) methods is input detection and estimation (IDE) method. The major idea in the IDE method is that at first unknown inputs \( u \) should be estimated, then the states will be estimated by using the estimated inputs. The basic IDE methods need additional attempt for detecting and estimating the maneuver start time, thus maneuver detection delay is inevitable.

A lot of literature tried to decrease the maneuver detection delay. For instance, in recently proposed modified input estimation (MIE) method (Khaloozadeh et al, 2009), states and unknown accelerations (maneuvers) are augmented with each other. By this modification original maneuvering model is transformed into non-maneuvering model then the original states and maneuvers are estimated simultaneously with a standard Kalman filter (KF). Although the MIE method has good performance in low and medium maneuvering case, it has two essential drawbacks; firstly, in this method for having state and maneuver estimations with suitable speed, it needs covariance resetting whenever a change in maneuver happens. Practically the covariance resetting cannot be done on time since the time of starting maneuver is undetermined.

Therefore, in high maneuver case or when maneuver changed constantly the MIE method does not have acceptable performance.

In order to solve this problem (Bahari et al. (2009a, b) and Beheshtipour et al. (2009)) have used Fuzzy forgetting factor or a Fuzzy fading memory to improve tracking accuracy for high maneuvering targets. However, the Fuzzy reasoning rules depend on some prior knowledge of the maneuvering targets, demands a high computational cost, and mostly leads to a poor real-time performance. In Tang et al. (2010) a new strong tracking MIE algorithm (STMIE) has been proposed using the idea of strong tracking filter (Zhou et al, 1996), and the formula of multiple fading factor matrices are derived based on the residuals. However, all mentioned methods are relatively low speed estimators in high maneuver case.

Another problem with the MIE method that none of mentioned papers solved is this fact that, the MIE method only can track a limited class of maneuvers. For example if maneuver term behaves like a ramp function, the MIE method will fail tracking. Moreover, with the rapid development of modern navigation technology, the maneuverability of aircrafts are growing stronger and stronger, so having the ability of tracking different maneuvers is an important feature that a good tracking method must have it.

In this paper for solving above mentioned problems, a novel target tracking model (called UBB model) and algorithm (called UBBM filter) are proposed. In UBB model process and measurement noises are considered as unknown but bounded (UBB) noises (Schweppel, 1973), that never have been used for MTT purpose. MTT methods with Bayesian
model require some knowledge on the statistical characteristics of the noise. Bayesian model are not useful most of the times, since these statistical characteristics of the noise are sometimes not available, time varying or incorrect, or impossible to estimate because of lack of adequate data. Also, in MTT problems most of the time it is better to consider unknown accelerations as uncertain stochastic processes but not white noise since white noise does not have time structure. Modeling uncertainties with UBB stochastic process (instead of white noise) is more appropriate since most uncertainties in physical systems have time structure and bounded values.

In this work based on UBB model a UBBM filter is proposed that can estimate the position, velocity, and acceleration of a target with unknown maneuver. UBBM filter does not require covariance resetting and it could handle all maneuver types. The numerical simulations of the proposed UBBM filter shows that it can quickly track every maneuvering target with bounded noise vectors with unknown statistical characteristics that may include modeling inaccuracies, discretization errors or computer round-off errors. \( \psi(k) \) represents the measurement noise and \( \omega(k) \) is unknown but bounded process noise. The only information about \( \psi(k) \) and \( \omega(k) \) are as follows

\[
\omega(k) \in \Omega_{\omega} \quad \psi(k) \in \Omega_{\psi}
\]

Also for initial condition following constraint is considered

\[
X(0) \in \Omega_X \quad \Rightarrow \quad X^T(0)\Sigma^{-1}(0)X(0) \leq 1, \forall n \in N
\]

Where \( \Omega_{\omega} \), \( \Omega_{\psi} \) and \( \Omega_{x} \) are ellipsoidal sets whose size and shape can vary with time. Also, \( Q(k) \), \( R(k) \) and \( \Psi(k) \) are positive definite matrices. The center of \( \Omega_{\omega} \), \( \Omega_{\psi} \) and \( \Omega_{x} \) are \( \bar{\Omega} \) and the orientations of \( \Omega_{\omega} \), \( \Omega_{\psi} \) and \( \Omega_{x} \) are respectively determined by the eigenvectors of \( Q(k) \), \( R(k) \) and \( \Psi(k) \), also the lengths of semimajor axes of \( \Omega_{\omega}(k), \Omega_{\psi}(k) \) and \( \Omega_{x}(k) \) are determined respectively by the lengths of the eigenvalues of \( Q(k), R(k) \) and \( \Psi(k) \).

