Experiment design plays a fundamental role in the practice of model identification. This is specially true in the case of MIMO systems, for which the large number of degrees of freedom available in the setup of experiments calls for careful planning in order to ensure the best possible excitation for the system while meeting the relevant operational requirements during experiments (such as, e.g., flight test constraints in aircraft and rotorcraft model identification). In this paper the problem of defining optimal input sequences for MIMO model identification is considered. The proposed method builds on previous results and allows the optimisation of suitable scalar functions of the Fisher information matrix by means of a computationally sound procedure for the design of both step-wise and multi-cyclic MIMO inputs. The design procedure is evaluated by designing MIMO experiments to estimate the parameters of a physical model for a rotorcraft platform.

1. INTRODUCTION

The aim of the input design problem is to compute an input sequence for system (1) which allows the estimation of the parameters \( \theta \) with the best possible accuracy (i.e., minimum variance) while at the same time ensuring that the state and output variables remain sufficiently close to the considered equilibrium, so as to limit the overall effect of the input perturbation on the system. For this reason, a number of constraints must be taken into account in the design of the experiment (see Klein and Morelli (2006)):

- amplitude/energy constraints on inputs, states and outputs;
- duration constraints on the experiments;
- constraints on the minimum sampling time \( T_s \);
- accuracy of the available instrumentation.

The paper is organised as follows. Section 2 is devoted to the formulation of the input design problem considered in this paper; in Section 3 the design method first proposed in Jauberthie et al. (2006b,a) for the design of piece-wise constant input signals is summarised, and the proposed modifications are presented; Section 4, on the other hand, deals with the design of optimal multi-sine inputs; finally, in Section 5, the case study dealing with input design and parameter estimation for the dynamics of a quadrotor helicopter is presented.

2. PROBLEM STATEMENT

Consider the nonlinear system

\[
\begin{align*}
\dot{x} &= f(x, u; \theta) \\
y &= h(x, u; \theta)
\end{align*}
\]

with \( u \in \mathbb{R}^n_u \), \( x \in \mathbb{R}^n_x \), \( y \in \mathbb{R}^n_y \) and the parameter vector \( \theta \in \mathbb{R}^p \), to which the equilibrium \((\bar{x}, \bar{y})\) with corresponding outputs \( \bar{y} = [\bar{y}_1 \bar{y}_2 \ldots \bar{y}_n] \) is associated.

The aim of the input design problem is to compute an input sequence for system (1) which allows the estimation of the parameters \( \theta \) with the best possible accuracy (i.e., minimum variance) while at the same time ensuring that the state and output variables remain sufficiently close to the considered equilibrium, so as to limit the overall effect of the input perturbation on the system.
In the following, the model of the measurement process defined by
\[ z(i) = y(iT_s) + v(i) \quad i = 1, 2, \ldots, N \]
will be adopted. In view of the above, the output sensitivities
with respect to the parameters can be computed as
\[ \frac{d}{dt} \left( \frac{\partial x}{\partial \theta_j} \right) = \frac{\partial f}{\partial x} \frac{\partial x}{\partial \theta_j} + \frac{\partial f}{\partial \theta_j} + \frac{\partial x(0)}{\partial \theta_j} = 0 \]
\[ \frac{\partial y}{\partial \theta_j} = \frac{\partial h}{\partial x} \frac{\partial x}{\partial \theta_j} + \frac{\partial h}{\partial \theta_j}, \quad j = 1, \ldots, p. \]
Defining the sensitivity matrix as
\[ S(i) = \left[ \frac{\partial y(i)}{\partial \theta} \right] = \begin{bmatrix} \frac{\partial y_1(i)}{\partial \theta_1} & \frac{\partial y_1(i)}{\partial \theta_2} & \cdots \\ \frac{\partial y_2(i)}{\partial \theta_1} & \frac{\partial y_2(i)}{\partial \theta_2} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}, \]
it is well known that the Fisher information matrix associated with the problem of estimating \( \theta \) under the above assumptions can be written as
\[ M = \sum_{i=1}^{N} S(i)^T R^{-1} S(i). \]
In the following, the cost function
\[ J = T^r \left[ M^{-1} \right] \]
for the experiment design problem will be considered, so that the problem can be concisely stated as the one of determining \( \hat{u} \) as the one of determining
\[ \hat{u} : \min_{\theta \in U} J, \]
subject to
\[ \xi_j^l \leq y_j(t) - \bar{y}_j \leq \xi_j^u \quad \forall t, \quad j = 1, 2, \ldots, n_o \]
\[ | u_k(t) - \bar{u}_k | \leq \mu_k \quad \forall t, \quad k = 1, 2, \ldots, n_i. \]
The problem statement is of course not complete unless the class \( U \) of considered input sequences is defined. In Section 3 the set of piece-wise constant inputs will be considered, while in Section 4 multisine inputs will be studied.

