Distributed Reference Management
Strategies for Networked Water
Distribution Systems

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Abstract: A distributed constrained supervisory strategy for large-scale spatially distributed systems is presented and applied to a networked water distribution system. The aim is at reconfiguring, whenever necessary in response to different supply downstream demands, the nominal set-points to the tank water levels so that all prescribed coordination and operational constraints are enforced during the system evolutions and sustainable equilibria are reached.

The proposed supervisory strategy follows a coordination-by-constraint approach and is based on Feed-Forward Command Governor (FF-CG) ideas. The effectiveness of the strategy is shown in the final example where the supervision of an eight-tank interconnected system is considered.

Keywords: Nonlinear Control, Distributed Predictive Control, Command Governors.

1. INTRODUCTION

Several papers have been presented in the literature on the control of water distribution systems with approaches based both on centralized (Mayne et al. [2000], Wahlin and Clemmens [2006], Begovich et al. [2007], Ocampo-Martinez et al. [2009]) and distributed (Georges [1999], Fawal et al. [1998]) MPC schemes. Although the above distributed approaches require several iterations to achieve the solution they, under mild assumptions (e.g. linearity of the subsystem models and convexity of the optimization problems to be solved), allow one to achieve control performance that are similar to those achievable by centralized solutions. The main drawback of such iterative schemes lies in the huge amount of iterations required, giving rise to decision times remarkably larger than those typically used in such control applications. In fact, in order to overcome this problem, in (Negenborn et al. [2009]) a non-iterative distributed MPC scheme is proposed for the control of irrigation canals.

Here, a novel distributed supervising strategy is presented in charge to manage the water level set-points of the downstream tanks of a water network which is assumed to be subject to several pointwise-in-time coordination and operational constraints. The proposed supervisory strategy is based on predictive control ideas used to synthesize Command Governor (CG) in more traditional contexts (see Gilbert et al. [1995]). The CG unit is an add-on control device superimposed to a primal compensated system that, whenever necessary, modifies the reference to the closed-loop system into a feasible one in order to enforce pointwise-in-time constraints.

The proposed solution is based on a recently presented alternative solution (Garone et al. [2011]) to the CG design problem, referred to as the FeedForward CG (FF-CG) approach, that, at the price of some additional conservativeness, is able to accomplish the CG task in the absence of an explicit measure of the state. The idea behind such an approach is that, if sufficiently smooth transitions in the set-point modifications are acted by the CG unit, one can have a high confidence on the expected value of the state, even in the absence of an explicit measure of it, thanks to the asymptotical stability of the system. The peculiarities of the FF-CG scheme make it an attractive solution for distributed frameworks because they alleviate the need to make the entire aggregate state, or substantially parts of it, known to all agents at each time instant. This can result in a lower amount of information exchanged between the involved agents.

Based on the above FF-CG approach, here a low-communication “parallel” distributed scheme based on ideas recently described in (Garone et al. [2010]) is discussed and its usefulness in a water distribution system application shown. In the proposed scheme, whenever possible, all agents are allowed to modify their own reference signals simultaneously (parallel). The main technical expedient behind such a scheme is the on-line determination of a suitable Cartesian Inner Approximation of the global constraint set (Bertsekas [2007]), which allows the agents to optimize independently and simultaneously their reference signals while ensuring the fulfillment of the global constraints.

* This paper was supported by the Belgian Network DYSCO (Dynamical Systems, Control and Optimization) funded by the Interuniversity Attraction Poles Program, initiated by the Belgian State, Science Policy Office.
2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

The aim of this paper is to develop a distributed CG strategy for interconnected linear systems subject to convex set-membership input and state-related constraints. To this end, consider a set of $N$ subsystems $\mathcal{A} = \{1, \ldots, N\}$. Each subsystem is a LTI closed-loop dynamical system regulated by a local controller which ensures stability and good closed-loop properties when the constraints are not active (small-signal regimes when the coordination is effective). Let the $i$-th closed-loop subsystem be described by the following discrete-time model

$$\begin{align*}
    x_i(t+1) &= \Phi_i x_i(t) + C_i^g g_i(t) + \sum_{j \in \mathcal{A} \setminus \{i\}} F_{ij} x_j(t) \\
    y_i(t) &= H^g_i x_i(t) \\
    c_i(t) &= H^c_i x_i(t) + L_i g(t)
\end{align*}$$

