Consensus via Distributed Adaptive Control

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Abstract: This paper concerns with consensus of a group of agents with unknown parameters. It is assumed that each agent can communicate only with its local neighborhoods. Then we propose a distributed adaptive control law based on model reference control strategy which uses only relative information between neighboring agents. It is shown that our adaptive control law attains a consensus if the communication graph is connected and bi-directional in some sense. Furthermore we interpret our proposing system with the notion of passivity, and show that the bi-directional information flow is required from the passivity for the system.

Keywords: adaptive control, multi-agent system, consensus, communication graph, passivity

1. INTRODUCTION

Multi-agent systems have distinguished features and wide range of applications, and attract many researchers (Arcak (2007); Bullo et al. (2009); Jadbabaie et al. (2003); Olfati-Saber and Murray (2004)) in the last few years. The features include fault-tolerance and flexibility, while the applications range over the fields of physics, biology, and engineering. Among a number of investigations on multi-agent systems, consensus problem is an active topic in recent years (Jadbabaie et al. (2003); Olfati-Saber and Murray (2004); Ren and Beard (2005)). Consensus means the agreement of the outputs or the state of all agents by negotiating with their local neighbors.

Most of existing papers on consensus problem studied the case where the dynamics of each agent is identified precisely. For example, in the researches such as Jadbabaie et al. (2003); Olfati-Saber and Murray (2004); Ren and Beard (2005), agents are assumed to be with single-integrator kinematics. More recent research Scardovi and Sepulchre (2009) assumes that each agent has an ideal linear dynamics while Arcak (2007) studies heterogeneous nonlinear agents based on passivity. However, in real situations, many dynamical systems to be controlled include constant or slowly-varying uncertain parameters and this is particularly indispensable for multi-agent systems. This means that multi-agent systems including an enormous number of agents must compose heterogeneous dynamics with unknown parameters. Unfortunately, a few papers on multi-agent system treat the difficulties of uncertain parameters. For example, Pereira and Hsu (2008) and Pereira et al. (2009) deal with a formation control of some specific agents of uncertain Euler-Lagrange systems using artificial potential functions.

With these backgrounds, in this paper, we treat the consensus problem of agents whose dynamics are unknown and described by rational functions, which is a standard framework in model reference adaptive control. Adaptive control is an approach to the control of such systems with unknown or time-varying parameters and has a long history (e.g. see Astrom and Wittenmark (1994); Landau and Tomizuka (1981); Narendra and Annaswamy (1989)). The model reference adaptive control (MRAC) is one of the main approaches for constructing adaptive controllers and widely studied after the idea was presented during 1950’s. This control may be regarded as an adaptive servo system in which the desired performance is expressed in terms of a reference model, which gives the desired response to a command signal. Many practical applications of MRAC are also reported by using the advantages of computer technologies.

In this paper, we consider the consensus problem among agents with unknown parameters. We deal with both leader-follower case and leaderless case, and propose a distributed adaptive control based on the MRAC scheme which only uses the outputs or the state variables of the neighboring agents. Then we derive sufficient conditions on a communication topology of agents to achieve the consensus. The conditions on the graph are connectivity and bi-directionality. We interpret our proposing systems by means of passivity and show that the bi-directional communication is required from the passivity of error system.

This paper is organized as follows: In Section 2, we give notions of agents, leader and the followers. A graph, which describes the communication topology among agents, and the related notations are also introduced. Problem formu-
lations are given in Section 3. Then we propose distributed adaptive control laws in Section 4. In Section 5, we give the main results on a communication topology and the passivity interpretation of a proposing adaptive system. In Section 6, we also show an extension of the main results for the case of full-state accessible case among the agents. Simulation results which show the efficiency of our proposing distributed adaptive control are given in Section 7. Section 8 summarizes the main conclusions and shows the future work.

