Planning of Rhythmic Forearm Movements in Humans: On Stability Achieved by Physical Property of Musculo-skeletal System

Taiga Yamasaki * Xin Xin *

* Okayama Prefectural University, 111 Kuboki, Soja, Okayama 719-1197, JAPAN (e-mail: {taiga, xxin}@cse.oka-pu.ac.jp)

Abstract: Comfortable amplitudes of rhythmic forearm movements in the horizontal plane have been reported to increase as periods increase. This paper attempts to explain this phenomenon by hypothesizing that the neural system prefers a movement that achieves high stability even under a feedforward control of the musculo-skeletal system. A muscular control model of the rhythmic forearm movement is constructed with a feedforward controller composed of an inverse muscle model and an inverse skeletal model. Stability analysis of the model with sinusoidal desired orbits with various periods and amplitudes shows that an experimentally observed period-amplitude relation is qualitatively similar to the conditions such that the stability of the model’s movement is relatively high compared with smaller amplitude movements.

Keywords: Human motor control, musculo-skeletal system, movement planning, stability, periodic, cyclic, rhythmic

1. INTRODUCTION

Comfortable amplitudes of rhythmic forearm movements of humans in the horizontal plane have been reported to increase as prescribed periods increase, and this has been explained by an autonomous nonlinear oscillator model: a hybrid model of the Van der Pol oscillator and the Rayleigh oscillator (Beek et al., 1996). However, the model cannot account for the physiological mechanism of human motor control.

Trajectories of the rhythmic forearm movements in the horizontal plane have been explained by the minimum-jerk model under the assumptions of boundary conditions (Nagasaki, 1991). However, it is still unclear how the boundary conditions or resulting amplitudes of the movements are decided. On the other hand, if a joint angle changes sinusoidally with time, a decrease of the amplitude can always reduce its jerk cost. In this case, a movement with smaller amplitude would be a better choice for saving the jerk cost. Similarly, the joint torque cost (Nubar and Contini, 1961), the torque change cost (Uno et al., 1989) (time derivative of torque), and the joint power cost, would also decrease with the amplitude of the movement. These raise the question of how the certain degree of amplitudes observed in the rhythmic forearm movements (Beek et al., 1996) can be understood.

The purpose of this paper is to explain the relation between the periods and the amplitudes of the rhythmic forearm movements with consideration of the physiological mechanism of the neuro-musculo-skeletal system. We hypothesize that, for a given period, rhythmic movements along the orbit achieving high stability in the sense of feedforward control of the musculo-skeletal system is selected by a neural controller. The open-loop stability depends on physical properties of the controlled system (i.e., the musculo-skeletal system) as well as the profiles of the desired orbit (such as amplitude). Note, however, that the assumption does not intend to claim that the feedforward control is only the control mechanism of this task. Even if feedback as well as feedforward controls are actually employed in human motor control, as it is widely believed (Kundel et al., 2000; Enoka, 2008; Wolpert et al., 1998), the ease of the feedforward control would be beneficial for the nervous system to reduce computational loads on a neural feedback controller that guarantees the stability of a desired movement, and would secure control performance in the presence of delays in the neural feedback loops.

In this paper, to test the hypothesis, we experimentally observe the ‘comfortable’ period-amplitude relation of rhythmic forearm movement in humans, and theoretically evaluate the stability of the periodic forearm movement of the musculo-skeletal model driven by an ‘imaginary’ feedforward controller.

This paper is organized as follows: Section 2 describes the experimental setup for the rhythmic forearm movements in the horizontal plane. Section 3 describes the model construction, composed of the musculo-skeletal system modeled by using Stroeve’s muscle model (Stroeve, 1999a,b), and a neural controller that calculates an inverse model of the musculo-skeletal system to drive the system along a desired movement in a feedforward manner. Section 4 defines the stability of the desired periodic movement. Section 5 provides results of the experimental and computational analysis, compares these results, and discusses possibility of increasing feedforward stability of periodic movements.
Fig. 1. Experimental setup.

by appropriate movement planning. Section 6 makes some concluding remarks.

