State observation and friction estimation in engine air path actuator using higher order sliding mode observers

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Abstract: This paper presents the application of higher order Sliding Mode observers (SMO) for state observation and friction estimation of an engine air path actuator. The actuator model incorporates LuGre Friction Model for dynamic friction modeling. The study has two objectives, to observe the system dynamics and the friction dynamics, and to use the equivalent output injection to estimate the parameters of the LuGre model. Cascaded sliding mode observers have been used to observe system and friction dynamics separately. Friction force has been estimated using the Equivalent output injection of the observer, considering friction as an unknown external input. Through this estimation, identification of the LuGre model parameters becomes simplified. Simulation and experimental results show the effectiveness of the proposed observation and identification scheme.

1. INTRODUCTION

The air path of a diesel engine is an important part of the system, responsible for maintaining the quality of air in the engine (Song and Byun [1999]). It is controlled by control valves, that are fast mechatronic actuators (Isermann et al. [2002], Song and Byun [1999]), which are nonlinear in nature, given to the change in temperature, external load perturbations, and aero-load dynamics (Scattolini et al. [1997], Rossi et al. [2000]). Friction plays a great part in the nonlinear behavior, causing stick-slip, dead zones and part wear (de Witt et al. [1995], Olsson et al. [1997], Rossi et al. [2000], Pavkovic et al. [2006]). Conventionally friction has been modeled as a static function, but this practice is physically incorrect as friction is dynamic in nature (de Witt and Lischinsky [1997]). State feedback control of the actuators is difficult, as they are commonly equipped with only a position sensor. Given the complex nature of friction, it means that for effective control, not only velocity, but friction state dynamics also need to be observed (de Witt and Lischinsky [1997]).

State observation problem has Misawa and Hendrick [1989] been developed actively within Variable Structure and Sliding mode theory (Utkin et al. [1999]). Seminary works on observation based on sliding mode and equivalent control can be found in Utkin [1992], Edwards and Spurgeon [1998], Drakunov [1992] and Drakunov and Utkin [1995]. These endeavors introduced the formulation of extended observability conditions for systems with unknown inputs, leading to ensured observation of systems (Utkin et al. [1999]). The main drawback of this technique is that the use of conventional sliding mode in the step by step realization requires filtration at each step (Davila et al. [2007]). A step further in SMO development were the hierarchical observers, in which filtration was replaced by super twisting algorithm (Bejarano et al. [2006], Levant [1993]). Davila et al. [2005] have introduced a second order SMO based on modification of the super twisting algorithm. This observer ensures finite time convergence without filtration (Fridman et al. [2006], Davila et al. [2006]). Another advantage of this observer is that the equivalent output injection can be used to gain further information, such as unknown inputs or perturbations (Davila et al. [2006]).

In this paper, we have extended the works of Davila et al. [2005] and Xie [2007] to develop a complete observer for engine air path mechatronic actuator. A nonlinear model of the actuator has been described to represent the actuator system (Ahmed et al. [2010b]). Two observers have been cascaded, the actuator dynamics have been observed through measured position, using the modified super twisting observer described by Davila et al. [2005] and friction dynamics have been observed by cascading the observed velocity to a first order sliding mode observer (Drakunov and Utkin [1995], Xie [2007]. An important feature of this study is that we have used the equivalent output injection property of the super twisting observer to estimate friction force. This estimate, coupled with a parameter estimation method, can compensate for variations in certain parameters of the LuGre model.

The paper has been arranged in the following manner; actuator and friction modeling has been presented in section 2. The sliding mode observer and output injection have been explained in section 3. Simulation results have been presented in section 4, and experimental results have been presented in section 5. Some conclusions have been discussed in section 6.

2. MODELING

In modern diesel engines, the swirl actuator is integrated in the air inlet manifold. As seen in figure 1, the actuator consists of a DC motor, a return spring and gearing. The significant number of mechanical parts result in friction and stick-slip, spring nonlinearities, as can be seen in the
static characteristic curve of the actuator in figure 2. The modeling hence depends upon estimation for parameter identification.