2. PRELIMINARIES

In this paper, we consider a group of dynamical subsystems, in which the subsystems are called agents and the whole system is called a multi-agent system. The agents are assumed to have their own dynamics from the input $u_i$ to the output $y_i$ such as

$$y_i = h_i(s)u_i, \quad h_i(s) = Z_i(s)/R_i(s), \quad i \in \mathcal{V},$$

where $h_i(s)$ denotes the transfer function of the agents and $\mathcal{V} := \{1, 2, \ldots, N\}$ denotes the set of indices of the agents.

We also consider a case in the following that one of the agents can be regarded as a leader and the others the followers. If there exists a leader in the group, the index of the leader is denoted by $\ell$, and without loss of generality, $\ell = 1$. The rest of the agents are the followers, and the set of the followers are denoted by $\mathcal{V}_f := \{2, \ldots, N\}$. The details of the leader and the followers are explained in later.

For each agent, a feedback controller is applied which can use the outputs (output feedback case) or the state variables (state feedback case) of some of the agents. More clearly, a controller $k_i$ for the $i$-th agent is defined such as $u_i = k_i(y_i; j \in \mathcal{N}_i)$ for the output feedback case and $u_i = k_i(x_i; j \in \mathcal{N}_i)$ for the state feedback case, where $\mathcal{N}_i$ represents a subset of indices of the neighboring agents of which the outputs or the state variables can be used by the controller $k_i$. The set of $\mathcal{N}_i$, $i \in \mathcal{V}$ constructs the communication network between the agents.

The communication topology of a network of agents can be represented by using a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where we regard $\mathcal{V}$ as a set of nodes representing the agents, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents edges between the nodes. A node $i$ corresponds to $i$-th agent while an edge $(i, j)$ corresponds to an information flow from agent $j$ to $i$. A term in-degree of a node $i$ means the number of edges of $(i, j)$ and a term out-degree of a node $i$ means the number of edges of $(j, i)$. Then, $\mathcal{N}_i$ can be regarded as the neighbors of agent $i$ and it can be represented as $\mathcal{N}_i = \{ j \in \mathcal{V} | (i, j) \in \mathcal{E} \}$.

An undirected graph can be considered as a special case of a directed graph, where an edge $(i, j)$ in the undirected graph corresponds to an edge $(i, j)$ and $(j, i)$ in the directed graph. A directed path is a sequence of edges in a directed graph of the form $(i_1, i_2), (i_2, i_3), \ldots$ where $i_j \in \mathcal{V}$. An undirected path is defined analogously. A directed graph is said to have a directed spanning tree if there exists at least one node having directed path to all other nodes. An undirected graph is said to be connected if there is an undirected path between every pair of distinct nodes.

A graph $\mathcal{G}$ can be represented by a matrix called graph Laplacian $L \in \mathbb{R}^{N \times N}$ whose elements are defined as follows: $L_{ij} = -1$ if $j \in \mathcal{N}_i$ and $L_{ii} = |\mathcal{N}_i|$, otherwise $L_{ij} = 0$. Laplacian $L$ has a trivial eigenvalue 0 with an associated eigenvector $1 := [1 \cdots 1]^T$.

3. FORMULATION

In this paper, we mainly consider an output consensus problem of agents with unknown parameters and afterward mention a case of a state consensus problem in Section 6. In both cases, we assume that only the outputs or the state variables of the neighbors are accessible. Then, we deal with following two cases: (i) leader-follower case where there exists a leader in that group, (ii) leaderless case where no leader agent exists in the group.

Hereafter we formulate the problem setting for the output accessible case.

Leader-follower case The dynamics of the leader is given by

$$y_\ell = h(\ell)r, \quad h(\ell) = Z(\ell)/R(\ell), \quad \ell \in \mathcal{V}_f,$$

where $h(\ell)$ is a SISO stable transfer function, $Z(\ell)$ and $R(\ell)$ are monic polynomials $\ell$ of order $\xi_\ell$ and $\eta_\ell$, respectively, $r(t) \in \mathbb{R}$ is the reference input, and $y(t) \in \mathbb{R}$ is the output. On the other hand, the dynamics of the followers is given by

$$y_i = h_i(s)u_i, \quad h_i(s) = Z_i(s)/R_i(s), \quad i \in \mathcal{V}_f, \quad u_i = r + k_i(y_j; j \in \mathcal{N}_i), \quad i \notin \mathcal{V}_f,$$

where $h_i(s)$ is the SISO transfer function of the followers, $Z_i(s)$ and $R_i(s)$ are monic polynomials of order $\eta_i$ and $\xi_i$, respectively, with unknown coefficients, $u_i(t) \in \mathbb{R}$ is the control input, $y_i(t) \in \mathbb{R}$ is the output, and $r(t) \in \mathbb{R}$ is the reference signal, and $k_i$ is a control function. Assume that the neighbor $\mathcal{N}_i$ includes $i$.