It is assumed that the target moves in a plane, which is a two dimensional case. Therefore state vector becomes as follows

\[
X(k) = \begin{bmatrix} x(k) & \psi_x(k) & y(k) & \psi_y(k) \end{bmatrix}^T
\]

Where \( x(k), \psi_x(k) \) and \( y(k), \psi_y(k) \) are respectively the target positions and velocities in \( x \) and \( y \) directions in Cartesian coordinate.

Also \( F(k), G(k) \) and \( H(k) \) are defined as follows

\[
F(k) = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad G(k) = \begin{bmatrix} T^2/2 \\ T \\ 0 \\ 0 \\ T^2/2 \end{bmatrix}, \quad H(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\]

3. STANDARD UBBM FILTER

For the non-maneuvering target motion model (1) and (2), basic UBBM filter formulas are as follows (Sweppe, 1973)

\[
\Omega_{\omega}(k+1) = \begin{bmatrix} X_0 & X(k+1|k+1)^T \end{bmatrix} \Sigma_{\omega}(k+1|k+1) \begin{bmatrix} X_0 & X(k+1|k+1)^T \end{bmatrix}^T \leq 1
\]

\[
\tilde{X}(k+1|k+1) = F(k)\tilde{X}(k+1|k) + K(k+1)
\]

\[
\Sigma(k+1|k+1) = \begin{bmatrix} 1 - \delta^2(k+1) \end{bmatrix} \Sigma(k+1|k+1) + H(k+1)R(k+1)H^T(k+1)
\]

\[
\Sigma(k+1|k) = \frac{1}{1 - \beta(k)} F(k) \Sigma(k|k) F^T(k) + G(k)Q(k)G^T(k)
\]

\[
\tilde{\Sigma}(k+1|k) = \frac{1}{1 - \rho(k+1)} \Sigma(k+1|k)
\]

\[
Q(k) = \frac{1}{\beta(k)} Q(k), \quad R(k) = \frac{1}{\rho} R(k)
\]

\[
\delta^2(k+1) = \frac{1}{|X(k+1|k+1) - H(k+1)F(k)\tilde{X}(k|k)|^2} \left\| H(k+1)\tilde{\Sigma}(k+1|k) H^T(k+1) [X(k+1|k) - H(k+1)F(k)\tilde{X}(k|k)] \right\|
\]

For any \( \rho(k+1), \beta(k) \)

\[
0 \leq \rho(k+1) \leq 1, \quad 0 \leq \beta(k) \leq 1
\]

\[
\tilde{X}(0|0) = 0, \quad \Sigma(0|0) = \Psi
\]

Where centre of the bounding ellipsoid estimate set \( \Omega_{\omega} \) is \( X \), and the size and shape of this set is determined by \( \Sigma \).
Although, UBBM filter yields an estimate which is a set rather than a single vector, it is quite reasonable to consider \( \mathbf{X} \) being the best estimation. \( \mathbf{X} \) is a min-max estimate in a problem minimizing the maximum error between \( \mathbf{X} \) and the true but unknown value \( \hat{\mathbf{X}} \).

### 4. PROPOSED UBBM FILTER

The target motion model in maneuvering case can be considered as:

\[
\mathbf{X}(k + 1) = \mathbf{F}(k)\mathbf{X}(k) + \mathbf{C}(k)\mathbf{u}(k) + \mathbf{G}(k)\mathbf{w}(k) \tag{16}
\]

\[
\mathbf{z}(k) = \mathbf{H}(k)\mathbf{X}(k) + \mathbf{v}(k) \tag{17}
\]

Where \( \mathbf{C}(k) \in \mathbb{R}^{n \times m} \) is the input matrix and \( \mathbf{u}(k) \in \mathbb{R}^m \) is the completely unknown maneuver (input). They are defined below:

\[
\mathbf{C}(n) = \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{u}(n) = [u_x(n) \ u_y(n)]^T
\]

The standard UBBM filter cannot be used for maneuvering targets because it does not consider maneuver. This section aims to design an estimation algorithm for system (16) and (17) such that:

a) a set that contains all possible value of the true state vector \( \mathbf{X} \) is quantified at each time step \( k \).

b) the output error vector \( \mathbf{z}(k) - \mathbf{H}(k)\mathbf{X}(k) \) is acceptable, i.e., it remains in the interior of the ellipsoid (4) enclosing all probable values of the vector \( \mathbf{v}(k) \):

\[
\left[\frac{\mathbf{z}(k) - \mathbf{H}(k)\hat{\mathbf{X}}(k)}{T}\right]^T \mathbf{R}^{-1}(k) \left[\frac{\mathbf{z}(k) - \mathbf{H}(k)\hat{\mathbf{X}}(k)}{T}\right] \leq 1, \forall n \in N
\]

c) \( \mathbf{u}(k) \) could be calculated.

d) to have a good estimation for state vector as quick as possible.