2.1 Actuator Model

A detailed and physically motivated simulation model has been defined in Ahmed et al. [2010a], in which motor-spring systems have been characterized by nonlinearities due to the presence of spring pre-compression and significant friction. Keeping all these factors in mind, the actuator can be modeled in the form of the following electrical and mechanical equations (Scattolini et al. [1997], Ahmed et al. [2010a]):

\[ V_a = i_a R_a + L_a \frac{di_a}{dt} + E_a \]  
\[ \frac{d\omega}{dt} = K_a \frac{(V_a - K_a \omega)}{J_{tot}} - \frac{K_{spc} \theta - (T_{pc} + T_f)}{J_{tot}} \]  
\[ E_a = K_{em} \omega = K_a i_a \]  
\[ T_m = K_a i_a \]  
\[ T_{spc} = K_{spc} \theta \]

2.2 Friction Model

Friction is a natural force that exists between two surfaces in contact, moving relative to each other. The actual phenomenon is difficult to characterize since it depends upon a variety of physical conditions, however it arises essentially from surface irregularities or asperities (Olsson et al. [1997], Marton and Lantos [2007]).

The LuGre Friction model is a dynamic model that treats asperities as elastic bristles. Motion, according to this model occurs when the bristles start ‘slipping’ (de Witt et al. [1995]). It allows for the static friction to be modeled separately, and also incorporates Stribeck effect in the model (Olsson et al. [1997], de Witt et al. [1995]). The LuGre model considers friction as a function of the bristle displacement, bristle velocity and the system velocity

\[ T_f = \begin{cases} T_f(z, \dot{z}, \omega) = \sigma_o z + \sigma_1 \dot{z} + \sigma_2 \omega \\ \dot{z} = \omega - \frac{\sigma_o |\omega| z}{p(\omega)} \end{cases} \]  

Where \( z \) is the average deflection of the bristles before slipping, \( \sigma_o \) is their stiffness coefficient and \( \sigma_1 \) is the damping coefficient which stabilizes the dynamics in the Stribeck region (de Witt et al. [1995]), \( \sigma_2 \) is the coefficient of viscous friction. The \( p(\omega) \) function models the Stribeck effect as a function of velocity. If \( \omega_s \) is the velocity at which Stribeck effect takes place (Stribeck velocity), then \( p(\omega) \) can be expressed as:

\[ p(\omega) = T_c + (T_s - T_c) e^{(-\omega/|\omega_s|)} \]  

2.3 Complete Model

As seen in equation (2), the system dynamics are represented by a second order system with a second member. Using equations (3) and (4), the complete model can be written as:

\[ \dot{x}_1 = \theta, \quad x_2 = \frac{d\theta}{dt}, \quad z, \quad \dot{z} \
\dot{x}_2 = -\frac{1}{J_{tot}} \left[ K_{spc} x_1 + \frac{K^2_a x_2 + T_f + T_{pc}}{R_a} + \frac{K_a}{R_a J_{tot}} u \right] \quad \dot{z} = x_2 + \frac{\sigma_o |x_2|}{p(x_2)} \]

The system can be divided in terms of states as follows:

\[ \dot{x}_2 = f(x_1, x_2, u) + T_f(z, \dot{z}, x_2) \]  

As we have seen that the friction state depends upon the velocity. The proposed decoupling in terms of actuator system and friction allows us to observe their associated dynamics separately, first the velocity \( (x_2) \), and then using the observed velocity, the bristle deflection \( (z) \). The model parameters used in simulations are given in table 1. These have been found experimentally, using the method described in (Scattolini et al. [1997], Ahmed et al. [2010a,c]). In the following sections, we will present a sliding mode observer for state estimation.

3. SLIDING MODE OBSERVER

Considering system (6), we see that \( x_2 \) and \( z \) are to be estimated through \( x_1 \). Since \( z \) depends upon \( x_2 \), the observers have been cascaded. The first observer estimates \( x_2 \) and considers friction as a disturbance, while the second observer estimates the friction state variable \( z \).
3.1 Actuator State Observer

The proposed second order observer has the form (Davila et al. [2005]):

\[
\begin{align*}
\dot{x}_1 &= \dot{x}_2 + z_1 \\
\dot{x}_2 &= f(x_1, \dot{x}_2, u) + z_2
\end{align*}
\]  

(7)

Where \(\dot{x}_1\) and \(\dot{x}_2\) are the correction variables and \(z_1\) and \(z_2\) are the output injections of the following form.

\[
\begin{align*}
z_1 &= \lambda |x_1 - \dot{x}_1|^{1/2} \text{sgn}(x_1 - \dot{x}_1) \\
z_2 &= \alpha \text{sgn}(x_1 - \dot{x}_1)
\end{align*}
\]