In the leader-follower case, our goal is to design the controller $k_i$ satisfying that all followers track the leader’s output as

$$\lim_{t \rightarrow \infty} (y_i(t) - y_\ell(t)) = 0, \quad \forall i \in \mathcal{V}_f.$$

We assume the followings:

Assumption 1. For all the $i \in \mathcal{V}_f,$

1. $n_i$ is known,
2. there exist $n'$ and $\bar{m}'$ satisfying $n_i = n'$, $\bar{m}_i = \bar{m}'$,
3. $\bar{m}' = n' - 1$,
4. $Z_i(s)$ is stable, and
5. $h(s)$ is strictly positive real and $\eta_i \leq n$,
6. coefficients of $Z(s)$ are known to every follower.

Assumptions 1–(1) to (5) are standard setting for MRAC (Narendra and Annaswamy (1989)), while Assumption 1–(6) means that every follower knows a part of the leader’s dynamics, zeros of $h(s)$ a priori. As described in later, our proposing output feedback controller is composed of a state estimation part and a state feedback part. The above assumption is required from the fact that a state feedback cannot replace the zeros of the plants.

Under Assumption 1, we consider the following problem:

1. We can easily extend our results to the case where the numerator $Z_i(s)$ is not monic, i.e. $h_i(s) = \kappa_i Z_i(s)/R_i(s)$, by assuming that the sign of high-frequency gain $\kappa_i$ is known.
Problem 1. Design a distributed adaptive control law (3) which achieves (4).

Remark 1. The standard adaptive control law for MRAC can achieve (4) if every follower can observe the leader’s output $y_i$ directly. However, in this paper, we assume that only a part of followers, who are neighbors of the leader, can observe the leader’s output.

Remark 2. The standard MRAC can be regarded as a special case of $N = 2$ in Problem 1.

Leaderless case In the leaderless case, we assume that the multi-agent system does not include a leader (1) and our goal is to design the controller (3) satisfying the output consensus of the all agents:

$$\lim_{t \to \infty} (y_i(t) - y_j(t)) = 0, \quad \forall i, j \in V.$$  \hspace{1cm} (5)

The following is assumed:

Assumption 2. For the all $i \in V$, Assumption 1-(1)–(4) are satisfied.

Under the above setting, we consider the following problem:

Problem 2. Design a distributed adaptive control law (3) which achieves (5).

4. PROPOSING ADAPTIVE CONTROL LAW

Now we give a distributed adaptive control law (3) concretely for Problem 1 or 2:

$$\dot{\omega}_{i,1}(t) = \Lambda_i \omega_{i,1}(t) + \beta_i u_i(t),$$

$$\omega_{i,2}(t) = \Lambda_i \omega_{i,2}(t) + \beta_i y_i(t),$$

$$u_i(t) = \theta^*_i(t) \omega_i(t),$$

where time-varying parameter $\theta_i(t) \in \mathbb{R}^{2n}$ and the controller state $\omega_i(t) \in \mathbb{R}^{2n}$ are defined by

$$\theta_i(t) := \left[ \begin{array}{cc} \theta^*_{i,1}(t) & \theta^*_{i,2}(t) \\ \theta_{i,0}(t) \\ \end{array} \right]^T,$$

$$\omega_i(t) := \left[ \begin{array}{cc} r(t) & \omega^*_{i,1}(t) \\ \omega^*_{i,2}(t) & y_i(t) \\ \end{array} \right]^T,$$

respectively, $\Lambda_i \in \mathbb{R}^{(n-1) \times (n-1)}$ is an asymptotically stable matrix such that

$$|sI - \Lambda_i|$$

includes $Z(s)$ as a factor in the leader-follower case or is any common stable function to the agents in the leaderless case, and $\beta_i \in \mathbb{R}^{n-1}$ is such that the pair $(\Lambda_i, \beta_i)$ is controllable.