2. EXPERIMENT

After informed consent was obtained, two subjects (22-23 yr old, healthy, and right-handed male) participated (22-23 yr old, healthy, and right-handed male) participated in the experiment. The subject sat on a chair. His right forearm was fixed to a wooden beam that rotates in the horizontal plane around a vertical axis at the elbow joint. The height and the position of the chair was adjusted so that subject’s shoulder, elbow and wrist were at the level of the horizontal beam, the center of the elbow joint was aligned at the rotational axis, and the shoulder angle was 60 deg (Figs. 1 and 2). The subject was instructed to move periodically the right forearm around prescribed elbow angle (60 deg) with his comfortable amplitude and a prescribed tempo by an auditory metronome. The subject practiced in about 5 min, and performed 3 trials for each of tempo 30, 60, 90, 120, 150, 180, 210 bpm (period 2.0, 1.0, 0.67, 0.50, 0.40, 0.33, 0.29 s) in a randomized order. In each experimental trial, the subject found his comfortable amplitude for a prescribed tempo, then he kept a steady movement after sending a cue to the experimenter, and angle and angular velocity were measured by a potentiometer and gyro-sensor, respectively, and recorded at 1 ms of a sampling interval.

3. MODEL

3.1 Musculo-skeletal system

Skeletal model (SM)

Dynamics of a simplified skeleton of the forearm (and the hand) is modeled by

$$J \ddot{\theta} = \tau$$

where $J$ is the moment of inertia of the forearm (and the hand) around the elbow joint, $\theta$ is the elbow angular position, $\dot{\theta}$ is the elbow angular acceleration, and $\tau$ is the elbow joint torque.

Muscle model (MM)

A muscle model around the elbow joint was adopted from Stroeve (1999a,b). In this model, active torque $\tau_a$ and passive torque $\tau_p$ were exerted on the elbow joint

$$\tau(\theta, \dot{\theta}, a) = \tau_a(\theta, \dot{\theta}, a) + \tau_p(\theta, \dot{\theta})$$

where $a(t) = [a_1(t), \ldots, a_4(t)]^T$ is activation of muscles, taking a value $a_i \in [0, 1]$ for $(i = 1, 2, 3, 4)$. The indices $i = 1, 2, 3, 4$ indicate monoarticular flexor, monoarticular extensor, biarticular flexor, and biarticular extensor, respectively (Fig. 2).

The following are details of $\tau_a$ and $\tau_p$:

$$\tau_a(\theta, \dot{\theta}, a) = \sum_{i=1}^4 \tau_{a,i}(\theta, \dot{\theta}, a_i) = \sum_{i=1}^4 \tau_i F_i(\ell_i, \dot{\ell}_i, a_i)$$

$$F_i(\ell_i, \dot{\ell}_i, a_i) = a_i \cdot F_{max,i}(\ell_i) \cdot F_{v,i}(\dot{\ell}_i, a_i)$$

$$\ell_i(\theta) = -r_i \theta + c_{1,i}$$

$$\dot{\ell}_i(\theta) = -r_i \dot{\theta}$$

$$F_{v,i}(\dot{\ell}_i, a_i) = \exp \left\{ \left( \frac{\ell_i - c_{2,i}}{c_{3,i}} \right)^2 \right\}$$

$$F_{v,i}(\dot{\ell}_i, a_i) = \begin{cases} 0 & (\ell_i < -v_{max,i}(\ell_i, a_i)) \\ c_{6,i} v_{max,i}(\ell_i, a_i) + c_{6,i} \dot{\ell}_i & (-v_{max,i}(\ell_i, a_i) \leq \dot{\ell}_i < 0) \\ c_{6,i} c_4 \dot{\ell}_i v_{max,i}(\ell_i, a_i) + c_6, \dot{\ell}_i & (0 \leq \dot{\ell}_i) \end{cases}$$

$$v_{max,i}(\ell_i, a_i) = c_5, a_i F_{v,i}(\ell_i(\theta) - 1) + c_4$$

$$\tau_p(\dot{\theta}) = -c_9 \dot{\theta} - c_{10} \text{sgn}(\theta - c_{12}) \times \text{exp}(c_{11}(\theta - c_{12}) - 1)$$

where $r_i$ is a moment arm of each muscle with respect to the elbow joint, $\ell_i$ is muscle length (of the contractile element), $F_i \geq 0$ is contraction force of each muscle, $c_{1,1}, c_{2,1}, \ldots, c_{8,1}, c_{9,1}, \ldots, c_{12}$ are constants (see Appendix for details), $F_{max,i}$ is a maximal contraction force of each muscle (constant), $F_{v,i}$ is a force-length relation, and $F_{v,i}$ is a force-velocity relation of muscles. The force-length and force velocity relations of the monoarticular flexor and the passive torque are plotted in Fig. 3. The parameters of the
muscle model are set according to Stroeve (1999a), except that a part of parameters are modified to more realistic values of the range of motion of the elbow joint as in Fig. 3.

