A model for performance evaluation and sensitivity analysis of seaport container terminals

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Abstract: In the last decades, the growth of the containerised freight demand has led to a fast development of seaport container terminals. As an evidence of the relevance and of the interest that such intermodal terminals have reached, consider the significant amount of existing scientific works facing the problem of optimising their performances at operative, tactic, and strategic levels. In this framework, the present paper provides a model to face the problem of finding the best typology, and number, of resources for a given seaport container terminal. Evidently, on one hand, such problem is subject to “static” constraints due, for instance, to budget, manpower and space limitations, and on the other hand, to “dynamic” constraints that rise when the sequence of operations in the terminal, and the relevant timing, are taken into account. The paper is organised as follows: after a brief bibliography review, the proposed PN and (max,+)-algebra models are introduced. Then, some simulation results and the relevant sensitivity analysis are discussed.

Keywords: Petri nets, (max,+)-algebra, container terminal performances.

1. INTRODUCTION

In the last decades, the growth of the containerised freight demand has led to a fast development of seaport container terminals. In this framework, several scientific works and projects have faced the problem of optimising the performances of container terminals at

(1) tactic and strategic level, where decisions that will influence the terminal performances on the middle and long time horizon are taken;
(2) operative level, where decisions on what to do in the immediate future are taken.

Problems of the first kind rise because, due to the changes in the world global market, seaport container terminals need keeping the handling equipment efficient and technologically advanced. In this framework, terminal managers have to develop and to optimise different aspects of terminals, such as berth and yard sizing, as well as to choose the handling systems that better suit the specific characteristics of the terminals themselves, from both the points of view of the handling capacity (number of containers per hour) and of the investment and management costs. On the other hand, problems of the second kind face the optimal allocation of the available resources, in terms of manpower and handling equipment, knowing the freight demand in the immediate future, or whenever something unexpected occurs.

Therefore, the above problems are complicated by the fact that container terminals are, among the different intermodal systems, those that probably are more subject to the need to continuously adapt the existing plants with respect to the increasingly efficient technologies available by the market. In other words, terminal managers should be able to choose, among the very large number of handling systems and relevant providers, those that best suits their real needs.

In this framework, the present paper provides a model allowing to face the problem of finding the best kind and number of resources in a container terminal by means of a simple sensitivity analysis. In addition, the long term target of the present work consists of the definition of an optimisation procedure for investments decision making in container seaports taking into account, on one hand, the “static” constraints due to budget, manpower and space limitations, and, on the other hand, the “dynamic” constraints that rise when the sequence of operations in the terminal, and the relevant timing, are considered.

For what concerns the proposed modelling approach, Petri nets and (max,+)-algebra are used in this work to provide an easy-to-build modular model, on one side, and an analytical tool for determining the performances of the terminal in different configurations, on the other side.

The paper is organised as follows: after a brief bibliography review, the PN and (max,+)-algebra models are introduced. Then the modular representation of terminal handling cycles is described. Finally, some simulation results and the relevant sensitivity analysis are discussed.

1.1 Bibliography review

As previously mentioned, the performed bibliographic review showed that, in recent years, the optimisation of various aspects of the seaport terminals has been a central theme for several studies belonging to different scientific areas. A substantial number of articles consider the so-called “operation planning” problems. In this framework, one of the main problems is related to the need of finding
the optimal allocation of the terminal resources given the freight traffic flow and the set of the available resources (Rodrigo et al. (2002)), or of reducing the number of container handling and of resources to be used in the terminal via an optimal handling schedule (Kim (1997); Kim and Bae (1998)).

Other works face, on the contrary, management problems that refer to longer horizons, such as the best definition of the terminal areas (Kim and Kim (1999, 2002); Gambardella et al. (2001); Imai et al. (2007)).

For what concerns the modelling formalism, Discrete Event Systems (DES) theory (Cassandras and Lafortune (2008)) appears to be a natural approach for describing the operations inside terminals, especially for simulation aims (Sha (2008)). In particular, Petri Nets (Murata (1989)) has been often applied to analyse the behaviour of container terminal both in terms of performance evaluation (Degano and Di Febbraro (2001); Hailin Zhang and Zhibin Jiang (2006); Maione and Ottomanelli (2005); Di Febbraro et al. (2006)), and of fault detection (Degano et al. (2001)).