The following algorithm for UBBM filter is proposed here:

\[
\mathbf{u}_x(n) = \begin{bmatrix} \hat{v}_x(k + 1 \mid k) - \bar{v}_x(k \mid k) \\ \bar{v}_x(k + 1 \mid k) - \hat{v}_x(k \mid k) \end{bmatrix}, \quad \mathbf{u}_y(n) = \begin{bmatrix} \hat{v}_y(k + 1 \mid k) - \bar{v}_y(k \mid k) \\ \bar{v}_y(k + 1 \mid k) - \hat{v}_y(k \mid k) \end{bmatrix}
\]

Where \( \hat{v}_x(k \mid k) \) and \( \bar{v}_x(k \mid k) \) are target velocities in \( x \) and \( y \) direction respectively. So, following equations are used for UBBM filter:

\[
\mathbf{X}(k + 1 | k + 1) = \begin{bmatrix} \mathbf{X} - \mathbf{X}(k + 1 | k + 1) \end{bmatrix} \begin{bmatrix} \Sigma^2(k + 1 | k + 1) \\ \mathbf{X} - \mathbf{X}(k + 1 | k + 1) \end{bmatrix} \leq 1 \tag{18}
\]

\[
\mathbf{X}(k + 1 | k + 1) = \mathbf{F}(k)\mathbf{X}(k) + \mathbf{C}(k)\mathbf{u}(k) + \mathbf{G}(k)\mathbf{w}(k) \tag{19}
\]

\[
\begin{bmatrix} \hat{v}_x(k \mid k + 1) \\ \hat{v}_y(k \mid k + 1) \\ \bar{v}_x(k \mid k + 1) \\ \bar{v}_y(k \mid k + 1) \end{bmatrix} = \begin{bmatrix} \hat{v}_x(k + 1 \mid k) - \bar{v}_x(k \mid k) \\ \hat{v}_y(k + 1 \mid k) - \bar{v}_y(k \mid k) \\ \bar{v}_x(k + 1 \mid k) - \hat{v}_x(k \mid k) \\ \bar{v}_y(k + 1 \mid k) - \hat{v}_y(k \mid k) \end{bmatrix}
\]

For any \( \beta(0 \mid k) \),\( \beta(k) \)

\[
\begin{align*}
\mathbf{X}(0 \mid 0) &= 0 \\
\Sigma(0 \mid 0) &= \Psi
\end{align*}
\]

### 5. SIMULATION RESULTS

In this section the theoretical developments in MTT are verified by numerical simulations. The effectiveness of the proposed UBBM filter is compared with the MIE method that has been proposed by Khalaoozad et al. (2009). In their approach, the maneuver is augmented with the state space vector in a new single state vector. In this scheme, the maneuvering model is changed into non maneuvering model. Then, the original state and maneuver vectors are estimated simultaneously with a standard Kalman Filter. The MIE algorithm given in Khalaoozad et al. (2009) is as follows:

\[
\begin{align*}
\mathbf{K}_{\text{aug}}(k + 1) &= \mathbf{P}_{\text{aug}}(k + 1 \mid k + 1) \mathbf{H}^T_{\text{aug}}(k + 1) \\
&\quad + \mathbf{G}_{\text{aug}}(k) \mathbf{R}^{-1}_{\text{aug}}(k + 1) \\
\mathbf{P}_{\text{aug}}(k + 1 \mid k + 1) &= \mathbf{P}_{\text{aug}}(k + 1 \mid k) - \mathbf{P}_{\text{aug}}(k + 1 \mid k) \mathbf{H}^T_{\text{aug}}(k + 1) \\
&\quad \times \mathbf{R}_{\text{aug}}(k + 1) \mathbf{P}_{\text{aug}}(k + 1 \mid k + 1) \mathbf{H}^T_{\text{aug}}(k + 1) \tag{29}
\end{align*}
\]

\[
\begin{align*}
\mathbf{P}_{\text{aug}}(k + 1 \mid k) &= \mathbf{F}_{\text{aug}}(k) \mathbf{P}_{\text{aug}}(k \mid k) \mathbf{F}^T_{\text{aug}}(k) + \mathbf{G}_{\text{aug}}(k) \mathbf{Q}_{\text{aug}}(k) \mathbf{G}^T_{\text{aug}}(k) \tag{30}
\end{align*}
\]

\[
\begin{align*}
\mathbf{X}_{\text{aug}}(k + 1 \mid k + 1) &= \mathbf{F}_{\text{aug}}(k) \mathbf{X}_{\text{aug}}(k \mid k) + \mathbf{K}_{\text{aug}}(k + 1 \mid k) \\
&\quad \times \mathbf{Z}_{\text{aug}}(k + 1 \mid k) - \mathbf{H}_{\text{aug}}(k + 1 \mid k) \mathbf{F}_{\text{aug}}(k) \mathbf{X}_{\text{aug}}(k \mid k) \tag{31}
\end{align*}
\]