Finally, note that the above problem statement relies implicitly on the following assumptions.

**Definition 2.1. (Local structural identifiability).** Let \( \theta^o \in \mathbb{R}^p \), the model structure \( M(\theta) \) defined by (1) is said to be locally identifiable in \( \theta^o \) if \( \forall \theta_1, \theta_2 \) in the neighborhood of \( \theta^o \) it holds that \( M(\theta_1) = M(\theta_2) \Rightarrow \theta_1 = \theta_2. \)

**Assumption 1.** The model structure \( M(\theta) \) defined by (1) is locally identifiable.

**Assumption 2.** The constraints (3)-(4) allow the definition of an informative enough dataset (in the sense of Ljung (1999)) for the model structure defined by (1).

### 3. PIECE-WISE CONSTANT INPUTS

In this Section, the procedure for the optimal design of piece-wise constant inputs proposed in Jauberthie et al. (2006b,a) will be summarised, together with some modifications which have been introduced to improve the numerics of the optimisation process.

For the sake of clarity we will initially focus on scalar inputs; the extension to the case of multiple inputs is discussed in Section 3.4. The class of input signals of interest is given by
\[ u(t) = \pi + \sum_{k=1}^{r} (\mu \epsilon_k - \mu \epsilon_{k-1}) H(t - \tau_k), \quad \epsilon_0 = 0 \]
where \( \mu \) is defined in (4) and \( H \) is the Heaviside function:
\[ H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0. \end{cases} \]
The input sequence is therefore divided in \( r \) intervals (stages), each starting at time \( \tau_k \), with \( k = 1, 2, \ldots, r \), and has an overall duration of \( T \) seconds. \( u \) can only take one of three admissible values, namely \( \pi + \mu, \pi - \mu \) or \( \pi \), which correspond, respectively, to \( \epsilon_k = 1, \epsilon_k = -1 \) and \( \epsilon_k = 0 \). Parameter \( \pi \) is chosen such that
\[ 0 \leq \tau_1 < \tau_2 < \ldots < \tau_r \leq T. \]
The optimisation problem associated with the optimal design of the input within the considered class is solved in Jauberthie et al. (2006b,a) via a two step procedure. In the first step, the \( \epsilon_k \)s are used as optimisation variables, leading to a piece-wise constant optimal input \( \hat{u} \). This solution is subsequently refined in the second step, in order to obtain a differentiable input sequence.