Gains of the neural feedback are adjusted in response to a task, such as phenomena known as the state-dependent reflex or phase-dependent reflex (Hultborn, 2001). However, the gains would be restricted considerably, since the feedback control can be destabilized by a feedback delay that is inevitable in the nervous systems. This may imply demand for reducing loads of the neural feedback for achieving the task.

This paper examines a possibility that a periodic orbit \( \ddot{\theta}(t) \) input to the control system (Fig. 4) is planned by the nervous system so that the stability of the orbit without using the neural feedback is relatively high. To increase the stability, the nervous system must rely on the physical feedback, such as visco-elasticity of the muscles.

Here, we consider a neural controller that controls the musculo-skeletal system in a feedforward manner without using the neural feedback, and evaluate the stability of the movement of the musculo-skeletal system under the feedforward control. To construct the feedforward controller that receives the desired orbit \( \ddot{\theta}(t) \) as input and sends the muscle activation \( a(t) \) as output, we devise the inverse models of the skeletal system (inverse skeletal model: ISM) and the muscle system (inverse muscle model: IMM) as in Fig. 5.

In response to the joint torque \( \ddot{\theta}(t) \) and the desired orbit \( (\ddot{\theta}(t), \dot{\theta}(t), \dot{\theta}(t)) \), the active and passive torques of the muscles to realize them are calculated as follows:

\[
\tau_w(\ddot{\theta}, \dot{\theta}) = -c_{\theta}\dot{\theta} - c_{10}\text{sgn}(\ddot{\theta} - c_{12}) \left\{ \exp(c_{11}|\ddot{\theta} - c_{12}|) - 1 \right\} \tag{12}
\]

\[
\tau_w(\ddot{\theta}, \dot{\theta}) = \tau(\ddot{\theta}) - \tau_w(\ddot{\theta}, \dot{\theta}) \tag{13}
\]

In response to given \( (\ddot{\theta}, \dot{\theta}, \ddot{\theta}) \) and \( \tau_w \), the muscle activity \( a(t) \) to realize them are calculated in the following two steps. Although there exists ill-posedness in each step, we solve it under assumptions described below.

**Step 1** There are infinite ways that divide a given active torque \( \tau_a \) into the torques exerted by the muscles.
\[ \mathbf{\tau}_{ac} = [\mathbf{\tau}_{a,1}, \ldots, \mathbf{\tau}_{a,4}]^T \] (Ill-posedness 1). We assume that \( \mathbf{\tau}_{ac} \) is selected such that under the constraints
\[ h_0(\mathbf{\tau}) = \mathbf{\tau}_{a,1} + \mathbf{\tau}_{a,2} + \mathbf{\tau}_{a,3} + \mathbf{\tau}_{a,4} - \mathbf{\tau}_a = 0 \] (14)
\[ h_1(\mathbf{\tau}_{a,1}) = \mathbf{\tau}_{a,1} \leq 0 \] (15)
\[ h_2(\mathbf{\tau}_{a,2}) = \mathbf{\tau}_{a,2} \leq 0 \] (16)
\[ h_3(\mathbf{\tau}_{a,3}) = \mathbf{\tau}_{a,3} \leq 0 \] (17)
\[ h_4(\mathbf{\tau}_{a,4}) = \mathbf{\tau}_{a,4} \leq 0 \] (18)

the evaluation function
\[ f(\mathbf{\tau}) = \sum_{i=1}^{4} \left( \frac{\tau_{a,i}}{V_i} \right)^2 \] (19)
is minimized, where \( V_i \) is the volume of the muscle (Assumption 1). The solution of this optimization problem is
\[ \mathbf{\tau}_{ac} = [\mathbf{\tau}_{a,1}, \mathbf{\tau}_{a,2}, \mathbf{\tau}_{a,3}, \mathbf{\tau}_{a,4}]^T \]
\[ = \begin{bmatrix} \frac{V_1^2 \tau_a}{V_1^2 + V_2^2} & \frac{V_2^2 \tau_a}{V_1^2 + V_2^2} & 0 & 0 \\ \frac{V_2^2 \tau_a}{V_2^2 + V_3^2} & \frac{V_3^2 \tau_a}{V_2^2 + V_3^2} & 0 & 0 \end{bmatrix}^T \]
\[ \text{(} \tau_a > 0 \text{)} \] (20)
\[ \text{and} \quad \text{(} \tau_a = 0 \text{)} \]