where $t \in \mathbb{Z}_+$, $x_i \in \mathbb{R}^{n_i}$ is the state vector (which includes the controller states under dynamic regulation), $g_i \in \mathbb{R}^{m_i}$ the manipulable reference vector which, if no constraints (and no CG) were present, would coincide with the desired reference $r_i \in \mathbb{R}^{n_i}$ and $y_i \in \mathbb{R}^{m_i}$ is the output vector which is required to track $r_i$. Finally, $c_i \in \mathbb{R}^{n_c}$ represents the local constrained vector which has to fulfill the set-membership constraint

$$c_i(t) \in C_i, \quad \forall t \in \mathbb{Z}_+,$$

$C_i$, being a convex and compact polytopic set. It is worth pointing out that, in order to possibly characterize global (coupling) constraints amongst states of different subsystems, the vector $c_i$ in (1) is allowed to depend on the aggregate states and manipulable reference vectors $x = [x_1^T, \ldots, x_N^T]^T \in \mathbb{R}^n$, with $n = \sum_{i=1}^N n_i$, and $g = [g_1^T, \ldots, g_N^T]^T \in \mathbb{R}^m$, with $m = \sum_{i=1}^N m_i$. Moreover, we denote by $r = [r_1^T, \ldots, r_N^T]^T \in \mathbb{R}^m$, $y = [y_1^T, \ldots, y_N^T]^T \in \mathbb{R}^m$ and $c = [c_1^T, \ldots, c_N^T]^T \in \mathbb{R}^n$, with $n_c = \sum_{i=1}^N n_c$, the other relevant aggregate vectors. The overall system arising by the composition of the above $N$ subsystems can be described as

$$\begin{align*}
    x(t+1) &= \Phi x(t) + G g(t) \\
    y(t) &= H^g x(t) \\
    c(t) &= H^c x(t) + L g(t)
\end{align*}$$

where

$$\Phi = \begin{pmatrix} \Phi_1 & & & & \\ & \ddots & & & \\ & & \Phi_N & & \\ & & & \ddots & \\ & & & & \Phi_1 \end{pmatrix}, \quad G = \begin{pmatrix} G_1 & & & & 0 \\ & \ddots & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \ddots \end{pmatrix},$$

$$H^g = \begin{pmatrix} H^g_1 & & & & 0 \\ & \ddots & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \ddots \end{pmatrix}, \quad H^c = \begin{pmatrix} H^c_1 & & & & 0 \\ & \ddots & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \ddots \end{pmatrix}, \quad L = \begin{pmatrix} L_1 & & & & \end{pmatrix}.$$  

It is further assumed that

A1. The overall system (3) is asymptotically stable.

A2. System (3) is off-set free i.e. $H^g (I_n - \Phi)^{-1} G = 0$.

Roughly speaking, the CG design problem we want to solve is that of locally determine, at each time step and for each agent $i \in \mathcal{A}$ associated to each subsystem, a suitable reference signal $g_i(t)$ which is the best approximation of $r_i(t)$ such that its application never produces constraints violation, i.e. $c_i(t) \in C_i, \forall t \in \mathbb{Z}_+, \ i \in \mathcal{A}$.

Classical centralized solutions of the above stated CG design problem have been achieved by finding, at each time $t$, a CG action $g(t)$ as a function of the current reference $r(t)$ and measured state $x(t)$

$$g(t) := \tilde{g}(r(t), x(t))$$

such that $g(t)$ is the best approximation of $r(t)$ under the condition $c(t) \in C$, where $C \subseteq \{C_1 \times \ldots \times C_N\}$ is the global admissible region. Here we will focus on a slightly different approach in which the explicit dependence on the state vector disappears. This is a convenient solution to be used in a distributed environment because it eliminates the need to share the state vector amongst the agents that, as well known (see Dunbar [2007]), is one of the main difficulties in implementing distributed schemes. Such an approach, hereafter referred to as FeedForward CG (FF-CG), will be described in the next Section.