### 3.1 First step

The first step of the optimisation procedure uses a dynamic programming approach in order to solve the design problem by considering \( r \) subproblems, one at a time. The optimal solution for the \( k \)-th subproblem will be used as a starting point to solve the subsequent one. As mentioned in Section 2, for practical purposes it is essential to include constraints on the system’s inputs and outputs in the design, in the form
\[ \bar{\pi} - \mu \leq u(t) \leq \bar{\pi} + \mu \quad \forall t. \]
and
\[ \bar{y}_j - \xi_j^l \leq y_j(t) \leq \bar{y}_j + \xi_j^u \quad \forall t, \quad j = 1, 2, \ldots, n_o. \]
At each time instant, the outputs must remain confined in the regions defined by (5) for the computed input sequence to be considered admissible. In order to enforce such constraints, the output space is discretised in "strips" of constant width. Denoting with \( u^k(t) \) an input (output) sequence up to the \( k \)-th stage, the algorithm operates iteratively over each stage applying all possible values of the input for the entire duration of a stage. Integrating the state equations of the system one can compute the response of the constrained outputs. In general, subject to the application of different input values the output will reach different "strips" of the corresponding output space; in order to reduce the dimensionality of the problem, at each stage the algorithm not only eliminates all non admissible solutions, but retains only one solution "per strip" by checking the corresponding values of the cost function. At each stage, the algorithm operates by iterating on each response recorded at the previous stage, and re-applying to the system all the retained input values. The cost function is associated with admissible solutions is updated using the formulas discussed in the following Section. Eventually, the algorithm terminates after \( r \) steps, returning the optimal solution \( \hat{u} \) associated with the optimal cost.
3.2 Computation of the cost function

In the framework of the above described algorithm, the cost function \(2\) can be computed as follows. Denoting with \(S(i) = \frac{\partial y(i)}{\partial \theta} \), define

\[
S_{k,j} = \begin{bmatrix}
S(i) \\
S(i+1) \\
\vdots \\
S(j)
\end{bmatrix} = \begin{bmatrix}
\frac{\partial y(i)}{\partial \theta} \\
\frac{\partial y(i+1)}{\partial \theta} \\
\vdots \\
\frac{\partial y(j)}{\partial \theta}
\end{bmatrix}
\]

and note that the Fisher information matrix can be written as

\[
M = \sum_{i=1}^{N} S(i)^T R^{-1} S(i) = S_{1,N}^T (I_N \otimes R^{-1}) S_{1,N}
\]

where \(\otimes\) denotes the Kronecker product, \(N\) is the number of samples of measured output and \(I_N\) denotes the \(N \times N\) identity matrix. Similarly, the Fisher information matrix for stage \(k\) can be written as

\[
M_k = \sum_{i=(k-1)L}^{kL} S(i)^T R^{-1} S(i) = S_{(k-1)L,kL}^T (I_L \otimes R^{-1}) S_{(k-1)L,kL}
\]

where \(L\) is the number of samples in each stage, i.e., \(L = N/r\). One then has that

\[
M = \sum_{k=1}^{n} M_k = M^n,
\]

where by \(M^n\) we denote the Fisher information matrix computed up to stage \(n\). It then follows that

\[
M^{k+1} = M^k + M_{k+1} = M^k + S_{kL,(k+1)L}^T (I_L \otimes R^{-1}) S_{kL,(k+1)L},
\]

Applying the matrix inversion lemma to (6) and recalling that

\[
(A \otimes B)^{-1} = A^{-1} \otimes B^{-1},
\]

one gets the recursion

\[
\Sigma^{k+1} = \Sigma^k - \Sigma^k S_{kL,(k+1)L}^T (I_L \otimes R) + S_{kL,(k+1)L} \Sigma^k S_{kL,(k+1)L}^T \Sigma^k
\]

with the initialisation

\[
\Sigma^1 = [S_{1,L}^T (I_L \otimes R^{-1}) S_{1,L}]^{-1},
\]

The above expression for the computation of the cost function has the advantage of requiring a matrix inversion at each stage instead of at each time instant. Re-applying the inversion lemma to (8) one gets

\[
\Sigma^{k+1} = \Sigma^k - \Sigma^k S_{kL,(k+1)L}^T R_L^{-1} S_{kL,(k+1)L} \\
(M^k + S_{kL,(k+1)L} R_L^{-1} S_{kL,(k+1)L})^{-1} S_{kL,(k+1)L} R_L^{-1}
\]

where \(R_L\) is defined as

\[
R_L = I_L \otimes R
\]

and in view of (7) it holds that

\[
R_L^{-1} = I_L \otimes R^{-1}
\]

Equation (9) has been used to compute the value of the cost function for each admissible input sequence.