Step 2 The muscle activity \( a_i \), to realize \( \mathbf{\tau}_{a,i} \) and \((\hat{\theta}, \dot{\theta}, \ddot{\theta})\) must satisfy
\[ \mathbf{\tau}_{a,i} = r_i a_i F_{\max,i} F_{\ell,i} (\ell_i(\hat{\theta})) F_{\nu,i} (\ell_i(\hat{\theta}), \ell_i(\hat{\theta}), a_i). \] (21)

Note, however, that if the torque exceeds its maximal value corresponding to the maximal force of the muscle, i.e., \( \tau_{a,i}/r_i > F_{\max,i} F_{\ell,i} F_{\nu,i} / |a_{i-1}| \), (21) does not have any solution \( a_i \) in the range of \([0,1]\). In this case, we assume that a desired orbit is not realizable. On the other hand, if \( \tau_{a,i} = 0 \) and \( r_i \hat{\theta} > c_{4,i} - c_{5,i} \), \( a_i \) cannot be obtained uniquely, since there exists infinite \( a_i \in [0,1] \) that satisfies \( F_{\nu,i} = 0 \) (Ill-posedness 2). We assume that if \( \tau_{a,i} = 0 \), \( a_i = 0 \) is selected for simplicity (Assumption 2).

Under the above assumptions, (21) is solved with respect to \( a_i \) as
\[ a_i = \begin{cases} \frac{-d_{2,i} + \sqrt{d_{2,i}^2 - 4d_{1,i}d_{3,i}}}{2d_{1,i}} & (\ell_i > 0, \tau_{a,i} \neq 0) \\ 0 & (\tau_{a,i} = 0) \\ -d_{5,i} + \sqrt{d_{5,i}^2 - 4d_{4,i}d_{6,i}} \frac{d_{5,i}}{2d_{4,i}} & (\ell_i < 0, \tau_{a,i} \neq 0) \end{cases} \] (22)

\[ d_{1,i} = F_{\max,i} F_{\ell,i} c_{5,i} c_{6,i} c_{7,i} \]
\[ d_{2,i} = F_{\max,i} F_{\ell,i} \left\{ c_{6,i} c_{7,i} (c_{4,i} - c_{5,i}) + c_{8,i} \ell_i \right\} - c_{5,i} c_{6,i} c_{7,i} F_{\nu,i} / r_i \]
\[ d_{3,i} = -c_{6,i} c_{7,i} \left( c_{4,i} - c_{5,i} \right) + \ell_i \] / r_i
\[ d_{4,i} = F_{\max,i} F_{\ell,i} c_{5,i} c_{6,i} \]
\[ d_{5,i} = F_{\max,i} F_{\ell,i} c_{6,i} \left( c_{4,i} - c_{5,i} + \ell_i \right) - c_{5,i} c_{6,i} F_{\nu,i} / r_i \]
\[ d_{6,i} = \left\{ c_{6,i} (c_{5,i} - c_{4,i}) + \ell_i \right\} / r_i. \]

Note that, using (13), (20), and (22), the muscle activity \( a_i \) can be described as the function of a desired orbit \( a_i(\hat{\theta}, \dot{\theta}, \ddot{\theta}) \) (see also Fig. 5).

3.3 Summary of the model

The dynamics of the musculo-skeletal system driven by the muscle activity input \( a = [a_1, \ldots, a_4]^T \) described above in the feedforward manner is shown as a non-autonomous and nonlinear system:
\[ J\ddot{\mathbf{\theta}} = \mathbf{\tau}_a (\hat{\theta}, \dot{\theta}, \dot{\theta}, a (\dot{\theta}(t), \ddot{\theta}(t), \dddot{\theta}(t))) + \tau_p(\theta, \dot{\theta}) \] (23)
which has a solution corresponding to the desired orbit \( \ddot{\theta}(t) \).