Fig. 1 shows the block diagram of our proposing controller for each agent, which has a same structure as the one of the standard MRAC. In Fig. 1, the proposing adaptive controller is described in the block with a dotted line.

For Problem 1, similar to the standard MRAC, there exists an ideal parameter $\theta^*_i \in \mathbb{R}^{2n}$ for each agent $i$ such that if $\theta_i(t) = \theta^*_i$, the transfer function from $r$ to $y_i$ equals to $h(s)$. However the ideal parameter $\theta^*_i$ is not known to each agent $i$. This means that each agent needs to update the parameter $\theta_i(t)$.

Now we propose the following updating law which is based on relative output errors between neighborhood agents:

$$\dot{\theta}_i(t) = \sum_{j \in N_i} \{ y_j(t) - y_i(t) \} \omega_i(t).$$  \hspace{1cm} (7)

The updating law (7) of agent $i$ only requires the outputs of neighbors $y_j, j \in N_i$. This means that the parameter tuning is in a distributed way.

5. MAIN RESULTS

In this section, we derive sufficient conditions on the communication graphs under which our proposing distributed adaptive control law attains the consensus in the multi-agent systems. Furthermore, we interpret the proposing system in terms of passivity and show that the conditions are required for it.

5.1 Conditions on Communication Graphs

For Problem 1 in which the group has a leader, following theorem holds:

Theorem 1. For the systems (1) and (2) with the communication topology $G$ under Assumption 1, the adaptive control law (6)–(7) solves Problem 1 if the following three conditions hold:

(1) For any follower agents $i, j \in V_f, (i, j) \in E$ implies $(j, i) \in E$,

(2) $G$ has a directed spanning tree from the leader,

(3) In-degree of the leader $\ell$ is 0.

We omit the proof from the page limitation.

Remark 3. The parameters $\theta_i(t)$ converges to the ideal value $\theta^*_i$ if the reference input $r(t)$ has a PE property.

For Problem 2 in which the group has no leader, the following corollary holds:

Corollary 1. For the systems (2) with the communication topology $G$ under Assumption 2, the adaptive control law (6)–(7) solves Problem 2 if the following conditions hold:

(1) $G$ is undirected,

(2) $G$ is connected.

We omit the proof from the page limitation.

Remark 4. Corollary 1 guarantees “consensus”, i.e., (5), among agents, however it does not necessarily guarantee the stability of the whole system. In fact, we have numerical examples which achieve the consensus however the whole system is unstable (We also have numerical examples in which the whole system is stable and the consensus is achieved. See Section 7 for the details). Whereas when the group includes a stable leader, its stability guarantees that of the whole system and this fact indicates the importance of the leader in the adaptive consensus systems.

5.2 On the Topology of the Graphs

Fig. 2 shows examples of graphs which satisfy the conditions in Theorem 1 and Corollary 1. The difference
between them is the existence of the leader and the edges between the leader and the followers. In the leader-follower case, the leader receives no information from the followers, which means that the edges associated with the leader are not bi-directional, while in the leaderless case, every communication is bi-directional. Theorem 1 and Corollary 1 also require connectivity. Obviously, this is not only sufficient but also necessary for the consensus, while the bi-directionality does not seem to be necessary in general.

However, in the next subsection, we show that the bi-directionality is required from the passivity for the whole system.

5.3 A Passivity Interpretation

In this subsection, we analyze our proposing system in terms of passivity and show that the bi-directional communication is required from the passivity for the whole system in some sense.