2. THE PROPOSED MODEL

The considered modelling approach consists of defining a discrete event model by means of Petri Nets (PNs), which allow to easily build modular terminal models, and by means of \((\max, +)\)-algebra (Heidergott (2007)), which provides a complete “linear framework” for analytically analysing the systems represented via a certain class of PNs.

Then, in the following sections, the basic definitions of PN and \((\max, +)\)-algebra will be recalled, although the reader may refer to the above cited references for more detailed descriptions.

2.1 Basics on Petri Nets and Timed Marked Graphs

In this section, the basic characteristics of PNs are introduced. In particular, the definition of the so-called Deterministic Timed Petri Net (DTPN) is here given.

Then, a DTPN is a 6-uple

\[
\text{DTPN} = \{P, T, \text{Pre}, \text{Post}, M(0), \Theta\}
\]

(1)

where \(P\) is a finite non-empty set of \(n\) places, and \(T\) is a finite non-empty set of \(m\) transitions. Then, \(\text{Pre} \in \mathbb{N}^{n \times m}\) is the so-called pre-incidence matrix whose generic element \(\text{Pre}_{i,j}\) is equal to the number of arcs joining \(p_i\) and \(t_j\), whereas the \(\text{Post} \in \mathbb{N}^{n \times m}\) is the so-called post-incidence matrix whose generic element \(\text{Post}_{i,j}\) is equal to the number of arcs joining \(t_j\) and \(p_i\). Finally, the vector \(\Theta = \{\theta_1, \theta_2, \ldots, \theta_j\}\) gathers the deterministic delays \(\theta_j\), \(\forall j \in T\), representing the time elapsed between the enabling and the firing of transition \(t_j\), \(\forall j \in T\).

For what concerns the state of the system, it is represented by the marking vector \(M(k) \in \mathbb{N}^n\), whose generic elements \(M_i(k)\), \(i = 1, 2, \ldots, m\), give the number of “tokens” in the places \(p_i\), after the occurrence of the \(k\)-th event.

As regards the graphical representation of PNs, places are drawn as circles, transitions are drawn as black boxes, and, finally, arcs are represented by arrows.

Furthermore, a Timed Marked Graph (TMG) is a PN in which there are, at most, a single input arc and a single output arc entering and exiting any place \(p_i \in P\) of the net. Such a class of PNs, which will be used in the following sections for modelling the terminal cycles, results to be very interesting since it allows to write a set of linear \((\max, +)\)-algebra equations, providing the firing times of all the relevant transitions, thus permitting quick analytical analyses of the properties of modelled system.

2.2 Basics on \((\max, +)\)-algebra

The \((\max, +)\)-algebra formalism is a modelling approach resulting from the observation that the firing times of any transition of a TMG can be computed simply by means of operations of addition and maximisation. Then, the basis of this formalism is the algebraic semi-ring \(\{\mathbb{R}, \oplus, \otimes\}\), where \(\mathbb{R} = \mathbb{R}_{\geq 0} \cup \{-\infty\}\) indicates the set of real non-negative numbers together with \(-\infty\), whereas the two operators \(\oplus\) and \(\otimes\) determine the binary operations of maximization and an sum, that is,

\[
x \oplus y = \max\{x, y\}
\]

(2)

\[
x \otimes y = x + y.
\]

(3)

The success that the \((\max, +)\)-algebra has earned is due to the clear similarities between linear algebra in the classic systems theory and \((\max, +)\)-algebra in the DES theory. In particular, it has been shown that many results on linear algebra still hold, with an adequate meaning transposition, in terms of \((\max, +)\)-algebra. For instance, by means of \((\max, +)\)-algebra it is possible to write the state equation

\[
\tau(k + 1) = A \otimes \tau(k)
\]

(4)

where the vector \(\tau(k) = [\tau_1(k), \tau_2(k), \ldots, \tau_m(k)]^T\) gathers the \(k\)-th firing times of all the transitions \(t_j \in T\), whereas the matrix \(A \in \mathbb{R}^{m \times m}\) is a constant matrix gathering appropriate combinations of the elements of the vector \(\Theta\). In addition, eigenvalues of matrix \(A\) still play equally fundamental. In fact, the performances of a generic DES represented via a \((\max, +)\)-algebra model only depend on the matrix \(A\), and can be analysed by computing the relevant (unique) eigenvalue. Then, the problem is to establish whether there are a scalar \(\kappa\) and a vector \(v\) such that:

\[
Av = \kappa v.
\]