4. STABILITY OF THE PERIODIC ORBIT

Let \( \theta = \hat{\theta} + \epsilon \) where \( \epsilon \) is small. Linearization of the motion equation (23) along the desired orbit \((\hat{\theta}, \dot{\theta}, \ddot{\theta})\) yields
\[ J\ddot{\mathbf{\theta}} + \epsilon \dot{\mathbf{\theta}} + \mathbf{\theta} = \mathbf{\tau}_a (\hat{\theta} + \epsilon, \dot{\theta} + \epsilon, \ddot{\theta} + \epsilon, a (\dot{\theta}(t), \ddot{\theta}(t), \dddot{\theta}(t))) \]
\[ + \tau_p(\theta + \epsilon, \dot{\theta} + \epsilon) \]
\[ \Rightarrow \mathbf{\theta} = \hat{\mathbf{\theta}}, \dot{\mathbf{\theta}}, a (\dot{\theta}(t), \ddot{\theta}(t), \dddot{\theta}(t)) \]
\[ + \epsilon \left[ \frac{\partial (\mathbf{\tau}_a + \tau_p)}{\partial \theta} \right] |_{\theta=\epsilon=\dot{\theta}=\ddot{\theta}} \]
\[ + \epsilon \left[ \frac{\partial (\mathbf{\tau}_a + \tau_p)}{\partial \dot{\theta}} \right] |_{\theta=\epsilon=\dot{\theta}=\ddot{\theta}} \]
\[ + \epsilon \left[ \frac{\partial (\mathbf{\tau}_a + \tau_p)}{\partial \ddot{\theta}} \right] |_{\theta=\epsilon=\dot{\theta}=\ddot{\theta}} \] (24)

which gives a variational equation
\[ J\ddot{\mathbf{\theta}} + B(\hat{\mathbf{\theta}}, \dot{\mathbf{\theta}}, \ddot{\mathbf{\theta}}) \dot{\mathbf{\theta}} + K(\hat{\mathbf{\theta}}, \dot{\mathbf{\theta}}, \ddot{\mathbf{\theta}}) \epsilon = 0 \] (25)

where
\[ K(\hat{\mathbf{\theta}}, \dot{\mathbf{\theta}}, \ddot{\mathbf{\theta}}) := \frac{\partial (\mathbf{\tau}_a + \tau_p)}{\partial \theta} \] (26)
\[ B(\hat{\mathbf{\theta}}, \dot{\mathbf{\theta}}, \ddot{\mathbf{\theta}}) := \frac{\partial (\mathbf{\tau}_a + \tau_p)}{\partial \dot{\theta}} \] (27)

Suppose a desired periodic orbit \( \hat{\theta}(t) = \hat{\theta}(t+T) \) of period \( T \) as a solution of the system (23). The desired orbit is shown to be stable if the eigenvalues \( \rho_1 \) and \( \rho_2 \) of the matrix
\[ \begin{bmatrix} e_{1}(T) & e_{2}(T) \\ \dot{e}_{1}(T) & \dot{e}_{2}(T) \end{bmatrix} \]
satisfy \( \max_j |\rho_j| < 1 \) (\( j = 1, 2 \)), where \((e_{1}(T), \dot{e}_{1}(T)), (e_{2}(T), \dot{e}_{2}(T))\) are integral values of the variational equation (25) from the initial states \((e(0), \dot{e}(0)) = (1,0), (0,1)\), respectively (Nayef and Balachandran, 1995).

In this paper, we quantify the ease of stabilization by \( \max_j |\rho_j| \), i.e., the smaller \( \max_j |\rho_j| \), the more stable the desired orbit.

5. RESULTS

5.1 Experimental results

The period and amplitude of the movement of measured data were defined practically by a time interval between successive peaks of flexion, and a half of an average of angle deviations of successive flexion-extension peaks and successive extension-flexion peaks, respectively.

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Fig. 6. Experimental results.

The relations between the period and the ‘comfortable’ amplitudes selected by the subjects were plotted in Fig. 6. The amplitudes varied among trials and between subjects. For instance, the data of the subject B at period 1.0 s seem to form two clusters, which shows variability among trials. The amplitude of subject B was larger than that of subject A in many cases, which shows variability between the subjects. On the other hand, the amplitude showed a tendency to increase as the period increase in both subjects.

5.2 Simulation results

The desired orbit was assumed as $\dot{\theta}(t) = A \sin(2\pi t/T) + \pi/3$ for simplicity in the model analysis. Then, stability of the desired orbits with various periods $T$ and amplitudes $A$ under the feedforward controller was analyzed.

Time responses of the system (23) to the sinusoidal desired orbit with $T = 0.5\,\text{s}$ and $A = 10\,\text{deg}$ were plotted in Fig. 7. The solid and dashed lines in the ‘Angle’ panel of the figure show the model’s angle and the desired angle, respectively. In the ‘Torque’ panel the solid and dashed lines show $\tau$ and $\tau_d$, respectively. In the ‘Flexor’ panel the solid and dashed lines show $\alpha_1$ and $\alpha_2$, respectively. In the ‘Extensor’ panel the solid and dashed lines show $\alpha_2$ and $\alpha_4$, respectively. In this condition, the orbit corresponding to the desired orbit was stable since $\rho = 0.609$, 0.0064. Actually, the angle deviation in the initial state in Fig. 7 appeared to decrease with time. The rotational elasticity and viscosity of the elbow joint appeared to change considerably through the period.