3.3 Second step

The second step of the design procedure aims at determining a differentiable input sequence "sufficiently close" to the optimal one determined in the first step. To this purpose, \(u\) is approximated with

\[
u(t) = \pi + \sum_{j=1}^{r} \frac{a_j \xi_j - a_{j-1} \xi_{j-1}}{1 + e^{K(t_j - t)}}
\]

where \(K\) is a sufficiently large integer, constants \(\xi_j\) take the optimal values computed in the first step and \(a_j, t_j\) are the optimisation variables of the second step. The cost function is unchanged, while the following constraints are considered:

\[
|a_j| \leq \mu, \quad j = 1, \ldots, r,
\]

\[
t_{j+1} - t_j \geq D, \quad j = 1, 2, \ldots, r,
\]

\[
t_1 \geq 0,
\]

\[
t_r \leq T,
\]

where \(\mu\) is defined in (4) and \(D \in (0, 1]\) is a parameter which can be chosen to limit the shift of the time instants \(t_j\) with respect to the duration of a stage. Finally, note that constraints on the system outputs must be retained to ensure that perturbing the new decision variables does not lead to a non admissible solution.

3.4 Extension to the multiple input case

In order to simplify the multiple input problem, it is assumed that each input channel is excited separately in time. To this purpose, a vector of time instants associated with switches from one input channel to the following one is defined:

\[
t_{sw} = [t_{sw1}^2, t_{sw2}^2, \ldots, t_{sw1}^n]^{-1}.
\]

In the case \(n_i = 2\) (system with two inputs), \(t_{sw} = t_{sw1}^1\), the admissible input value for time instants before \(t_{sw}^1\) are given by

\[
\begin{bmatrix}
\pi_1 + \mu_1 \\
\pi_2
\end{bmatrix}, \begin{bmatrix}
\pi_1 \\
\pi_2
\end{bmatrix}, \begin{bmatrix}
\pi_1 - \mu_1 \\
\pi_2
\end{bmatrix}
\]

while for time instants after \(t_{sw}^1\) we have

\[
\begin{bmatrix}
\pi_1 \\
\pi_2 + \mu_2
\end{bmatrix}, \begin{bmatrix}
\pi_1 \\
\pi_2
\end{bmatrix}, \begin{bmatrix}
\pi_1 \\
\pi_2 - \mu_2
\end{bmatrix}
\]

The above expressions can be generalised to \(n_i > 2\).

4. MULTISINE INPUTS

A significant portion of the literature on input design is devoted to the case of multisine inputs. The attention, however, is mainly focused on the problem of choosing the relative phases of the individual harmonics so that suitable parameters of the signal, such as, e.g., the crest factor considered in Rivera et al. (2009)) are optimised, under the assumption that the user has specified the set of individual frequencies to be excited by the input. In this work, the goal will be to use both phases and amplitudes as design variables in an optimisation problem similar to the one discussed in the previous Section, taking into account the model dynamics and the constraints on the relevant input and output variables.

As in the previous Section, a scalar multisine input is first considered, which can be defined as

\[
u(k) = \lambda \sum_{i=1}^{N_i} \sqrt{2\alpha_i} \cos(\omega_i(kT_s + \phi_i)),
\]

where \(\lambda\) is the scaling factor, \(\alpha_i\) the amplitudes of individual frequencies, \(\omega_i\) the corresponding frequencies, \(T_s\) the sampling time, and \(\phi_i\) the relative phases.
where \( N_f \) is the number of harmonics in the signal, \( N \) is the number of samples, \( \omega_i, \alpha_i \) and \( \phi_i \) are, respectively, the angular frequency, the Fourier coefficient and the initial phase, \( \lambda \) is a scale factor and \( T_s \) is the sampling interval. The angular frequencies \( \omega_i \) are assumed to be equally spaced according to

\[
\omega_i = \frac{2\pi i}{NT_s}.
\]

(11)

The spectrum of the signal (10) is then given by

\[
\Phi(\omega_i) = \frac{\lambda^2 \epsilon_i N}{2}, \quad i = 1, 2, \ldots, N_f,
\]

and can be shaped by the designer by choosing suitable values for the parameters \( \lambda, \epsilon_i, N \) and \( N_f \). Denote with \( \omega_L \) and \( \omega_U \) an upper and a lower bound to the range of frequencies to be covered by the input spectrum \( \omega_L \leq \omega \leq \omega_U \). For each frequency in this range, the designer can specify suitable amplitudes to achieve the desired power distribution.