Notice that the observer structure has been based only on the function representing the actuator parameters \((f(x_1, \dot{x}_2, u))\), and the friction model has not been included. In the following subsections, it will be shown that the equivalent output injection would lead to estimation of the unobserved friction as an unknown input. In order to analyze the observer’s convergence, let us consider the error dynamics. Taking \(\tilde{x}_1 = x_1 - \dot{x}_1\) and \(\tilde{x}_2 = x_2 - \dot{x}_2\), we obtain the following error equations.

\[
\begin{align*}
\dot{\tilde{x}}_1 &= \dot{\tilde{x}}_2 - \lambda |\tilde{x}_1|^{1/2} \text{sgn}(\tilde{x}_1) \\
\dot{\tilde{x}}_2 &= f(x_1, x_2, \dot{x}_2, u) - \text{sgn}(\tilde{x}_1)
\end{align*}
\]  

(8)

Where \(f(x_1, x_2, \dot{x}_2, u) = f(x_1, x_2, u) - f(x_1, \dot{x}_2, u) + \xi(x_1, x_2, u)\).

Here, the function \(\xi\) represents any disturbances and perturbations in the system. If the system states are bounded, then there exists a constant \(f^+\) such that

\[|f(x_1, x_2, \dot{x}_2, u)| < f^+\]

for all possible \(x_1, x_2\).

Remark 1: If the system acceleration is bounded, \(f^+\) can be found as double the maximum possible acceleration of the system (Davila et al. [2005, 2006]). Let \(\alpha\) and \(\lambda\) satisfy the following constraints:

\[
\alpha > f^+ \\
\lambda > \sqrt{\frac{2}{\alpha + f^+ (1 + p)}} (1 - p)
\]

(10)

Where \(p\) is a constant such that \(0 < p < 1\).

Theorem 1: (Davila et al. [2005]) Let the parameters of the observer (7) be selected according to the constraints given in (10). If (9) exists for system (6), then the variables of the observer (7) converge in finite time to the states of system (6).

Proof of this theorem has been described in detail in (Davila et al. [2005, 2006]).

3.2 Friction state observation

As seen in equation (3), friction dynamics depend upon the velocity state \(x_2\). Considering (3), we can see that the observer described in the previous section is not suitable for the observation of friction states. This is because neither state, \(z\), or \(\tilde{z}\) can be measured. We have seen that the only measured variable is the position, \(x_1\). Therefore, a first order sliding mode observer has been used for friction observation, which depends upon position error dynamics. This strategy, has also been used by Xie [2007] for friction state observation for adaptive control purpose.

Let us define \(e = x_1 - x_{1\text{ref}}, \dot{e} = \dot{x}_1 - \dot{x}_{1\text{ref}}\). The sliding surface for this observer can be defined by the following Hurwitz polynomial:

\[s = \dot{e} - Ce\]

Where \(C > 0\)

The friction state estimate \(\hat{z}\) can be obtained through the stribbeck velocity function \(p(x_2)\), observed velocity from the actuator state observer \(\dot{x}_2\), and the error \(s\). The observer has the following structure

\[
\hat{z} = x_2 - \frac{|x_2| \hat{z}}{p(x_2)} - \alpha \text{sgn}(s)
\]

(11)

Where \(\alpha > 0\)

The error dynamics can be found through the friction model dynamic equation (3) and equation (11) as

\[
\dot{\hat{z}} = -\frac{|x_2|}{p(x_2)} \hat{z} - \alpha \text{sgn}(s)
\]

Where \(\hat{z} = z - \hat{z}\)

Details on the observer’s convergence can be found in Drakunov and Utkin [1995].

3.3 LuGre Model Parameteric Variation Estimation

Parameter identification of the LuGre model is the greatest difficulty in its application. As can be seen in equation (3), the model depends upon six parameters. Furthermore, the behavior of friction in a mechanical system changes under changing environmental conditions (e.g. temperature) and also as the parts in contact grow older. Hence nominal model parameters, identified in a certain condition and at
a certain time in the actuator’s life, are not reliable to represent friction in the part under different operational conditions.

