Robust Tracking Control Of Uncertain Dynamic Nonlinear Systems
Via Type-2 Fuzzy Sliding Mode Approach

A. Al-khazraji*, A. Hamzaoui.

* University of Reims Champagne Ardenne, Centre de Recherche en STIC, IUT de Troyes, France, (e-mails: ayman.hussain@univ-reims.fr, abdelaziz.hamzaoui@univ-reims.fr).

Abstract: This paper presents a novel sliding mode control algorithm for an nth order nonlinear system suffering from parameters uncertainty and subjected to external perturbation. Using the proposed approach, the system will be on the sliding surface at the initial time (t=0). The idea is to eliminate the reaching phase and hence, a considerable amelioration of system robustness can be achieved. Moreover, two type-2 fuzzy systems is introduced to construct the indirect adaptive controller. The problems related to adaptive fuzzy controllers like singularity is resolved with the guarantee that all signals of closed-loop system are uniformly ultimately bounded. The proposed controller has many advantages, such as satisfactory control performance under a wide range of operating conditions and parameter variations, a faster response than conventional controller for nonlinear systems and suppressed chattering phenomenon. A simulation example is considered to confirm the efficiency, excellent performance, and robustness of the proposed approach.

1. INTRODUCTION

Sliding mode control (SMC) has been recognized as a robust and efficient control method for complex, high order or nonlinear dynamical systems. The principal goal of the SMC technique is to force the system state to follow a certain prescribed manifold, known as the sliding surface. Once the manifold is reached, the system is forced to remain on it thereafter while converging to the origin of error plane. The main drawback of the SMC is the requirement of a discontinuous control across the sliding manifold. In practical systems, this leads to a phenomenon termed as chattering (Hussain et al., 2007), (Hussain et al., 2008). Chattering involves high-frequency control switching and may lead to excitation of unmodelled high-frequency system dynamics. Smoothing techniques such as boundary layer (Lei et al., 2010) have been employed in order to prevent chattering. However, such approach leads to a loss of asymptotic stability and a controller that can guarantee final tracking accuracy only to a certain vicinity of the demand.

From the other hand, fuzzy logic has been known to be an efficient tool for controlling ill-defined or parameter-variant plants. By generalizing fuzzy rules, a fuzzy logic controller (FLC) can cope well with severe uncertainties (Hussain et al., 2011). These controllers have been used in a wide variety of applications in engineering, such as: aircraft/spacescraft; automated highway systems; autonomous vehicles; washing machines; process control; robotics control and decision-support systems.

A new generation, called type-2 fuzzy logic system (T2FLS), has started to acquire a remarkable attention in term of uncertainties exploiting. It has been shown that T2FLS has the potential to outperform the classical type-1 FLS (T1FLS) (Ezziani et al., 2008), (Al-khazraji et al., 2010). In spite of these applications, the FLC can’t ensure alone the robustness against external perturbation and high parametric variations.

Over the last decade, the apparent similarities between the sliding mode and fuzzy controllers in diagonal form have been noticed. This fact has subsequently motivated considerable research effort in combining the two topologies (Manceur et al., 2010a) in a manner that serves to reduce the limitations of the sliding mode, while still maintaining the guarantees of global uniform stability (Manceur et al., 2010b).

In this paper, we propose a robust tracking control of uncertain dynamic nonlinear systems via type-2 fuzzy sliding mode method. This approach tries to combine the flexibility and the intelligence of fuzzy logic with the efficiency of SMC. The main contributions presented in this work are: (i) the reaching phase of SMC is eliminated (ii) the switching function of SMC is replaced by a smooth nonlinear function to get a soft control signal rather than hard fluctuation between positive and negative values, (iii) the resulting closed-loop system is proved to be uniformly ultimately bounded and the tracking error can converge to the arbitrarily small neighbourhood of the origin, and (iv) the used mathematical development permits to avoid the possible controller singularity problem and to overcome the control gain constraints encountered in the classical adaptive fuzzy control laws.

