Control of a Vibrating Axisymmetric Membrane using Piezoelectric Transducers

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Abstract: In this paper, the problem of the active vibration control of a thin and flexible disc is addressed. The mechanical structure tackled here is equipped with two piezoelectric circular patches: one of them works as a sensor and the other is used as an actuator. Both are fixed on the disc, one on each side, and centered according to its axis of symmetry. The purpose of this work is to design a controller allowing the active damping of the most vibrating modes in a specified bandwidth. Robustness issues against neglected dynamics are discussed. After describing the problem, we first discuss on the model properties, derived from a finite element analysis, particularly about the structure’s symmetry consequences. Then, we propose a control method leading to the reduction of several modes of vibration. Numerical simulations are proposed to analyze the modelling and the vibration control efficiency.

Keywords: vibration control, axisymmetric membrane, piezoelectric transducers, robust control

1. INTRODUCTION

The problem of active vibration damping of thin mechanical structures is a topic that has received a great attention by the control community since several years. Especially when actuators and sensors are based on piezoelectric materials. Piezoelectric materials have several interesting features and what make them so popular is that they are electromechanical transducers because they transform electrical phenomena into mechanical ones, and reciprocally. So, they can either be used as sensors or as actuators. Among the different categories of piezoelectric material, lots of people focused on PZT based ceramics for their quite good electromechanical coupling properties. Indeed, they allow better mechanical deformations for a given electric field than piezoelectric polymers such as PVDF ones. PZT based ceramics are often used as force actuators since they offer very high mechanical loading capabilities. They are also used as displacement sensors and offer accurate positioning capabilities (for example in nanopositioning problems [e.g. Sebastian and Salapaka, 2005]). For mechanical structures that are deformable, piezoelectric materials find their utilities as strain sensors or strain actuators. With an appropriate controller, they permit to achieve shape control [e.g. Park and Mills, 2005, Juan et al., 2007]) or the active damping of multi-modal vibrations thanks to their very large bandwidth. Moreover, their behavior is quite linear when they work in a specific range of use. That is why piezoelectric materials have aroused great interest for the instrumentation of thin mechanical structures. Such equipped structures are often associated with the keyword smart structures. In this area, the major challenge is the design of controllers able to damp the most vibrating modes in a specified low frequency bandwidth while ensuring robustness against high frequency modes, outside the bandwidth of interest, often unmodelled or weakly modelled. Such smart structures enter the category of Distributed Parameter Systems, described by Partial Differential Equations (PDEs), known to lead to infinite dimensional systems. The inherent feature of this kind of systems is that they arise robustness issues when they are tackled with finite dimensional control tools.

Many works have concerned the vibration control problem of the “Euler-Bernoulli beam” equipped with one rectangular piezoelectric actuator and sometime, another one, identical and collocated, but used as sensor. See for example Chen et al. [2004], Banks et al. [2002] where one edge of the beam is clamped whereas the other remains free. Other works dealt with the problem of vibration control for laminated rectangular plates [e.g. Köggl and Bucalem, 2005] or complex plate like structures [e.g. Tliba and Abou-Kandil, 2003, Tliba et al., 2005]. In this paper, we address the problem of the active vibration control in continuous time, of an axisymmetric membrane equipped with two collocated piezoelectric circular patches. One of them is used as a strain sensor and the other is used as a mechanical deformation actuator. They are fixed on the structure so that they keep the axis of symmetry of the disc.

The paper is organized as follows. In the first part, we give more details on the system under consideration and explain the objectives of the control problem. In the second part, we briefly discuss about how we obtained the input-output dynamical model. We analyze the model’s properties from considerations of symmetry. The third part is devoted to the control approach that we considered to lead to the vibrations reduction of the axisymmetric membrane.

2. PROBLEM DESCRIPTION

2.1 System under consideration

The system considered here is described in Fig. 1. It is a composite membrane composed by a brass disc with a clamped circular edge. This disc is embedded into a mobile support moving only along the z axis. The moving support is subjected to an unknown acceleration, noted w(t) in the sequel. This flexible membrane is equipped with two PZT-based piezoelectric

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patches: one used as an actuator and the other used as a sensor. The sensor’s thickness is 0.7 mm. It is greater than the actuator’s thickness which is 0.4 mm. An explanation about why should the thickness of a piezoelectric sensor must be greater than the actuator’s one can be found in Tliba [2004]. These circular patches are supposed to be rigidly bonded on the disc, one on each side, and centered according to the axis of symmetry. All the physical parameters of the materials used here can be found in Minazara-Erambert et al. [2005]. The main dimensions of the composite membrane are given in Fig. 1.