In this section, we will present a method to estimate variations in the \( \sigma_o, \sigma_1 \) and \( \sigma_2 \) parameters of the LuGre model, using nominal parameter values, the equivalent output injection of the actuator state observer, and the estimated states obtained from the friction state observer. Let us consider system (8) again. It had been mentioned earlier that the advantage of finite time convergence of the second order sliding mode is that it ensures the existence of a time constant \( t_o > 0 \), such that for all \( t > t_o \)

\[
0 \equiv \ddot{x}_2 \equiv F(x_1, x_2, \dot{x}_2, u) + \xi(x_1, x_2, u) - \alpha \text{sgn}(\dot{x}_1)
\]

As indicated earlier, \( \xi(x_1, x_2, u) \) contains all external disturbances and perturbations. Therefore, considering friction as an external force, it can be estimated using \( \dot{z}_{eq} \).

Thus \( \xi(x_1, x_2, u) = T_f + \Delta F(x_1, x_2, u) \) (12)

It can be seen, considering (8) that since \( x_2 = \dot{x}_2 \),

\[
F(x_1, x_2, \dot{x}_2, u) = F(x_1, x_2, u) - F(x_1, \dot{x}_2, u) = 0
\]

Then the equivalent output injection becomes

\[
\dot{z}_{eq} = \alpha \text{sgn}(\dot{x}_1) = \xi(x_1, x_2, u)
\]

Ideally, in sliding mode theory, \( \dot{z}_{eq} \) is expected to change at a high (infinite) frequency. Since such a high frequency is not realizable in reality, the state oscillates near the intersection, causing \( \dot{z}_{eq} \) to switch at a finite frequency. This oscillation has high and low frequency components. Utkin et al. [1999] have shown that motion in sliding mode is determined by the low-frequency (i.e. slow) components of oscillation. It is hence reasonable to consider the equivalent control to be near the low-frequency component of the real control. The equivalent control \( \dot{z}_{eq} \) can hence be determined by filtering out the high frequency components using a low pass filter. In our study, we have considered that the nominal model is totally known, i.e. \( \Delta F(x_1, x_2, u) = 0 \). The equivalent output injection hence becomes

\[
\dot{z}_{eq} = T_f
\]

Once we have estimated the friction force actually acting on the actuator, and we have observed friction states, parametric variations can be estimated. For the sake of differentiating between terms, let us define \( \dot{z}_{eq} = T_{fa} \) as the friction force estimated through the actuator state observer and \( T_{fo} \) as the friction force calculated using the observed dynamics and nominal values of the LuGre model parameters.

\[
\begin{align*}
T_{fa} &= \sigma_{oA} \dot{z} + \sigma_{1A} \ddot{z} + \sigma_{2A} \dot{x}_2 \\
T_{fo} &= \sigma_{oG} \dot{z} + \sigma_{1G} \ddot{z} + \sigma_{2G} \dot{x}_2
\end{align*}
\]

Defining

\[
\sigma = [\sigma_o \ \sigma_1 \ \sigma_2], \ \varphi(t) = \begin{bmatrix} \dot{z} \\ \ddot{z} \\ \dot{x}_2 \end{bmatrix}
\]

We obtain

\[
\begin{align*}
T_{fa} &= \sigma_A \varphi(t), \ T_{fo} = \sigma_O \varphi(t) \\
\Delta T_f(t) &= (\sigma_A - \sigma_O) \varphi(t) = \Delta \sigma \varphi(t)
\end{align*}
\]

Where \( \Delta T_f(t) = (T_{fa} - T_{fo}) = (\dot{z}_{eq} - T_{fo}) \) and \( \varphi(t) \) are known due to the guaranteed finite time convergence of the observers. Equation (16) is in the form of a linear regression model as expressed in (Soderstrom and Stoica [1989]). Here \( \Delta \sigma \) is the vector to be estimated. Let us develop a linear regression algorithm from eq. (16). Post-multiplying both sides by \( \varphi(t)^T \) gives us a square matrix for late inversion. The average values of eq. (16) can then be expressed as

\[
\frac{1}{T} \int_0^T \Delta T_f(t) \varphi(t)^T dt = \Delta \sigma \frac{1}{T} \int_0^T \varphi(t) \varphi(t)^T dt
\]

\[
=> \Delta \sigma = \left[ \int_0^T \Delta T_f(t) \varphi(t)^T dt \right]^{-1} \left[ \int_0^T \varphi(t) \varphi(t)^T dt \right]^{-1}
\]