where
\[
X_{aG}(k) = \begin{bmatrix}
X(k) \\
u(k)
\end{bmatrix},
F_{aG}(k) = \begin{bmatrix}
F(k) & C(k) \\
0 & I
\end{bmatrix},
G_{aG}(k) = \begin{bmatrix}
G(k) \\
0
\end{bmatrix},
w_{aG}(k) = w(k)
\]

\[
z_{aG}(k) = H_{aG}(k)X_{aG}(k) + v_{aG}(k)
\]

\[
H_{aG}(k) = [H(k)\Phi(k) H(k)C(k)]
\]

\[
v_{aG}(k) = H(k)G(k)w(k) + v(k + 1)
\]

\[
Q_{aG}(k) = E\{w_{aG}(k)w_{aG}^T(k)\} = Q(k)
\]

\[
R_{aG}(k) = E\{v_{aG}(k)v_{aG}^T(k)\} = H(k)G(k)Q(k)G^T(k)H^T(k) + R(k)
\]

\[
T_{aG}(k) = E\{w_{aG}(k)v_{aG}^T(k)\} = Q(k)G^T(k)H^T(k)
\]

In the simulations in order to have a fair comparison between MIE method and the proposed UBBM filter, process and measurement noises are considered as zero mean white processes. Since the MIE method is usable for white noise processes. In the examples, \( T = 1 \) second, \( g = 9.8 m/s^2 \), \( Q_w = 1 \), and \( R_v = 10 \). Initial conditions are \( X(0) = \begin{bmatrix} 200 m & 20 m/s & 100 m & -15 m/s \end{bmatrix}^T \), \( \dot{X}(0|0) = [0 \ 0 \ 0 \ 0]^T \) and parameters \( \rho = 0.8, \beta = 0.8 \). \( \sum(0|0) = I \) and \( P(0|0) = 10I \) where \( I \) is the identity matrix with appropriate dimensions.

**Example 1:** in this example, a constant acceleration model is considered such that for \( t \leq 100s \); \( u = [0 \ 0] m/s^2 \). The target begins to maneuver at 100th second with high maneuver \( u = [20 \ -10]^T m/s^2 \).

Figs. 1-4 compare the estimated positions, velocities and accelerations of the target estimated by the MIE method and the proposed method in the first example. The first row of Figs. 1-4 depicts respectively the actual positions, velocities and accelerations in x and y directions in dash-dot line, the estimated positions, velocities and accelerations by the MIE method in solid line and estimated values by the proposed UBBM filter in dashed line. The second row depicts respectively the positions, velocities and accelerations estimation errors in x and y directions by MIE method in solid line and the proposed UBBM filter in dashed line. For \( t < 100s \) (non-maneuvering stage) both of the methods could track the states of the target correctly without steady state error but when target begins to maneuver, the MIE method tracking performance decreases and it tries to estimate the states very slow such that until \( t = 500s \) it has big estimation errors. Figures show that the proposed UBBM filter could track the target very quickly without steady state error.
Example 2: in this example, a non-constant acceleration is considered. Before 100th there is no maneuver and after 100th the target begins to maneuver with $u = [2 \ 1]^T \times (t - 100) \text{m/s}^2$.

Figs. 5-8 compare the results of the MIE method and the UBBM method for the second example. The first row of Figs. 5-8 depicts respectively the actual positions, velocities and accelerations in x and y directions in dash-dot line and the estimated positions, velocities and accelerations by MIE method in solid line and the proposed method in dash line. The second row depicts respectively the positions, velocities and accelerations estimation errors in x and y directions by MIE method in solid line and for the proposed UBBM filter in dashed line. For $t < 100s$ (non-maneuvering stage) both of the methods could track states of the target correctly without steady state error but when target begins to maneuver, the MIE method could not track the target and the estimation errors increase gradually. Also, it can be seen that the proposed filter tracks the target fast with no steady state error.
6. CONCLUSIONS

In this paper a new MTT method based on UBB model has been proposed. Firstly a UBB model for target motion is developed and then a new MTT algorithm called UBBM filter is given. By UBBM filter target tracking in both maneuvering and non-maneuvering cases is possible. It can be seen that by using the proposed filter the target positions, velocities and accelerations were estimated very well without steady state errors. Also, the UBBM filter does not have any limitation on the type of the unknown maneuver and it can be used for a large diversity of input types. Moreover, it does not require maneuver detection and covariance resetting that are necessary in previous MTT methods. Therefore, UBBM filter reduces the tracking time. Simulation results are provided to confirm the theoretical development and to compare the filter with MIE method. Results show the high performance of the UBBM filter for both maneuvering and non-maneuvering targets with different inputs.

7. REFERENCES