For the general case of a system with \( n_i \) inputs we will resort to the general form for a multiple input multistruct in Rivera et al. (2009):

\[
u_j(k) = \sum_{i=1}^{n_i(n_s+n_a)} \delta_{ji} \cos(\omega_i k T_s + \phi_j^\omega)+
+ \lambda_j \sum_{i=n_i(n_s+n_a)+1}^{n_i(n_s+n_a)+1} \sqrt{2} \alpha_{ji} \cos(\omega_i k T_s + \phi_j^\epsilon) +
+ \sum_{i=n_i(n_s+n_a)+n_a}^{n_i(n_s+n_a)+n_a} a_{ji} \cos(\omega_i k T_s + \phi_j^\alpha)
\]

(12)

where, in addition to the above defined quantities, \( n_s, n_a, n_a \) are, respectively, the number of harmonics at low, medium and high frequency, \( \delta_{ji}, \alpha_{ji}, a_{ji} \) are the Fourier coefficients of the harmonics at low, medium and high frequencies and \( \phi_j^\omega, \phi_j^\epsilon, \phi_j^\alpha \) are the corresponding phases. The grid of angular frequencies \( \omega_i \) is defined as in (11). Note that the above design input includes components with frequencies below \( \omega_L (n_s) \) and above \( \omega_U (n_a) \). The \( n_s \) low frequency components can be used to model low frequency components of the input due to the nominal operation of the system. The \( n_a \) high frequency components, on the other hand, cover the range between \( \omega_L \) and the Nyquist frequency; their number is related with number of samples in the input sequence by:

\[
\frac{N}{2} = n_i(n_s + n_a + n_a).
\]

In order to ensure that the \( n_s \) components of the input vector are uncorrelated, the following constraints between the Fourier coefficients of the harmonics forming the signal (12) are enforced:

\[
\delta_{ji} = \begin{cases} 
\neq 0 & i = j, n_i + j, \ldots, n_i + (n_s - 1) + j \\
0 & 1 \leq i \leq n_i n_s 
\end{cases}
\]

\[
\alpha_{ji} = \begin{cases} 
\neq 0 & i = n_i n_s + j, n_i (n_s + 1) + j, \ldots, n_i n_s + n_a - 1 + j \\
0 & 0 \leq n_s n_a \leq i \leq n_i (n_s + n_a) 
\end{cases}
\]

\[
a_{ji} = \begin{cases} 
\neq 0 & i = n_i (n_s + n_a) + j, \ldots, n_i (n_s + n_a - 1) + j \\
0 & n_i (n_s + n_a) \leq i \leq n_i (n_s + n_a + n_a) 
\end{cases}
\]

The experiment design method leaves to the user the choice of the following parameters:

- \( \omega_L, \omega_U \);
- the sampling period \( T_s \);
- the numbers of harmonics \( n_s \) and \( n_a \);
- the actual duration of the experiment \( N \) (in samples).

Rivera et al. (2009) provides guidelines for the choice of such parameters (omitted for brevity). The parameters left to the optimisation procedure, on the other hand, are the amplitudes and the phases of the harmonics forming each component of the input vector. Concerning the initialisation of the optimisation algorithm, uniform amplitudes can be considered, while the classical Schroedter formula can be used to initialise the phases (see Schroedter (1970)).

5. CASE STUDY: MODEL IDENTIFICATION FOR A QUADROTOR UAV

In this Section, the results obtained by applying the experiment design methods described in Sections 3-4 to the problem of estimating the parameters of a quadrotor UAV will be presented and discussed. More precisely, some background on the dynamics of quadrotor helicopters will be provided and the model considered for experiment design will be presented. Subsequently, the designed inputs and the corresponding parameter estimation results will be illustrated.