The remaining part of this paper is organized as follows: Section 2 is dedicated to the formulation and the investigation of the single input single output (SISO) system tracking control problem. In section 3, we present the synthesis of the proposed controller. The efficiency of the proposed approach is shown in section 4 using a simulation example. Finally, section 5 concludes the paper.
2. PROBLEM STATEMENT

Consider the following $n$th order non-linear SISO system, described by:

$$
\begin{align*}
\dot{x}^{(n)} &= f(x) + g(x)u + d \\
y &= x
\end{align*}
$$

(1)

where $f(x)$ and $g(x)$ are non-linear, uncertain, continuous and bounded functions, $x = [\dot{x}, \ddot{x}, \ldots, x^{(n-1)}]^T$ is the state vector, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ represent the input and output of the plant respectively, and $d$ represents the unknown external disturbances assumed to be bounded and verifies $|d| \leq a_d$ where $a_d$ is a positive constant. The elements of $x$ are assumed to be available for measurement. It is also assumed that the system remains in the forced region, and therefore $g(x) \neq 0$. In this work, we assume that $g(x) > 0$. However, the analysis can be adapted easily for $g(x) < 0$.

Accordingly, there is a positive unknown constant $b$ verifies $g(x) \geq b > 0$. It should be noted that the parameter $b$ is required only for the mathematical analysis while its true value is not needed neither for the simulation nor for the real time implementation of the controller.

The control objective is to synthesize and implement a fuzzy robust controller to force the system output to track a given bounded desired reference trajectory $y_d$, i.e., the tracking error $e = y_d - y$ must converge toward zero.

We also assume that the $i^{th}$ derivative of the desired reference trajectory $(y_d^{(i)})_{i=1,\ldots,n}$ is bounded. Consequently, there is a positive constant $\kappa$ such that

$$
\left|y_d^{(i)}\right| \leq \kappa.
$$

Before using SMC, we remind that the sliding process includes two phases: the first is the approaching phase, when the sliding surface is non-zero value ($S \neq 0$), and the second is the sliding phase when the sliding surface equal to zero ($S = 0$). To ensure the robustness of the closed loop system, we consider the following sliding surface:

$$
S = -\lambda_i e - \lambda_{i-1}e - \ldots - \lambda_{2}e(2) - \lambda_{1}e(1) = e(n-1) + \sum_{i=1}^{n-1} \lambda_i e(i-1)
$$

(2)

where the coefficients $\lambda_i$ are chosen such as all the roots of the corresponding polynomial are located in the open left half plane. The classical sliding-mode approach can be given in the following control law (Al-khazraji et al., 2010):

$$
u = u_{eq} - g^{-1}(x)u,
$$

(3)

where

$$
u_{eq} = g^{-1}(x)\left[-f(x) + y_d^{(i)} + \sum_{i=1}^{n-1} \lambda_i e(i-1)\right]
$$

(4)

$$u_{eq} = K_d \text{sign}(S)
$$

(5)

The controller in (3) contains different constraints such as:

1) the knowledge of both $f(x)$ and $g(x)$ functions which is not always possible.
2) the presence of the unknown perturbation makes the calculation of the scalar $K_d$ in (5) difficult.
3) the use of the sign function in (5) provokes the chattering phenomena which can damage the system.
4) the sliding surface given in (2) produces two major disadvantages:

i). during the approaching phase, the system is sensitive to uncertainties and disturbances which may provoke the phenomenon of chattering in the neighbourhood of the surface.

ii). choosing the values of $\lambda_i$ to get a fast system response implies significant initial control energy during the transient response; however, very fast response can lead to overshoot or even system instability.

The proposed approach has the objective to resolve these problems.

3. PROPOSED APPROACH

The idea of this work is to find an expression that allows us to put the system on the sliding surface at the initial time ($t=0$) which can be satisfied using the following sliding surface:

$$
S = e(n-1) + \sum_{i=1}^{n-1} \lambda_i e(i-1) - \tau
$$

(6)

where $\tau = e^{-\tau}(e(n-1)(0) + \sum_{i=1}^{n-1} \lambda_i e(i-1)(0))$

The effect of elimination the reaching phase has been demonstrated practically in (Al-khazraji et al., 2010).