![Fig. 1. Axisymmetric composite membrane (dimensions on the right) inserted in the device which is subjected to vibrations (Computer Aided Design figure on the left)](image)

2.2 Control objectives

This system is SISO. It has only one control input and one measured output. The controlled input noted \( u(t) \) is the voltage applied across the piezoelectric actuator. The measured output noted \( y(t) \) is the electric voltage delivered by the piezoelectric sensor. The disturbance input is the signal \( w(t) \) defined above, corresponding to the total acceleration applied to the clamped circular edge of the structure. The controlled output that we consider, noted \( z_p(t) \), is the \( z \) component of the acceleration of a point located at the center of the disc and on the upper side of the sensor. The main control objective is to minimize the energy of the closed-loop controlled output \( z_p(t) \) for any disturbance input \( w(t) \) of finite energy. This specification is translated into an expected reduction of at least 15 dB of the peaks of resonance of the modes excited by \( w(t) \) in the closed loop transfer between \( w \) and \( z_p \). Furthermore, we have to avoid the “spillover” phenomenon. It means that the sought controller should never excite high-frequency modes in closed-loop. To this end, we look for a linear output feedback controller \( \mathcal{C} \). This control problem can be formulated like an \( \mathcal{H}_\infty \) synthesis problem that we shall describe in the section 4.1.

3. MODELLING OF THE COMPOSITE MEMBRANE

3.1 A distributed parameter system

A PDE, describing the dynamical deformations of a simple membrane -that is, without any piezoelectric component bonded on it- in polar coordinates, can be found for example in [Gérardin and Rixen, 1997, p. 233]. Unfortunately, for our system, with 2 collocated patches, one bonded on each face, we can hardly address the control problem with an analytical approach because of the complex geometrical topology and the heterogeneity of the materials composing the composite membrane. Indeed, this feature makes more difficult the obtaining of the dynamical inputs-outputs model especially for the control input to the measured output relation. Even under some simplificative assumptions, the resulting model would not be accurate enough compared to a real experimental behavior.

3.2 Finite dimension model

The Finite Element Method (FEM) provides a numerical approach of the problem of the composite membrane modelling, able to take into account the complexity of the topology as well as the heterogeneity of the materials. Following a method proposed in Tliba and Abou-Kandil [2005], one can obtain an accurate description of the inputs-outputs dynamical behavior. One can also refer to Komzisk [2005] and Tliba [2004] for more details on how to use FEM to derive a finite dimensional model of the inputs-outputs dynamical behavior.

The FEM, combined with modal analysis, permit to get a high order but accurate model in state-space form, devoted to analysis purposes (1). The analysis model describes the inputs-outputs dynamical behavior in the bandwidth \([0 \rightarrow 3500Hz]\). A reduced order one, derived from the analysis model and devoted to the controller synthesis is obtained thanks to a truncation technic, with keeping of the static gains of the analysis model. It differs from (1) by the presence of non-zero feedthrough terms between output \( y \) and inputs \( w \) and \( u \). The analysis model is of order 88 and contains 76 uncontrollable and unobservable modes. These modes all have the particularity to not be axisymmetric as illustrated in Fig. 3 for few modes. The reduced model is of order 8, including the first 4 axisymmetric modes, all controllable and observable (see the modal shapes Fig. 2).

Let \( x_p \in \mathbb{R}^n \) be the state vector of the system, whatever its order. It means either \( n_p = 88 \) for the analysis model or \( n_p = 8 \) for the synthesis model.

\[
\mathcal{F} \begin{cases}
\dot{x}_p(t) = A_p x_p(t) + B_{p,w} w(t) + B_{p,u} u(t) \\
z_p(t) = C_p z_p(t) + D_{p,w} w(t) + D_{p,u} u(t) \\
y(t) = C_{p,y} x_p(t) + D_{p,w} w(t) + D_{p,y} u(t)
\end{cases}
\]

The Bode frequency response between the control input and the measured output is given Fig. 4 and the one between the disturbance and the controlled output \( z \)-acceleration of the center of the membrane) is given Fig. 5. The main features of the complex poles composing the open-loop reduced order model are summarized in Tab. 1.