At first, by employing the methodology of the standard MRAC (e.g., Narendra and Annaswamy (1989)), with the transfer function $h(s)$ of the leader, the dynamics of $i$-th agent including our proposing adaptive controller can be represented as

$$
\begin{cases}
\dot{x}_i = A x_i + B \left( r + \phi_i^T \omega_i \right), \\
y_i = C x_i
\end{cases}
$$

where $x_i(t) \in \mathbb{R}^n$ is a state of $i$-th agent’s system and $\phi_i(t) := \Theta_i(t) - \Theta^* \in \mathbb{R}^{2n}$ is the parameter error from the leader.

From (8), the output of agent $i$ can be represented as

$$
y_i = h(s) r + h(s) (\phi_i^T \omega_i).
$$

The output error between agent $i$ and $j$, $e_{ij} := y_i - y_j$, can be written as

$$
e_{ij} = h(s) (\phi_i^T \omega_i - \phi_j^T \omega_j).
$$

Then, the whole error system of the all agents or the leader can be summarized into a block diagram in Fig. 3(a) where $\hat{L}$ for the leader-follower case or $L$ for the leaderless case denotes the Laplacian matrix which represents the communication topology among agents.

In both cases, $\hat{L}$ and $L$ should be symmetry for the consensus and then they can be decomposed into a product of incidence matrix $D \in \mathbb{R}^{N \times M}$ where $M$ denotes the total number of edges (Arcak (2007)):

$$
\hat{L} \text{ or } L = DD^T.
$$

With this matrix $D$, we can replace the block with the dotted line in Fig. 3(a) by Fig. 3(b).

Fig. 2: Examples of communication graphs satisfying the conditions in Theorem 1 and Corollary 1 between the leader and the followers. In the leader-follower case, the leader receives no information from the followers, which means that the edges associated with the leader are not bi-directional, while in the leaderless case, every communication is bi-directional. Theorem 1 and Corollary 1 also require connectivity. Obviously, this is not only sufficient but also necessary for the consensus, while the bi-directionality does not seem to be necessary in general.

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$$
\begin{cases}
\dot{x}_i = A x_i + B \left( r + \phi_i^T \omega_i \right), \\
y_i = C x_i
\end{cases}
$$

where $x_i(t) \in \mathbb{R}^n$ is a state of $i$-th agent’s system and $\phi_i(t) := \Theta_i(t) - \Theta^* \in \mathbb{R}^{2n}$ is the parameter error from the leader.

From (8), the output of agent $i$ can be represented as

$$
y_i = h(s) r + h(s) (\phi_i^T \omega_i).
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The output error between agent $i$ and $j$, $e_{ij} := y_i - y_j$, can be written as

$$
e_{ij} = h(s) (\phi_i^T \omega_i - \phi_j^T \omega_j).
$$

Then, the whole error system of the all agents or the leader can be summarized into a block diagram in Fig. 3(a) where $\hat{L}$ for the leader-follower case or $L$ for the leaderless case denotes the Laplacian matrix which represents the communication topology among agents.

In both cases, $\hat{L}$ and $L$ should be symmetry for the consensus and then they can be decomposed into a product of incidence matrix $D \in \mathbb{R}^{N \times M}$ where $M$ denotes the total number of edges (Arcak (2007)):

$$
\hat{L} \text{ or } L = DD^T.
$$

With this matrix $D$, we can replace the block with the dotted line in Fig. 3(a) by Fig. 3(b).

In Fig. 3(b), the strictly positive realness of $h(s)$ obviously implies that of the blocked part. Furthermore, we can show the following (hereafter in this subsection, we also represent $\hat{L}$ by $L$ in short):

**Proposition 1.** Let $G$ be a graph and $L \in \mathbb{R}^{N \times N}$ be a corresponding Laplacian matrix. Then, $h(s)L$ is passive for all positive real function $h(s)$ if and only if $G$ is undirected.

We omit the proof from the page limitation.