The time derivative of (6) can be rewritten as:

$$
\dot{S} = e(n-1) + \sum_{i=1}^{n-1} \lambda_i e(i) + \tau
$$

(7)

Accordingly, the proposed surface eliminates the approaching phase and replaces the disadvantages mentioned previously by two contributions

1. The solution of system sensitivity problem and fragile robustness during the approaching phase, since such phase is removed.
2. The reduction of convergence time as we have only the sliding phase.

Section 2 has shown that control law in (4) guarantees Stability and robustness of the closed-loop system if the nominal model of the plant is well known. If this is not the case, it is proposed to approximate the unknown functions $f(x)$ and $g(x)$ by two bounded adaptive type-2 fuzzy logic systems (T2FLS). The T2FLS output can be easily bounded using saturation function. According to the descriptions found in (Al-khazraji et al., 2010), the approximators will be in the following form:
\[ \hat{f}(x) = \hat{\phi}_f^T \psi_f, \]  
(8)

\[ \hat{g}(x) = \hat{\phi}_g^T \psi_g, \]  
(9)

Hence, the control law in (3) and (4) become respectively

\[ u = u_{eq} - \hat{g}^{-1}(x)u_{re}, \]  
(10)

\[ u_{eq} = \hat{g}^{-1}(x) \left[ -\hat{f}(x) + \gamma(x) + \sum_{i=1}^{d} \lambda_i e^{(i)} \right], \]  
(11)

To avoid discontinuities occurring with the control signal in (5), a third bounded T2FLS has been added to replace the \( K_f \) by an adaptive gain. Furthermore, the sign term is modified to get a soft variation when \( S = 0 \) thanks to the hyperbolic tangent function as illustrated in (12) and figure (1).

\[ u_{re} = \hat{K}_{fuzz} \tanh(S) \]  
(12)

![Graphical representation of hyperbolic tangent](image)

At the moment, most of the proposed approaches are based on increasing the gains until a satisfactory behaviour is achieved. Only few consider the possibility of decreasing the gains as the uncertainty reduce. It has recently well clarified that the chattering phenomenon is not due to the discontinuous control only but to the unmodelled dynamics of actuators and sensors (Boiko et al., 2007). Therefore the use of a high gain control instead of the sign function requires that the disturbance vanishes on the sliding surface, otherwise chattering can appear as well, and unpredictably, because of uncertainty. Therefore, the main idea behind the design of \( K_{fuzz} \) is that when the system state run away from the sliding surface \( (S \dot{S} > 0) \) for a large value of \( \left| S \right| \), controller gain should be increased in order to return the state toward the surface. In contrast, when the system state approaches the sliding surface \( (S \dot{S} < 0) \) for a large value of \( \left| S \right| \), controller gain should be decreased to avoid chattering. The gain \( K_{fuzz} \) is a strictly positive value obtained from T2FLS (13).

\[ \hat{K}_{fuzz}(S, \dot{S}) = \hat{\phi}_k^T \psi_k \]  
(13)

The sliding surface input is described using three fuzzy sets (N, Z, P) while its derivative is described using five membership functions (NB, NS, Z, PS, PB). For the output variable, we consider three fuzzy sets (S, M, B) as shown in table (1).

<table>
<thead>
<tr>
<th>S ( \Delta S )</th>
<th>NB</th>
<th>NS</th>
<th>Z</th>
<th>PS</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>B</td>
<td>B</td>
<td>M</td>
<td>S</td>
<td>M</td>
</tr>
<tr>
<td>Z</td>
<td>B</td>
<td>M</td>
<td>S</td>
<td>M</td>
<td>B</td>
</tr>
<tr>
<td>P</td>
<td>M</td>
<td>S</td>
<td>M</td>
<td>B</td>
<td>B</td>
</tr>
</tbody>
</table>

Using (10) to calculate \( y^{(m)} \) and (1) to calculate \( y^{(n)} \), (7) can be written as follows:

\[ \dot{S} = (\hat{f} - f)(\hat{g} - g)u - \hat{K}_{fuzz} \tanh(S) - d + \tau \]  
(14)