Quadrotor helicopters are a very popular architecture for the development of UAV platforms (see, e.g., Castillo et al. (2005) and the references therein), in view of their favorable dynamic characteristics (see Das et al. (2009)). More precisely, quadrotors are operated with the rotors running clockwise and counterclockwise in pairs, so that the net effect of gyroscopic effects and aerodynamic torques in trim conditions is minimised. The vertical motion is controlled by thrusting the four motors simultaneously, while pitch and roll are commanded by increasing/decreasing the speed of the rear/front and left/right motors. Finally, yaw control is obtained by combining the two above actions in a coordinated way.

The motion is usually represented in terms of the inertial coordinates \( x, y, z \) and of the center of mass in an inertial reference frame and of the Euler angles \( \psi, \theta, \phi \) representing the attitude of a suitably defined body frame (the principal inertia axes are usually adopted for the sake of simplicity). A general dynamic model for the 6 degrees of freedom (DOFs) of a quadrotor model can be summarised as follows, for, respectively, the linear dynamics

\[
\ddot{x} = (\cos \psi \cos \phi \sin \theta + \sin \psi \sin \phi) \frac{b}{m} U_1
\]

(13)

\[
\ddot{y} = (\sin \psi \cos \phi \sin \psi \sin \phi) \frac{b}{m} U_1
\]

(14)

\[
\ddot{z} = (\cos \theta \cos \phi) \frac{b}{m} U_1 - g,
\]

(15)

where \( m \) is the mass of the vehicle; the angular dynamics, expressed in terms of the body components \( p, q \) and \( r \) of the inertial angular rate

\[
\dot{\phi} = \frac{(I_y - I_z)}{I_x} q r + \frac{b l U_2 - J_m q}{I_x} \Omega_R
\]

(16)

\[
\dot{\psi} = \frac{(I_z - I_x)}{I_y} p r + \frac{b l U_3 + J_m p}{I_y} \Omega_R
\]

(17)

\[
\dot{\psi} = \frac{(I_x - I_y)}{I_z} p q + \frac{d}{I_z} U_4,
\]

(18)
where \( I_x, I_y, I_z \) are the principal moments of inertia, \( J_m \) is the moment of inertia of the individual rotor, \( l \) is the radius of the quadrotor and \( b \) is an aerodynamic parameter; the relation between \( p, q \) and \( r \) and the derivatives of the Euler angles
\[
\dot{\phi} = p \sin(\phi) \tan(\theta) q + \cos(\phi) \tan(\theta) r
\]
\[
\dot{\theta} = \cos(\phi) q - \sin(\phi) r
\]
\[
\dot{\psi} = \frac{\sin(\phi)}{\cos(\theta)} q + \frac{\cos(\phi)}{\cos(\theta)} r.
\]
In the above equations the control variables \( U_i, i = 1, \ldots, 4 \) are related with the angular rates \( \Omega_i, i = 1, \ldots, 4 \) of the four rotors by
\[
U_1 = \Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2
\]
\[
U_2 = \Omega_1^2 - \Omega_2^2
\]
\[
U_3 = \Omega_2^2 - \Omega_3^2
\]
\[
U_4 = \Omega_2^2 + \Omega_3^2 - \Omega_1^2 - \Omega_4^2,
\]
and \( \Omega_R = \Omega_1 + \Omega_3 - \Omega_2 - \Omega_4 \).