(5)

where the eigenvalue \(\kappa\) represents the period of the system once it has reached a steady state. In other words, \(\kappa\) indicates how quickly the system “visits” the same state in a steady state, being this time in general greater than the interval between two successive fires of each transition in a steady state. Therefore, from a computational point of view, the value of the eigenvalue \(\kappa\) can be simply computed as

\[
\kappa = \bigoplus_{j=1}^m (\text{tr}A^j)^{1/j},
\]

where \(\text{tr}A^j\) is the trace of the matrix \(A^j\), computed in the sense of \((\max, +)\)-algebra. Evidently, when the eigenvalue \(\kappa\) is small, the relevant modelled system quickly repeats the same operations.
3. THE “SHIP TO YARD” CYCLE MODEL

In this section, the model of the considered “ship to yard” import cycle is described. Then, the considered containers movements considered in such a cycle are:

1. the loading/unloading of container to/from ships, trucks, and trains, and
2. the transfer of containers between different areas of the terminal,

which can be performed using different kinds of handling systems, such as:

- portainers;
- trailers;
- straddle carriers;
- Rail Mounted Gantry (RMG);
- etc.

As regards the Petri net model of the considered cycle, it consists of representation of the sequence of operations starting with the unloading of containers from the ship, until their storage in the stocking area. In particular, the considered operations are

1. the unloading of containers from the ship, by means of a portainer;
2. the transfer of containers from the quay to the yard, by means of a trailer;
3. the storage of containers in the yard, by means of a straddle carrier;
4. the handling system “repositioning”, after the completion of an operation, so as to be ready to begin another operation.

Note that some of the above operations requires the synchronisation, that is the contemporary presence, of the handling system and of a container. This is the reason why they will be represented in the PN model via transitions, as described in the following.

3.1 The PN model

Consider the PN model reported in Fig. 1, where places and transitions have the meaning reported in Tab. 1 and Tab. 2, respectively, and representing the sequence of operations characterising the import cycle of container to be unloaded from a ship and stored in the yard. As above mentioned, it is worth noting the presence of three synchronisations representing the need of the contemporary availability of the portainer and of the container to be unloaded from the ship ($t_1$), of the trailer and of the container to be transferred to the yard ($t_2$), and, finally, of the straddle carrier and of the container to be stored in the yard ($t_4$), respectively.

Note that the model in Fig. 1 may also represent the “yard to train” sequence of operations, starting with the picking up of a container in the yard, and ending with the its placing on the train. Such an “alternative” meaning of the PN in Fig. 1 holds when of places and transitions have the meanings reported in Tab. 3, and Tab. 4, respectively. Nevertheless, the model structure remains the same. Moreover, it is possible to show that the same structure also applies to the models of “yard to truck” and “yard to ship” cycles. Then, such an observation, shows the structure also a pplies to the models of “yard to truck” and transitions have the meanings reported in Tab. 3, and Tab. 4, respectively. Moreover, it is possible to show that the same structure also applies to the models of “yard to truck” and “yard to ship” cycles. Then, such an observation, shows
Table 3. Alternative meaning of places in Fig. 1.

<table>
<thead>
<tr>
<th>Place</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1</td>
<td>the container is in the yard</td>
</tr>
<tr>
<td>p2</td>
<td>the straddle carrier is available</td>
</tr>
<tr>
<td>p3</td>
<td>the container is picked up by the straddle carrier</td>
</tr>
<tr>
<td>p4</td>
<td>the trailer is available</td>
</tr>
<tr>
<td>p5</td>
<td>the container is placed on the trailer</td>
</tr>
<tr>
<td>p6</td>
<td>the container has reached the yard</td>
</tr>
<tr>
<td>p7</td>
<td>the RMG is available</td>
</tr>
<tr>
<td>p8</td>
<td>the trailer is returning to the yard</td>
</tr>
<tr>
<td>p9</td>
<td>to pick up another container</td>
</tr>
<tr>
<td>p10</td>
<td>the container is placed on the train</td>
</tr>
<tr>
<td>p11</td>
<td>the RMG can return to pick up another container</td>
</tr>
<tr>
<td>p12</td>
<td>the straddle carrier is preparing itself</td>
</tr>
<tr>
<td>p13</td>
<td>to pick up another container</td>
</tr>
</tbody>
</table>