3. THE FEEDFORWARD CG APPROACH

In this section we will introduce the FF-CG as a possible solution to the CG problem. To this end, from the classical CG approach of Bemporad et al. [1997], let us recall, for a given $\delta > 0$, the sets:

$$C^\delta := C \sim B_\delta,$$

$$W^\delta := \{g \in \mathbb{R}^m : c_\delta \in C^\delta\}$$

where $B_\delta$ is the ball of radius $\delta$ centered at the origin and $C \sim E$ is the Pontryagin set difference defined as $\{a + e : a \in A, e \in E\}$. In particular, $W^\delta$, which we assume non-empty, is the convex and closed set of all constant commands $g$ whose corresponding equilibrium points $c_\delta := H^g (I_n - \Phi)^{-1} G g + L g$ satisfy the constraints with margin $\delta$. Let introduce also the virtual evolutions of the c-variable

$$\dot{c}(k, x(t), g(t)) := H^- \Phi^k (x(t) - x_{g(t)}):$$

$$\dot{c}(k, x(t), g(t)) = c_{g(t)} + H^- \Phi^k (x(t) - x_{g(t)}).$$

Because $g(t) \in W^\delta$ and, in turn, $c_{g(t)} \in C^\delta$ at each time $t$, then, a sufficient condition to ensure that the constraints are satisfied, although in a quite arbitrary and conservative way, is to ensure that the transient component is confined into a ball of radius $\rho_{g(t)}$

$$\|H^- \Phi^k (x(t) - x_{g(t)})\| \leq \rho_{g(t)}, \quad \forall k \geq 0$$

and the border of $C_{\rho_{g(t)}} := \arg \max \rho_{g(t)}$

subject to $B^\rho_{g(t)} \subseteq C$}

where $B^\rho_{g(t)}$ represents the ball of radius $\rho$ centered at $c_{g(t)}$. Assuming $C$ a polytope, $\rho_{g(t)}$ can be easily computed as the minimal distance between $c_{g(t)}$ and the border of $C$, i.e.

Then, the FF-CG design problem translates into the problem of defining an algorithm that is able to select, at each time $t$, a reference value $g(t)$ such that (8) holds true for all $k \geq 0$. This has been achieved in (Garone et al. [2011]) by selecting a suitable integer $\tau$, referred to as a Generalized Settling Time, and computing the FF-CG actions only every $\tau$ sampling steps while maintaining them constant between two subsequent updatings. As a consequence, if condition (8) were holding true at time
t − τ and a certain command \(g(t − τ)\) were constantly applied to the system, the transient contribution from \(t\) onwards could be bounded as follows

\[
\|H^\tau \Phi^k x_{\Delta g(t)}\| \leq \rho_k g(t−τ) + \Delta g(t) − \gamma \rho_k g(t−τ), \forall k \geq 0,
\]

where \(\Delta g(t) = g(t)−g(t−τ)\), and \(x_{\Delta g(t)} = x_{g(t)}−x_{g(t−τ)}\). Finally, we can formulate the FeedForward CG selection algorithm as follows

**The FF-CG Algorithm**

**REPEAT at each time**

1.1 IF \((t = \kappa \tau, \kappa = 1, 2, \ldots)\)

1.1.1 SOLVE

\[
g(t) = \arg \min_g \| g − r(t) \|^2_\Psi
\]

**SUBJECT TO**

\[
\begin{bmatrix} g \in \mathcal{W}^d \\
(g − g(t − τ)) \in \Delta \mathcal{G}_g(t−τ)
\end{bmatrix}
\]

1.2 ELSE \(g(t) = g(t − 1)\)

2.1 APPLY \(g(t)\)

where \(\Phi > 0\) is a weighting matrix and \(\Delta \mathcal{G}_g\) is defined as the set of all the possible \(\tau\)-step incremental commands from \(g\) ensuring inequality (10) to hold true

\[
\Delta \mathcal{G}_g = \{\Delta g : \|H^\tau \Phi^k (I − \Phi)\| \mathcal{G}_g\| \leq \rho_k \alpha_{\Delta g} − \gamma \rho_k, \forall k \geq 0\}.
\]

It is worth to note that the sets \(\mathcal{W}^d\) and the generalized settling time \(\tau\) can be computed off-line from the outset. Finally the following properties can be proved

**Proposition 1.** Let assumptions A1-A2 be fulfilled. Consider system (3) along with the FF-CG selection rule and let an admissible command signal \(g(0)\) in \(\mathcal{W}^d\) be applied at \(t = 0\) such that (8) holds true. Then:

1) the minimizer of (11), computed every \(\tau\) steps, uniquely exists and can be obtained by solving a convex constrained optimization problem;

2) constraints are fulfilled for all \(t \in \mathbb{Z}_+\);

3) the overall system is asymptotically stable and whenever \(r(t) \equiv r\), the sequence of \(g(t)\) converges in finite time either to \(r\) or to its best steady-state admissible approximation: \(g(t) \to 0 \equiv \arg \min_g \| g − r \|^2_\Psi\).