4. \( \mathcal{H}_\infty \) DESIGN WITH POLE PLACEMENT CONSTRAINTS

The \( \mathcal{H}_\infty \) synthesis, like the \( \mathcal{H}_2 \) synthesis, is quite well suited to problems where the disturbance is a random signal, say a white (or colored) noise, with a given Power Spectral Density, and where the \( \mathcal{L}_2 \)-norm of the controlled output has to be minimized. See [e.g. Tliba et al., 2005] for an illustration of an \( \mathcal{H}_\infty \) controller design with experimental results applied to a rectangular plate like structure. The method used here to design an \( \mathcal{H}_\infty \) linear controller with pole placement constraint is based on the use of the reduced order model in order to obtain a low order controller able to satisfy the control objectives. This model is completed with additional inputs and outputs channels weighted by some specific filters. The weighting filters permit
to manage the several objectives expected in closed loop. The standard setup of the augmented plant is depicted in Fig. 7 where 3 filters have to be tuned. The theoretical backgrounds used to address this problem are those proposed in Chilali and Gahinet [1996]. Thus, we shall present only the design procedure in the sequel.

4.1 Controller design procedure

Let $s$ be the complex Laplace variable. Filters are denoted by transfer functions $W_b(s)$, $W_{perf}(s)$ and $W_{r-off}(s)$.

Filter $W_b(s)$ is used to specify the features of the Power Spectral Density of the random perturbation $w$. We basically often chose a low-pass filter to “color” the white noise modeling this random input. The filter’s cutoff frequency is 300Hz and is of order 1. The Power Spectral Density is $0.01g^2/Hz$ where $g$ denotes the earth’s gravity.

Filter $W_{perf}(s)$ is used to set the level of attenuation of resonant peaks in the accelerometric frequency response (i.e. between input $w$ and output $z_a$). It is a proper and low-pass filter of order 1, with a cutoff frequency set to 1000Hz and $-15dB$ for the low-frequency gain and $-60dB$ for the high-frequency gain.

Filter $W_{r-off}(s)$ is a high-pass filter of order 2 devoted to the robustness with respect to neglected modes of high frequencies in the synthesis model. A method for tuning parameters was proposed in Tliba [2006] for another application. The cutoff frequency is set to 5000Hz. The low-frequency gain is $-25dB$ and the high-frequency gain is 40dB.

To enforce the damping effect and avoid the well known pole-zero cancellation of the central $\mathcal{H}_\infty$ compensator, [Gahinet and Apkarian, 1994], we introduce some geometrical constraints on the location of the closed-loop poles in the complex plane as depicted in Fig. 6. The 3 parameters defining this convex sector are set to $\theta \approx 87.13^\circ$ (corresponding to a minimum iso-damping of 0.05), $\alpha = -25$ and $\rho = 5 \times 10^5$.

Finally, the augmented plant of Fig. 7 is of order 12. This $\mathcal{H}_\infty$ synthesis problem with pole placement constraint is composed...
Fig. 6. (ρ, θ, α) convex sector (green) of the complex plane targeted in closed-loop.

The formulation of the $\mathcal{H}_\infty$-control is turned into a nominal performance objective problem, where all additive uncertainties are considered like perturbations. The controller is obtained by solving the optimal $\mathcal{H}_\infty$ control problem using convex optimization tools:

$$K_{opt}(s) = \arg \left\{ \min_{K(s) \in \mathcal{R}} \gamma \right\}$$

under $K(s) \in \mathcal{R}$ and $\{p_i \in C\}_{i=1,2,...} \subset \mathcal{D}(\rho, \theta, \alpha)$

where $\gamma = \| F_1 \{ P(s), K(s) \} \|_{\infty}$. $F_1 \{ P(s), C(s) \}$ denotes the closed-loop transfer matrix between input $[w_y, w]^T$ and output $[z_u, z_a]^T$, $\{p_i \in C\}$ are the closed loop poles.