| Table 4. Alternative meaning of transitions in Fig. 1. |

<table>
<thead>
<tr>
<th>Transition</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>t1</td>
<td>the straddle carrier picks up</td>
</tr>
<tr>
<td>t2</td>
<td>the straddle carrier places</td>
</tr>
<tr>
<td>t3</td>
<td>the container on the train area</td>
</tr>
<tr>
<td>t4</td>
<td>the RMG picks up the container from the trailer</td>
</tr>
<tr>
<td>t5</td>
<td>the trailer moves back to the yard</td>
</tr>
<tr>
<td>t6</td>
<td>the RMG places the container on the train</td>
</tr>
<tr>
<td>t7</td>
<td>the RMG returns to pick up another container</td>
</tr>
<tr>
<td>t8</td>
<td>the straddle carrier returns</td>
</tr>
</tbody>
</table>

that it is easy to build complex cycles by simply joining “elementary modules” such as the one depicted in Fig. 1.

To conclude, it is worth noting that the proposed PN model results to be a TMG. Then, it is possible to analytically analyse the properties of the modelled system, via a set of state equations which provide the sequence of the firing times of all the transitions. Then, to this aim, in the next section, the relevant (max,+)-algebra based model is described.

3.2 The (max,+)-algebra model

In this section, the state equations providing the firing times of all the transitions in Fig. 1 are introduced. In doing so, it has been assumed that

- $\tilde{\tau}_i(k)$ is the time instants at which the $k$th firing of token enters the place $p_i$, $\forall p_i \in P$;
- $\tau_j(k)$ is the time instant at which the transition $t_j$, $\forall t_j \in T$ fires for the $k$th time.

With these assumptions, it is possible to write the equations

$$
\tau_1(k) = \max \{ \tilde{\tau}_1(k), \tilde{\tau}_2(k) \} + \theta_1
$$

$$
\tau_2(k) = \max \{ \tilde{\tau}_3(k), \tilde{\tau}_4(k) \} + \theta_2
$$

$$
\tau_3(k) = \tilde{\tau}_5(k) + \theta_3
$$

$$
\tau_4(k) = \tilde{\tau}_5(k) + \tilde{\tau}_7(k) + \theta_4
$$

$$
\tau_5(k) = \tilde{\tau}_6(k) + \theta_5
$$

$$
\tau_6(k) = \tilde{\tau}_6(k) + \theta_6
$$

$$
\tau_7(k) = \tilde{\tau}_1(k) + \theta_7
$$

$$
\tau_8(k) = \tilde{\tau}_2(k) + \theta_8
$$

which, being $p_i^* \in t_j$, the set gathering the input places of transitions $t_j$, may be expressed in the concise form

$$
\tau_j(k) = \bigoplus_{p_i^* \in t_j} \tilde{\tau}_i(k) \otimes \theta_j, \quad k = 1, 2, \ldots, \forall t_j \in T
$$

and giving the $k$th firing time of all the transitions $\in T$, depending on the time they are enabled, that is, on the time $\max_{p_i^* \in t_j} \{ \tilde{\tau}_i(k) \}$.

Then, since the relations

$$
\tau_1(k + 1) = (\tau_1(k) \otimes \theta_1) \oplus (\tau_6(k) \otimes \theta_6)
$$

$$
\tau_2(k + 1) = [\tau_1(k) \otimes (\theta_1 + \theta_2)] \oplus (\tau_5(k) \otimes (\theta_1 + \theta_2))
$$

$$
\tau_3(k + 1) = [\tau_1(k) \otimes (\theta_2 + \theta_3 + \theta_4)] \oplus (\tau_5(k) \otimes (\theta_2 + \theta_3 + \theta_4))
$$

$$
\tau_4(k + 1) = [\tau_1(k) \otimes (\theta_2 + \theta_3 + \theta_6)] \oplus (\tau_5(k) \otimes (\theta_2 + \theta_3 + \theta_6))
$$

$$
\tau_5(k + 1) = [\tau_1(k) \otimes (\theta_2 + \theta_3 + \theta_6 + \theta_7)] \oplus (\tau_5(k) \otimes (\theta_2 + \theta_3 + \theta_6 + \theta_7))
$$

$$
\tau_6(k + 1) = [\tau_1(k) \otimes (\theta_2 + \theta_3 + \theta_6 + \theta_7)] \oplus (\tau_5(k) \otimes (\theta_2 + \theta_3 + \theta_6 + \theta_7))
$$

$$
\tau_7(k + 1) = [\tau_1(k) \otimes (\theta_2 + \theta_3 + \theta_6 + \theta_7)] \oplus (\tau_5(k) \otimes (\theta_2 + \theta_3 + \theta_6 + \theta_7))
$$

$$
\tau_8(k + 1) = [\tau_1(k) \otimes (\theta_2 + \theta_3 + \theta_6 + \theta_7)] \oplus (\tau_5(k) \otimes (\theta_2 + \theta_6 + \theta_8))
$$

by substituting Eq. (8) into Eq. (6), and by expressing the resulting equations in the (max,+)-algebra formalism.