For details please refer to (Garone et al. [2011]).

### 4. DISTRIBUTED PARALLEL FF-CG

Here we introduce a distributed CG schemes based on the presented FF-CG approach assuming that the agents are connected via a communication network. Such a network is modeled by a communication graph: an undirected graph \(\mathcal{G} = (\mathcal{A}, \mathcal{B})\), where \(\mathcal{A}\) denotes the set of the \(N\) subsystems and \(\mathcal{B} \subset \mathcal{A} \times \mathcal{A}\) the set of edges representing the communication links amongst agents. More precisely, the edge \((i, j)\) belong to \(\mathcal{B}\) if and only if the agents governing the \(i\)-th and the \(j\)-th subsystems are able to directly share information within \(\tau\) sampling times. The communication graph is assumed to be connected, i.e. for each couple of agents \(i \in \mathcal{A}, j \in \mathcal{A}\) there exists at least one sequence of edges connecting \(i, j\) with the minimum number of edges connecting the two agents denoted by \(d_{i,j}\).

The set of all agents with a direct connection with the \(i\)-th agent will be referred to as Neighborhood of the \(i\)-th agent \(N_i = \{j \in A : d_{i,j} = 1\}\).

The two key points to be considered in building up such a kind of strategy are:

a) the definition of the information set available to each agent

b) the determination of a set of decentralized "selection rules" such that the composition of all feasible local commands satisfies global constraints (12).

Moreover, we will assume that each agent acts as a gateway in redistributing data amongst the other, no directly connected, agents. Then, at each time instant \(t\), the \(i\)-th agent has knowledge of the following vector:

\[
\xi_i(t−τ) = [g_1^T(t−d_{i,1}τ), \ldots, g_i^T(t−τ), \ldots, g_N^T(t−d_{i,N}τ)]^T.
\]

It results that the most recent information regarding the applied commands, which all agents share at each decision time \(t\), is given by the vector

\[
\xi(t−τ) = [g_1^T(t−d_{\max,1}τ), \ldots, g_N^T(t−d_{\max,N}τ)]^T
\]

where \(d_{\max,i} = \max_{j \in A} d_{i,j}\).

Then, the main idea behind the proposed selection rule is that of generating, every \(τ\) steps and on the basis of the information shared by all the agents of the network, a set of decoupled alternative and equivalent constraints (one for each agent) such that their local fulfillment implies the fulfillment of the global constraints (12). In other words, at each computation time step we substitute the admissible region (12) with its Set Cartesian Decomposition (Bertsekas [2007]). If such a decomposition is opportunely performed, the problem decouples and each agent will have simply to fulfill the inclusion into a local set of the form

\[
\Delta g_i(t) \in \Delta G_{v,i}(t), i = 1, \ldots, N
\]

with \(\Delta G_{v,i}(t) \subseteq \mathbb{R}^{n_i}, i = 1, \ldots, N\), convex and compact sets containing \(0_{n_i}\) for all \(t \geq 0\).

It remains to show how to generate local decoupled constraints that ensure global constraints satisfaction. The first step is to observe that if constraints (14) are satisfied at each time step, then we could define the set of all possible feasible values for \(g(t)\), computed on the basis of the common information vector \(\xi(t−τ)\), as follows

\[
\Xi(t−τ) = \{\xi(t−τ)\} + \left(\begin{array}{c}
\Delta G_{v,1}(t−τ) \times \ldots \times \Delta G_{v,N}(t−τ) \times \ldots \times \Delta G_{v,N}(t−d_{\max,1}τ) \\
\ldots \times \Delta G_{v,N}(t−d_{\max,N}τ)
\end{array}\right)
\]

where the operator + denotes the Pontryagin set sum defined as \(X + Y := \{z = x + y : \forall x \in X, \forall y \in Y\}\). On the basis of the above set of feasible values for \(g(t−τ)\), the set of all admissible aggregated command variations can be easily computed as follows

\[
\Delta \mathcal{G}_{\Xi(t−τ)} = \{\Delta g \in \mathcal{G}_d, \forall g \in \Xi(t−τ)\} \cap \{\Delta g + g \in \mathcal{W}^d, \forall g \in \Xi(t−τ)\}
\]