By adding and subtracting the optimal values of the fuzzy systems \( (\hat{f}^*, \hat{g}^* \text{ and } \hat{K}_{fuzz}^*) \) which correspond to the minimum approximation error, and after some manipulations, we can obtain the following equation:

\[ \dot{S} = -f - \hat{g}u - \hat{K}_{fuzz} \tanh(S) + \epsilon \]  
(15)

where

\[ \hat{f} = -f - \hat{g} = -\hat{\phi}_f^T \psi_f = \hat{\phi}_f^T \psi_f, \]  
\[ \hat{g} = -(\hat{g} - g)u = -\hat{\phi}_g^T \psi_g + u = \hat{\phi}_g^T \psi_g + u \]  
\[ \hat{K}_{fuzz} = -\hat{K}_{fuzz}^* \]  
\[ \epsilon = -d - (\hat{f} - f^*) - (g - g^*)u + \tau \]

According to the universal approximation theorem, it has been proven that linear combinations of the fuzzy basis functions are capable of uniformly approximating any real continuous function on a compact set to arbitrary accuracy (Wang et al., 1992). Thus, \( (f - f^*) \) and \( (g - g^*)u \) is bounded since the control signal is bounded physically and mathematically as it consists of bounded signals (11-12). On the other hand, the term \( \tau \) converges to zero as \( t \) converges to infinity. Thus the term \( \epsilon \) is also bounded.

To study the stability of the overall system using the proposed approach, we will consider the following Lyapunov function:

\[ V = \frac{1}{2} S^2 + \frac{1}{2\gamma_f} \psi_f^T \hat{\psi}_f + \frac{1}{2\gamma_g} \psi_g^T \hat{\psi}_g + \frac{1}{2\gamma_k} \psi_k^T \hat{\psi}_k \]  
(16)

where \( \gamma_f, \gamma_g \) and \( \gamma_k \) are positive constants representing the learning rates of the conclusion part of \( \hat{f}, \hat{g} \) and \( \hat{K}_{fuzz} \) in the T2FLS fuzzy systems respectively.

The time derivative of \( V \) will be as follows:

\[ \dot{V} = SS + \frac{1}{\gamma_f} \psi_f^T \hat{\psi}_f + \frac{1}{\gamma_g} \psi_g^T \hat{\psi}_g + \frac{1}{\gamma_k} \psi_k^T \hat{\psi}_k \]  
(17)

Substituting (14) in (17) and after replacing \( f^* \), \( \hat{g} \) and \( \hat{K}_{fuzz} \) by their equivalent in we \( \hat{S}(x, t) \) can get.

\[ \dot{V} = S(e - \hat{K}_{fuzz}^* \tanh(S)) - \frac{1}{\gamma_f} \left( \psi_f^T + \gamma_f S \hat{\phi}_f^T \hat{\psi}_f \right) \]  
\[ - \frac{1}{\gamma_g} \left( \psi_g^T + \gamma_g S \hat{\phi}_g^T \hat{\psi}_g \right) \]  
\[ - \frac{1}{\gamma_k} \left( \psi_k^T + \gamma_k S \hat{\phi}_k^T \hat{\psi}_k \right) \]  
(18)
By choosing the following adaptation laws:

\[
\begin{align*}
\dot{\hat{f}} &= -\gamma_f S \hat{f}^T, \\
\dot{\hat{g}} &= -\gamma_g S \hat{g}^T, \\
\dot{\gamma} &= -\gamma_k S \hat{g}^T \tanh(S)
\end{align*}
\]

(19)

the derivative of the Lyapunov function becomes:

\[
\dot{V} = S \left( \varepsilon - \hat{K}_{fuzz}^T \tanh(S) \right)
\]

(20)

If we choose the optimal value of \( \hat{K}_{fuzz} \) to be greater or equal to maximum value of \( \varepsilon \) such that \( \hat{K}_{fuzz}^T \geq \| \varepsilon \|_{\max} \), we can rewrite (20) as follows:

\[
\dot{V} \leq \| \varepsilon \| - \| \hat{K}_{fuzz}^T \|.
\]

(21)

The \( \tanh(S) \) function has two roles: i) providing a soft transition instead of hard one in the classical sign function, and ii) to scale the sliding gain when \( S \) has a small value. Such controller allows decreasing the gain if and only if the uncertainty is reduced. This concept is true also for conclusion part updating of the sliding gain \( \hat{f}, \hat{g} \) in (19).