Then, by considering the firing delays reported in Tab. 2, it is possible to determine, by means of equation Eq. (5), that the eigenvalue $\kappa$ is equal to 130 s. Hence, with this handling systems configuration and delays, the “ship to yard” import cycle unloads a container from the ship and store it in the yard about every two minutes.
4. SENSITIVITY ANALYSIS

In this section some considerations about the sensitivity analysis (Saltelli et al. (2008)) of the cycle time \( \kappa \) are discussed. In particular, the sample frequencies of the values of \( \kappa_h \) computed by means of \( H \) repeated applications of Eq. (5), are reported in some histograms, thus allowing to comprehend the effect of the uncertainty of the parameters \( \theta_j \) on the cycle time. To do so, the firing times associated with the transitions have been assumed to be normal stochastic variables, with given mean and variance, that is,

1. \( \theta_1 \in N(100, 25) \);
2. \( \theta_2 \in N(10, 25) \);
3. \( \theta_3 \in N(35, 49) \);
4. \( \theta_4 \in N(10, 25) \);
5. \( \theta_5 \in N(35, 49) \);
6. \( \theta_6 \in N(90, 900) \);
7. \( \theta_7 \in N(30, 100) \);
8. \( \theta_8 \in N(15, 25) \);

being \( N(\mu, \sigma^2) \) a generic normal stochastic variables with mean \( \mu \) and variance \( \sigma^2 \). Note that in such an analysis, the PNs model still remains deterministic, but is repeatedly applied, together with the relevant \((\max,+)\) algebra representation, by varying the transition delays accordingly with the extraction of the normal stochastic variables above described. Note that, from an operative point of view, the matrix \( A \) simply results to be a stochastic matrix, whose elements are extraction of the above normal stochastic variables. Moreover, in order to keep the approach simple, only one transition delay has been considered stochastic at a time.

Then, the results of the sensitivity analysis are reported in Fig. 2, Fig. 3, Fig. 4, Fig. 5, Fig. 6, and Fig. 7, where it is possible to observe that the cycle time \( \kappa \) is very sensitive (great sample variance) to the uncertainty of the parameters \( \theta_4 \) and \( \theta_7 \), which represent the times associated with transitions \( t_3 \) (collecting from trailer) and \( t_9 \) (yard handling system available). On the other hand, the other operation timings have a negligible influence on the cycle time (small sample variance). In addition, the cycle time have resulted to be insensitive (null variance) with respect to the uncertainties of the duration of the operations modelled by transitions \( t_3 \) and \( t_5 \), that is, to the movement of the trailer from bay to the yard and back. This is the reason why the relevant histograms are not reported in the paper.

In addition, the histograms reported in the above figures also show that, except for the operations that take place in the yard, when the stochastic extractions of duration of the operations represented by the transitions is smaller than the nominal one reported in Tab. 2, the cycle time \( \kappa \) is limited by a lower bound equal to 130 s. In other words, decreasing the values of \( \theta_1, \theta_2, \) and \( \theta_8 \), does not decrease the system period \( \kappa \).

Then, from the robustness point of view, such results suggest that it is more convenient to concentrate the available resources to optimise the operations that take place in the yard, that is, to reduce the relevant uncertainty, as well as the relevant nominal value, while it would be certainly less convenient to invest resources in the other operations.

5. CONCLUSIONS

In this paper a model for the performance evaluation of container terminals has been described. To do so, a PNs representation of import cycles of the terminal has been described, also pointing out the modularity of the proposed approach that allows to represent, with the same model, different kind of cycles, maybe characterised by different resources, both in terms of their quantity and typology.

Then, once noted that the obtained model resulted to be a TGM, a \((\max,+)\)-algebra formalisation of the terminal cycles has been derived, so as to analytically compute the performances of the systems, such as the “nominal”
Finally, some results about a realistic case study has been discussed.

Work is in progress to define an optimisation problem based on the presented model, and the relevant solution algorithm, so as to maximise the performances of the terminal import/export cycles, as well to minimise the relevant variance.

REFERENCES