Finally, observe that the (approximated) cartesian decomposition giving rise to the agent-wise decoupled constraints (14) should satisfy the following set inclusion condition

\[
\Delta G_{v,1}(t) \times \ldots \times \Delta G_{v,N}(t) \subseteq \Delta \mathcal{G}_{\Xi(t−τ)}
\]

\[
0_i \in \Delta G_{v,1}(t), \ldots, 0 \in \Delta G_{v,N}(t)
\]

Note also that, because \(\Xi(t−τ)\) is a common information shared by all agents in the network, once an opportune objective function is defined all agents will be able to independently determine the same collection of sets \(\Delta G_{v,i}(t), i = 1, \ldots, N\). We can finally describe the Parallel FF-CG procedure to be performed every \(\tau\) steps:

1.1 Each agent determines the collection of sets \(\Delta G_{v,i}(t)\), \(i = 1, \ldots, N\) as the solution of its instance of the following optimization problem
max \( \Delta G_{v,i}(t), i = 1, \ldots, N \) subject to (17), (18), (19)

where \( V(\cdot) \) denotes a possible measure of the volume of a set (in order to achieve good dynamical properties it would desirable that \( \Delta G_{v,i}(t) \times \Delta G_{v,N}(t) \) would be as large as possible)

2.1 each agent chooses its own reference by solving the following convex optimization problem

\[
\begin{align*}
g_i(t) &= \arg \min_{g_i} \| g_i - r_i(t) \|_{Y_i}^2, \\
\text{subject to } (g_i - g_i(t - \tau)) &\in \Delta G_{v,i}(t)
\end{align*}
\]

(20)

where \( V(\cdot) \) denotes a positive if \( V(t) \geq 0 \)
0 if \( V(t) < 0 \)

The following local and global constraints are to be enforced at each time instant

A1-A2-A3 Each point belonging to \( \partial(W^a) \) is viable, \( \partial(W^b) \) denoting the border of \( W^b \)

For space limitations no other details are given here on the fulfillment of A3. The characterization of viable points, a computational way of checking if the viability property is satisfied by the polyhedral set \( W^b \) at hands and a geometrical method allowing one to compute suitable inner approximations of \( W^b \) satisfying A3 are presented in Casavola et al. [2011], where a proof that all internal points of \( W^b \) are viable is also presented.

Finally, we can present some properties enjoyed by the P-FFCG scheme.

Theorem 1. Let assumptions A1-A2-A3 be fulfilled. Consider system (3) as the composition of \( N \) subsystems in the form (1), along with the distributed P-FFCG selection rule, and let an admissible aggregate command signal \( g(0) = [g_1^T(0), \ldots, g_N^T(0)]^T \in W^b \) be applied at \( t = 0 \) such that (8) holds true. Then

1) for each agent \( i \in \mathcal{A} \), at each decision time \( t = kr, k \in \mathbb{Z}_+ \), the minimizer in (20) uniquely exists and can be obtained by solving a convex constrained optimization problem;

2) The overall system acted by agents implementing the P-FFCG supervisory policy never violates the constraints, i.e. \( c(t) \in \mathcal{C} \) for all \( t \in \mathbb{Z}_+ \).

3) Whenever \( r_i(t) \equiv r_i, \forall i \in \mathcal{A} \) with \( r_i \) constant set-points the sequence of \( g(t) = [g_1^T(t), \ldots, g_N^T(t)]^T \) asymptotically converges either to \( r=r_1^T(t), \ldots, r_N^T(t) \) if \( r \in \mathcal{W}^b \) or to a point \( \hat{r} \). In the case that \( d_{\max} = 1 \) such a point is Pareto-Optimal.

Remark 1 - The proof of Theorem (1), here omitted for brevity, is available at Casavola et al. [2010].

5. A EIGHT-TANK WATER DISTRIBUTION SYSTEM APPLICATION

Let us consider the water tank network depicted in Figure 1. The system consists of the interconnection of four cascaded two-tank models. Each cascaded subsystem is described by the following non-linear equations

\[
\begin{align*}
\rho S_1^2 h_1^i &= -\rho A_1^1 \sqrt{2gh_1^i} + u_i \\
\rho S_2^2 h_2^i &= -\rho A_2^2 \sqrt{2gh_2^i} + \rho A_1^1 \sqrt{2gh_1^i} + \sum_{j \in S^i} \rho A_2^1 \sqrt{2gh_1^j}
\end{align*}
\]