**Remark 5.** We can also show that consensus is achieved by using the passivity theorem. Let $W \in \mathbb{R}^{(N-1)\times N}$ be with orthonormal rows that are each orthogonal to $1 \in \mathbb{R}^N$:

$$
W 1 = 0, \quad WW^T = I_{N-1}.
$$

By the definition of $W$ in (11), $WLW^T$ preserves the all eigenvalues of $L$ except the one at zero, then its smallest eigenvalue is

$$
\lambda_1(WLW^T) = \lambda_2(L)
$$

which means that nonsingularity of the matrix $WLW^T$ is equivalent to connectivity of the graph and equivalent to strictly positive realness of $h(s)LW^T$.

Let the state error of $x(t) \in \mathbb{R}^n$ be $\epsilon(t) := Wx(t) \in \mathbb{R}^{n(N-1)}$. By the properties of $W$ in (11), $\epsilon(t) = 0$ if and only if the consensus is achieved. The feedforward part is $h(s)LW^T$ which is strictly positive real, and the feedback part from $\sum_{j \in \mathcal{N}_i} e_{ij}(t)$ to $-\phi_i^T(t)\omega_i(t)$ is passive. Therefore, we can prove that the origin $\epsilon(t) = 0$ is asymptotic stable by the passivity theorem.

6. FULL STATE ACCESSIBLE CASE

In the following of this paper, we show an extension of the previous results for the case that the state of each agent is accessible to its neighbors.

6.1 Formulation and Distributed Adaptive Controller

**Leader-follower case** The dynamics of the leader is given by

$$
\dot{x}_l = Ax_l + Br, \quad x_l(t) \in \mathbb{R}^n, r(t) \in \mathbb{R}^m,
$$

where $x_l(t) = x_1(t)$, $r(t) = u_1(t) \in \mathbb{R}^m$ denotes the reference input, and $A$ is assumed to be stable. On the other hand, the dynamics of the followers is given by

$$
\dot{x}_i = A x_i + Bu_i, \quad x_i(t) \in \mathbb{R}^n, u_i(t) \in \mathbb{R}^m, \forall i \in \mathcal{V}_f
$$

where $u_i(t) \in \mathbb{R}^m$ denotes the control input. Assume that the neighbor $\mathcal{N}_i$ includes $i$. The objective is to design the controller $k_i$ satisfying that the all followers track the leader’s state:

$$
\lim_{t \to \infty} (x_i(t) - x_l(t)) = 0, \quad \forall i \in \mathcal{V}_f.
$$

We assume the following:

**Assumption 3.** For each follower $i \in \mathcal{V}_f$:

1. $A_i, i \in \mathcal{V}_f$ is unknown,
2. there exists a feedback gain $K^*_i \in \mathbb{R}^{n \times m}$ satisfying
   $$
   A_i + BK_i^* = A,
   $$
3. $P = P^T > O$ satisfying
   $$
   PA + A^TP < O
   $$

is known,
When \(|V_f| = 1\), Assumption 3 is a standard setting for MRAC.

**Remark 6.** This assumption means that A of the leader is not necessarily known to the followers. For example, if we only know that the stable matrix A is symmetry, \(P = I\) is enough.

Under Assumption 3, we consider the following problem:

**Problem 3.** Design a distributed adaptive control law (14) which achieves (15).

**leaderless case.** In this case, we assume that the multi-agent system does not include a leader (12) and our goal is to design the controller (14) satisfying the consensus of the state variables of all the agents:

\[
\lim_{t \to \infty} (x_i(t) - x_j(t)) = 0, \quad \forall i, j \in V. \tag{18}
\]

We consider the following assumption on each agents:

**Assumption 4.** For the all \(i \in V,\)

1. \(A_i, i \in V\) is unknown,
2. there exists a feedback gain \(K_i^* \in \mathbb{R}^{n_i \times m}\), stable \(A_i\), and known \(B\), where \(A\) and \(B\) are controllable, satisfying

\[
A_i + BK_i^{*T} = A, \tag{19}
\]

3. \(P = P^T > O\) satisfying

\[
PA + A^TP < O \tag{20}
\]

is known.