Where \( \hat{\Delta} \sigma \) is the estimation of \( \Delta \sigma \). The following relationships hold for any nonsingular square matrix (Davila et al. [2006]).

\[
\begin{align*}
\Gamma^{-1}(t) &= I \\
\Gamma^{-1}(t) \hat{\Gamma}(t) + \hat{\Gamma}(t) \Gamma^{-1}(t) &= 0
\end{align*}
\]

If \( \Gamma(t) = \left[ \int_0^T \varphi(t) \varphi(t)^T dt \right]^{-1} \), then using the abovementioned relationships, we can represent the dynamics of \( \Delta \sigma \) as

\[
\dot{\Delta} \sigma = \Delta \sigma \Gamma^{-1}(t) \hat{\Gamma}(t) + \Delta T_f(t) \varphi(t)^T \Gamma(t)
\]

\[
=> \Delta \sigma = -\Delta \sigma \Gamma^{-1}(t) \hat{\Gamma}(t) + \Delta T_f(t) \varphi(t)^T \Gamma(t)
\]

And the dynamics of \( \hat{\Gamma}(t) \) can be expressed by

\[
\hat{\Gamma}(t) = -\Gamma(t) \left[ \varphi(t) \varphi(t)^T \right] \Gamma(t)
\]

The convergence conditions of the estimation have been derived by Davila et al. [2006], in the following theorem.

\[
\sup \| t \hat{\Gamma}(t) \| < \infty \quad \frac{1}{T} \int_0^T \varepsilon(t) \varphi(t)^T dt \rightarrow 0 \text{ as } t \rightarrow \infty
\]

**Theorem:** The algorithm (18), (19) ensures the convergence of \( \Delta \sigma \) to \( \hat{\Delta} \sigma \) under condition (20).

Proof of this theorem can be found in (Davila et al. [2006]).

4. SIMULATION RESULTS

The system model (5) was simulated using the parameters given in table 1. The system was controlled using second
order sliding mode control based on super twisting algorithm. Details on the controller design can be found in Ahmed et al. [2010b].

Figure 3 shows the performance of the actuator state observer. Both the accuracy and the finite time convergence properties are visible in the observation errors of \( x_1 \) and \( x_2 \). In figure 4 the performance of friction state observer has been shown and the result of friction estimation has been presented. It can be seen that the filtered equivalent output injection signal represents the friction force very accurately.

The results of parameter identification based on the estimated friction force are given in Table 2. It can be seen that the estimated variation is very close to the parameter variation used in simulation. The convergence is shown in figure 5.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal</th>
<th>Varied</th>
<th>( \Delta \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lugre Stiffness Coefficient</td>
<td>2800</td>
<td>2753</td>
<td>46</td>
</tr>
<tr>
<td>Lugre Damping Coefficient</td>
<td>53</td>
<td>45</td>
<td>7</td>
</tr>
<tr>
<td>Lugre Viscous Coefficient</td>
<td>0.012</td>
<td>0.015</td>
<td>-0.003</td>
</tr>
</tbody>
</table>

Table 2. LuGre Model Parameters

The experimental setup consists of an actuator test bench which is controlled by LabView, using the National Instruments CompactRio system. The system has a high resolution dynamometer and encoder. The dynamometer can measure torque up to 1N.m with a resolution of 1mN.m. The incremental encoder can measure angles with a resolution of 0.088°.

The convergence can be seen in figure 6. The nominal values are those identified during modeling, and the identified...
values are those identified during operation. It can be seen that \( \sigma_0 \) varies significantly from the nominal value, while the other parameters are not much affected. In fact, as \( \sigma_0 \) - which models the static friction component- is the most significant parameter as static friction is the major nonlinearity in the actuator, as seen in the characteristic curve (figure 2).

![Sliding Mode Observer](image)

**Fig. 6. Experimental Convergence**

### 6. CONCLUSION

In this paper, a cascaded sliding mode observer scheme has been used to observe the states of a nonlinear engine air path actuator. The actuator model incorporates a dynamic friction model. Two observers have been cascaded to observe actuator dynamics and the consequent friction dynamics. A scheme for estimation of parametric variations in the friction model has been described, based on the equivalent output injection of the observers. The effectiveness of this work can be seen in the results. Future development of this work involves application of the parameter identification scheme to a complete feedback-linearization based control scheme, as has been proposed in Ahmed et al. [2010b].

### REFERENCES