With specific reference to the parameter estimation problem at hand, assuming that \( m \) and \( l \) are known a priori with the necessary accuracy and noting that \( b \) can be estimated in a dedicated experiment on the vertical axis (see (15)), the goal is to estimate the principal moments of inertia \( I_x, I_y, I_z \) and the scalar parameter \( d \), so only three out of the four control inputs need to be excited, while \( U_1 \) is held at the constant value \( \bar{U}_1 \). Therefore, neglecting the gyroscopic terms in (16) and (17) (i.e., assuming that \( J_m \ll I_x, I_y \)) it is possible to focus on the angular dynamics only and use in input design the following model
\[
\dot{\phi} = p \sin(\phi) \tan(\theta) q + \cos(\phi) \tan(\theta) r
\]
\[
\dot{\theta} = \cos(\phi) q - \sin(\phi) r
\]
\[
\dot{\psi} = \frac{\sin(\phi)}{\cos(\theta)} q + \frac{\cos(\phi)}{\cos(\theta)} r.
\]
\[
\ddot{z} = - g + (\cos(\theta) \cos(\phi)) \frac{b}{m} \bar{U}_1
\]
\[
\dot{p} = \frac{I_y - I_z}{I_x} q r + \frac{lb}{I_x} U_2
\]
\[
\dot{q} = \frac{I_z - I_x}{I_y} p r + \frac{lb}{I_y} U_3
\]
\[
\dot{r} = \frac{I_y - I_x}{I_z} p q + \frac{d}{I_z} U_4
\]
subject to the constraints \( |\phi| < 0.35 \) rad, \( |\theta| < 0.35 \) rad, \( |\psi| < 0.35 \) rad, \( |\ddot{z}| < 1 \) m/s², which have been defined in order to maintain the quadrotor sufficiently close to the considered trim condition (hover). Note, in particular, that the output \( \ddot{z} \) is not informative as far as the parameters of interest are concerned, but is only retained in the design of the experiments in order to enforce a constraint on the maximum vertical acceleration.

The input design methods outlined in Sections 3-4 have been applied to the quadrotor model presented in the previous Section. In particular, both a piece-wise constant input and a multi-sinusoidal one have been designed and their performance in providing informative data has been tested in simulated experiments. For the design of the multisine input, the angular frequency range of interest has been chosen as \( \omega_L = 2 \) rad/s, \( \omega_U = 10 \) rad/s, the sampling period as \( T_s = 0.063s \) and the numbers of harmonics as \( n_s = 6 \) and \( n_a = 0 \). All the considered input sequences have a duration \( T = 9.45 \) s, derived from the guidelines in Rivera et al. (2009) as function of the range of angular frequencies of interest. The optimal piece-wise constant input has been designed by dividing the above duration in \( r = 15 \) stages.

The optimal piece-wise constant input and the optimal multisine are illustrated, respectively, in Figures 1 and 2. For the sake of comparison, a set of three linear frequency sweeps covering the angular frequency range from \( \omega_L = 2 \) to \( \omega_U = 10 \) have also been designed (see Figure 3).
TABLE 2. Parameter estimates and percentage estimation errors obtained for each of the considered inputs.

<table>
<thead>
<tr>
<th></th>
<th>True values</th>
<th>( d )</th>
<th>( I_x )</th>
<th>( I_y )</th>
<th>( I_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweep</td>
<td>1.045 \cdot 10^{-5}</td>
<td>4.75 \cdot 10^{-3}</td>
<td>4.75 \cdot 10^{-3}</td>
<td>8.56 \cdot 10^{-5}</td>
<td></td>
</tr>
<tr>
<td>Multisine</td>
<td>1.110 \cdot 10^{-5}</td>
<td>5.003 \cdot 10^{-3}</td>
<td>4.945 \cdot 10^{-3}</td>
<td>8.969 \cdot 10^{-5}</td>
<td></td>
</tr>
<tr>
<td>Piecewise</td>
<td>1.033 \cdot 10^{-5}</td>
<td>4.844 \cdot 10^{-3}</td>
<td>4.821 \cdot 10^{-3}</td>
<td>8.734 \cdot 10^{-5}</td>
<td></td>
</tr>
</tbody>
</table>

6. CONCLUDING REMARKS

The problem of defining optimal input sequences for MIMO model identification has been considered. The proposed method builds on results available in the literature (see, e.g., (Jauberthie et al 2006) and (Rivera et al 2009)) and allows the optimisation of suitable scalar functions of the Fisher information matrix by means of a computationally sound procedure for the design of both step-wise and multi-cyclic MIMO inputs. The proposed procedure has been evaluated by designing MIMO experiments to estimate the parameters of a physical model for a quadrotor helicopter platform.