Under Assumption 4, we consider the following problem:

**Problem 4.** Design a distributed adaptive control law (14) which achieves (18).

The distributed adaptive control law for (14) is given by

\[
u_i(t) = r(t) + K_i^{T}(t)x_i(t), \tag{21}
\]

\[
K_i(t) = -x_i(t) \sum_{j \in N_i} (x_i(t) - x_j(t))^T PB. \tag{22}
\]

The proposing adaptive multi-agent system can be described by a block diagram in Fig. 4. In Fig. 4, \(L \in \mathbb{R}^{N \times N}\) denotes the Laplacian matrix which describes the communication topology between the agents. The feedback gain of each agent \(K_i\) is updated via output of the block \(L \otimes I_n\), which means that the tuning rate is proportional to the relative error of the states between the neighboring agents.

**6.2 Conditions on Communication Graphs**

For Problem 3 in which the group has a leader and the followers, following theorem holds:

**Theorem 2.** For the systems (12) and (13) with the communication topology \(\mathcal{G}\) under Assumption 3, the adaptive control law (21)–(22) solves Problem 3 if the conditions of Theorem 1 hold.

We omit the proof from the page limitation.

**Remark 7.** The feedback gain \(K_i\) converges to the ideal gain \(K_i^*\) if the reference input \(r\) has a so-called PE property.

The similar result for Problem 4 is also given as follows:

**Corollary 2.** For the systems (13) with the communication topology \(\mathcal{G}\) under Assumption 4, the adaptive control law (21)–(22) solves the Problem 4 if the conditions in Corollary 1 hold.

We omit the proof from the page limitation.

**Remark 8.** Similar to Corollary 1, this corollary guarantees “consensus” in the sense of (18) among agents, however it does not necessarily guarantee the stability of the whole system.

**7. SIMULATIONS**

In this section, we show simulation results for Theorem 1 and Corollary 1. We consider a case of \(N = 4\) and set the transfer functions in (2) as

\[
h_2(s) = \frac{s + 10}{(s + 2.0)(s + 3.0)}, \quad h_3(s) = \frac{s + 7.4}{(s + 4.9)(s + 9.7)};
\]

\[
h_2(s) = \frac{s + 0.1}{(s + 0.2)(s + 3.3)}, \quad h_3(s) = \frac{s + 2.8}{(s + 6.2)(s + 4.8)}.
\]

We set \(\ell = 1\) and \(V_f = \{2, 3, 4\}\) for the leader-follower case and \(V = \{1, 2, 3, 4\}\) for the leaderless case. Matrices \(A_i\) and \(\beta_i\) in each controller are set as

\[
A_i = -1, \quad \beta_i = 1.
\]

The communication graphs for Theorem 1 and Corollary 1 are set as in Fig. 2, respectively.

Fig. 5 shows step and sinusoidal responses of each agents. We set the reference input as \(r(t) = 10 \sin t\), respectively. Fig. 5(a) and 5(b) show the leader-follower case while Fig. 5(c) and 5(d) show the leaderless case.

In each simulations in Fig. 5, we can observe that the consensus is achieved (Note that in the leaderless case, although Corollary 1 does not necessarily guarantee “the
In this paper, we considered the consensus problems among agents with unknown parameters. We proposed a distributed adaptive control law which is based on the standard MARC strategy, however uses only neighbors’ outputs or state variables. We derived sufficient conditions for communication graph to achieve the consensus. The condition is the connectivity and the bi-directionality of the graph. Furthermore, we interpret our proposing adaptive system in terms of passivity, and show that the bi-directionality of the graph is required from the passivity for the error system. Numerical examples show the efficiency of our proposing distributed adaptive control.

Our future work includes the analysis of the consensus value in the leaderless case and the extension to the directed graph case and time-varying graph case.

8. CONCLUSIONS AND FUTURE WORKS

In this paper, we considered the consensus problems among agents with unknown parameters. We proposed a distributed adaptive control law which is based on the standard MARC strategy, however uses only neighbors’ outputs or state variables. We derived sufficient conditions for communication graph to achieve the consensus. The condition is the connectivity and the bi-directionality of the graph. Furthermore, we interpret our proposing adaptive system in terms of passivity, and show that the bi-directionality of the graph is required from the passivity for the error system. Numerical examples show the efficiency of our proposing distributed adaptive control.

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