4.2 Optimization results

The optimum $\mathcal{H}_\infty$ controller is obtain with a controller of order 12, for the minimum $\mathcal{H}_\infty$, cost: $\gamma = 0.7876$. It means the control objectives are robustly achieved. Looking more closely at the results reveals that the $\mathcal{H}_\infty$ norm for the transfer function between $W(s)$ and $Z_u(s)$ is 0.5195, which means that the damping objective was robustly satisfied. Moreover, the $\mathcal{H}_\infty$ norm for the transfer function between $W_c(s)$ and $Z_u(s)$ is 0.26281, which means that the robustness specification w.r.t. high-frequency unconsidered modes was also satisfied.

The $\mathcal{H}_\infty$ optimal controller frequency response is illustrated on Fig. 8. It is a stable controller of order 12. We can observe that its cutoff frequency is around 5000 Hz and that its high frequency roll-off is 40 dB per decade, as specified by the filter $W_{eff}(s)$ on the standard setup Fig. 7.

Fig. 8. Optimum $\mathcal{H}_\infty$ controller frequency response.

The closed-loop poles' features are summarized in Tab. 1. This table show the efficiency of the $\mathcal{H}_\infty$ optimal controller which permitted to damp the vibrating modes as expected. The closed-loop poles do lie within the green convex sector of the complex plane which corresponds to the $(\rho, \theta, \alpha)$ convex sector $\mathcal{D}(\rho, \theta, \alpha)$, as shown in Fig. 9.

<table>
<thead>
<tr>
<th>Mode n°</th>
<th>Frequency (Hz)</th>
<th>Damping Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>119.98</td>
<td>120.01</td>
</tr>
<tr>
<td>2</td>
<td>497.3</td>
<td>497.55</td>
</tr>
<tr>
<td>3</td>
<td>1177.8</td>
<td>1178.4</td>
</tr>
<tr>
<td>4</td>
<td>2186.6</td>
<td>2187.8</td>
</tr>
</tbody>
</table>

Table 1. Damping coeff., and natural frequency of the 4 modes composing the reduced order model: open-loop vs closed-loop.

5. SIMULATION RESULTS

In this section, the closed-loop performances of the previous $\mathcal{H}_\infty$ controller are analyzed through few responses.
Fig. 9. On the left: Root locus: × open-loop poles, + closed-loop poles, ◦ open-loop zeros, −− convex sector constraint. On the right: Zoom on the region of the low-frequency modes.

Fig. 10 illustrates the closed-loop accelerometric frequency response compared to the open-loop one. One can note the reduction of at least 20dB on each peak of resonance. This corroborates the previous analysis of the $H_\infty$ costs.

The left figure of Fig. 11 shows the control voltage signal of the piezoelectric actuator when the set-up of Fig. 1 is subjected to a rectangular shock like signal, of magnitude 5g and with a duration of 10ms. The analysis model of order 88 was used to run this simulation. The voltage required at the beginning is quite high so that, in experimental situation, the control signal may saturates to a lower level.

The right figure of Fig. 11 shows a comparison of the open-loop and the closed-loop acceleration signal $z_a(t)$ when the disturbance is the rectangular shock-like described above. One can observe an enhanced time response of the signal in closed-loop.

Fig. 12 shows the open-loop frequency response in Nichols diagram. It allows to compare the controller properties (minimum stability margins) applied both to the reduced order model and the analysis one. In both cases, the gain margin is 3.12dB, the phase margin is $-12.3$ deg and the delay margin is of $8.65ms$. Although these values might seem a little bit small, in practice they appear to be sufficient since the steady-state gain of the model is often overestimated compared to the experiment one. What is important to be emphasized is that stability and performances are kept when the controller is applied to the analysis model of full order.

6. CONCLUSION & FUTURE WORKS

We have just shown a work dealing with the active vibration control of an axisymmetric membrane piezo-actuated, using robust control tools to design an efficient controller satisfying the control problem. In future works, we will consider the problem of the actuator’s saturation and the design of an anti-windup compensator that permits to recover some linear performances presented in this paper.

REFERENCES

Fig. 11. On the left: Time response of the closed-loop control signal to a shock like disturbance $w(t)$ (black-). On the right: Time response of the closed-loop (red-) and uncontrolled (blue-) acceleration signal $z_a(t)$ to a shock like disturbance $w(t)$ (black-).

Fig. 12. Reduced order (blue-) vs full-order (magenta-) open-loop frequency response in Nichols diagram.