Consequently, equation (21) implies that \( \dot{V} \leq 0 \) and hence, the sliding surface is attractive; that is, when the system is on this surface, it remains there as demonstrated in (Utkin, et al., 1977). In this case, \( S = 0 \), and the system is governed by the equivalent control \( u_\alpha \). Since the gains \( \lambda_i \) used to determine \( u_\alpha \) are calculated so that the Hurwitz criterion of stability is satisfied, the origin of the sliding surface \( e = e^{(1)} = \cdots = e^{(n-1)} = 0 \) is asymptotically stable (Utkin, et al., 1977); i.e. the tracking error converges towards zero in a finite time.

4. SIMULATION EXAMPLE

This section illustrates the robust tracking of the proposed controller for the mass-spring-damper system depicted in figure (2). The dynamic system can be described with the following equation:

\[
y' = u - f_k(x) - f_B(x) - f_C(x) + d
\]

(22)

where \( y \) designates the generalized coordinate, \( x = [y, y']^T \), \( f_k(x) = 2y \) denotes the spring force, \( f_B(x) = 2y \) denotes the damping force, \( f_C(x) = 0.1x\sin(y) \) is the friction force and \( d \) is external disturbance.

The learning rate should allow each T2FLS to comfortably the universe of discourse for \( \hat{f} \) and \( \hat{g} \) approximators. For \( \hat{K}_{fuzz} \), we can consider the double of \( \hat{f} \) interval; [0, 5]. The sliding parameter is chosen to satisfy (2) and to make a trade-off between convergence speed and sharpness of applied controller. Here, we choose that \( \lambda_i = 5 \). The constants \( \gamma_f, \gamma_g \) and \( \gamma_k \) are chosen \( \gamma_f = 50, \gamma_g = 1 \) and \( \gamma_k = 10 \). The learning rate should allow each T2FLS to be modified according to its maximum participation in the control law equation. The adjustable parameter vectors are initialized to small random values.

Figures (3) and (4) show the simulation results obtained for external disturbance \( d = 0.25\sin(3t) \).

The reference trajectory is divided into three parts:

\[
y_d = \sin(t) \quad \text{for the interval } [0, 9.425] \text{ second}.
\]

\[
y_d = -\sin(2t) \quad \text{for the interval } [9.425, 17.32] \text{ second}.
\]

\[
y_d = -(\pi/30)\sin(t) \quad \text{for the interval } [17.32, 30] \text{ second}.
\]

The initial angular position and velocity are chosen to be 0.5 and zero respectively. We remark that the system attains the reference trajectories, figure (3)-(4), even when the initial conditions are far from the reference ones. The short response time for the three different trajectories (amplitude/frequency) is obtained thanks to the good performance of T2FS.

Figure (4) also shows that the tracking error for angular position and velocity converges rapidly toward zero for both cases. The perfect sliding process is noticeable such that once the error (in position or velocity) attains zero, it stays there and if we modify suddenly the desired trajectory then each T2FLS will be adapted as in (18) to rectify this effect and eventually error values will again converge to zero.

Figure (5) also depicts the control action calculated using the proposed method in which the chattering phenomenon and
the abrupt variations appearing in classical sliding mode control are eliminated.

Comparing our work to the one presented in (Lin et al., 2008), we can state that the proposed method allows better performance with a guaranteed robustness due to the use of sliding mode control. Furthermore, the variable sliding mode gain allows reducing the controller energy and, hence, simplifies the real-time implementation.

5. CONCLUSION

The effectiveness the proposed approach (fast and robust tracking, small error, chattering free controller) is well confirmed in the obtained simulation results. To improve current work, the authors intend to find a formal method to optimize the selection of $tanh(S)$ function slope in (12), the exponential decay rate in (6), and $\lambda$ in (2). Such optimization will be useful for real time implementation that we aim to carry out later.

6. ACKNOWLEDGEMENT

This work is developed for the project CPER-MOSYP and supported by both the Champagne-Adrdenne Region and the European found FEDER.

7. REFERENCES


Dr. A. Al-khazraji would like to notify that A. Hussain is his old surname in reference publications.