Results of the stability analysis for various periods $T$ and amplitudes $A$ were summarized in Fig. 8. The number of conditions of the periods and amplitudes, equally spaced over the range, were 400 and 240, respectively. The gray scale and the contour lines of the figure show $\max_j |\rho_j|$, which takes smaller values for more stable movement. The black region shows unrealizable movements requiring too large muscle forces beyond their maximal values. It appears that $\max_j |\rho_j| < 1$ is satisfied in the wide area of the periods and amplitudes, i.e., the desired periodic orbits can be stabilized even by the feedforward controller in many cases. The region with large amplitude ($A > 50$ deg) and a long period ($T > 1.0\,\text{s}$) showed high stability. There exists a ‘valley’ with relatively high stability, lying between $(A,T) \simeq (15,0.25)$ and $(A,T) \simeq (30,2.0)$. It is interesting to note that the stability of the movements with extremely small amplitude was lower than that of the movements with moderate or large amplitudes.

5.3 Discussion

Comparing the experimental results with the simulation results, the ‘comfortable’ period-amplitude relation ob-
served in the experiment (Fig. 6) was qualitatively similar to the ‘valley’ that showed relatively high stability in the model analysis (Fig. 8). This can support the hypothesis that the nervous system selects movements that are easy to stabilize in the feedforward control. However, the region with large amplitude ($A > 50$ deg) and a long period ($T > 1.0$ s) showed higher stability. This can suggest that the nervous system does not select the most stable movement.

Incidentally, if the nervous system prefers the most stable movement under the feedforward controller, it is reasonable to suppose that the controller should promote muscular co-contraction, since the co-contraction is widely believed to enhance the joint stiffness. However, it was supported by surface electromyography (EMG) that the co-contraction of muscles around the elbow joint was very small during the task (our unpublished observation).

If the wave forms are common, movements with smaller amplitudes require relatively smaller amplitudes of torque ($\tau$), power ($\dot{\tau}$), jerk ($\ddot{\tau}$), and torque change ($\tau$). However, the subject has not selected movements that can save these values, which may contradict the previous cost-minimization models (Hogan, 1984; Nagasaki, 1991; Uno et al., 1989). On the other hand, this paper revealed that the small amplitudes also lead to relatively low stability of movements as shown in Fig. 8. This could be a reason why the nervous system does not select movements with small amplitudes.

6. CONCLUSION

The relation between the periods and the amplitudes of the rhythmic forearm movement in the horizontal plane was studied based on the experiment and the model analysis. The stability of the rhythmic movements is evaluated in the musculo-skeletal model driven by the neural feedforward controller. The comparison of the experimental and computational studies showed that the measured period-amplitude relation was qualitatively similar to the condition such that the stability of the model’s movements was relatively high compared with smaller amplitude movements.

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REFERENCES


Appendix A. DETAILS OF PARAMETERS

The physical constants $c_{1,i}, c_{2,i}, \ldots, c_{8,i}, c_9, \ldots, c_{12}$ in this paper corresponds to parameters in Stroeve (1999a) as follows:

$$c_{1,i} = \ell_r - r_1(\theta_1 - \theta_{r1}) + r_2 \theta_{r2} - \ell_t$$
$$c_{2,i} = \ell_{cen} = \ell_{min} + L_{opt}(\ell_{max} - \ell_{min})$$
$$c_{3,i} = \ell_{esh} = L_{sh}(\ell_{max} - \ell_{min})$$
$$c_{4,i} = V_{vm}$$
$$c_{5,i} = V_{vm} V_{er}$$
$$c_{6,i} = V_{sh}$$
$$c_{7,i} = V_{sh}$$
$$c_{8,i} = V_{ml}$$
$$c_9 = B_2$$
$$c_{10} = \frac{T_{max}}{\exp(P E_{sh}) - 1}$$
$$c_{11} = \frac{P E_{sh}}{P E_{XH}}$$
$$c_{12} = \theta_{r2}$$

The parameters about the passive torque were $c_9 = 0.200$, $c_{10} = 1.52 \times 10^{-7}$, $c_{11} = 13.3$, and $c_{12} = 1.05$. See Stroeve (1999a) for meanings and other values of the parameters.