where \( u_i \) is the water flow supplied by the pump whose command is the voltage \( V_i, i \in \mathcal{A} := \{1, \ldots, 4\} \). Moreover, \( S^i, q = 1, 2 \), are the tank sections, \( h_1^i \), the water levels in the tanks, \( A_1^q \), the sections of pipes connecting the tanks, and \( g \) and \( \rho \) the gravity constant and the water density respectively. Their values are specified in Tables 1-2. \( S^i \), represents the set of subsystems which provide water to the downstream tank of \( i \)-th subsystem; in our case \( S^1 = \{2\}, S^2 = \{3\}, S^3 = \{4\} \) and \( S^4 = \emptyset \). Each subsystem has a related decision maker or agent with the task of regulating the levels \( h_1^i(t) \), \( i \in \mathcal{A} \) by acting properly on the incoming water flows \( u_i(t) \) and by exchanging information with other agents by means of the communication graph depicted in Figure 2. A simple static equation is used to model the relationship between the input voltage \( V_i(t) \) and the incoming mass of water.

![Fig. 1. A four cascaded two-tank water system](image)
Fig. 2. Communication graph

<table>
<thead>
<tr>
<th>Subsystem 1</th>
<th>Tank 1 Value</th>
<th>Tank 2 Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>2500 cm$^3$</td>
<td>2500 cm$^3$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>4 cm$^2$</td>
<td>8 cm$^2$</td>
</tr>
<tr>
<td>$h_1$</td>
<td>80 cm</td>
<td>70 cm</td>
</tr>
<tr>
<td>$h_1'$</td>
<td>1 cm</td>
<td>1 cm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subsystem $i \in {2,3,4}$</th>
<th>Tank 1 Value</th>
<th>Tank 2 Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_i$</td>
<td>2500 cm$^3$</td>
<td>2500 cm$^3$</td>
</tr>
<tr>
<td>$A_i$</td>
<td>8 cm$^2$</td>
<td>8 cm$^2$</td>
</tr>
<tr>
<td>$h_i$</td>
<td>80 cm</td>
<td>70 cm</td>
</tr>
<tr>
<td>$h_i'$</td>
<td>1 cm</td>
<td>1 cm</td>
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Table 1. Tanks and constraints values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$</td>
<td>980 cm/sec$^2$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$10^{-3}$ Kg/(cm$^3$)</td>
</tr>
<tr>
<td>$V_{\text{max}}$</td>
<td>$V_{\text{c}}$</td>
</tr>
<tr>
<td>$T_c$</td>
<td>0.8 sec</td>
</tr>
</tbody>
</table>

Table 2. Parameter values

$|h_{1i} - h_{3i}| \leq 5$ cm, $|h_{2i} - h_{3i}| \leq 5$ cm, $|h_{1i} - h_{2i}| \leq 5$ cm

The system is linearized around the equilibrium $\bar{V}_i = \bar{u}_{i, \text{eq}} = 2$, $i \in A$. $h_1' = 32$ cm and discretized with sampling time $T_c = 0.8$ sec. In addition, for each subsystem a local pre-compensator consisting in a decentralized LQ output feedback controller (Zhu and Pagilla [2006]) is implemented in such a way that the offset property $A_2$ is guaranteed. The reported simulations investigate the behavior of the overall system when the desired set-points to the water levels of the downstream tanks have the profiles depicted in Figure 4 (red dashed line). At the beginning, the desired references $r_i$, $i \in A$ are set at the equilibrium $r_i = 32$ cm, $i \in A$. At time $t = 30$ sec, the reference $r_1$ related to the downstream tank of subsystem 1 is changed from 32 cm to 42 cm. At the same time, also the reference $r_2$ is modified from 32 cm to 34 cm. These values are kept constant until time instant $t = 400$ sec when they are changed back to their initial values. Simultaneously, the desired references $r_3$ and $r_4$ change their levels at time $t = 300$ sec from 32 cm to 27.85 cm and to 28.5 cm respectively and hold these new values until time $t = 800$ sec when they are brought back to the previous levels. In Figures 5-6, the constrained vector responses can be observed. It is important to note how such a vector violates the constraints at several time instants when a CG device is not used. On the contrary, this never happens when any kind of CG unit is used. In particular, the responses of the classical CG, the FF-CG and the P-FFCG units are all reported for comparisons.

In Figure 3, the evolutions of the downstream water levels are depicted while Figure 4 shows the various CG actions.

Fig. 3. Water levels in the downstream tanks

Fig. 4. CG actions

Fig. 5. Coordination constraints

The standard CG and the FF-CG centralized schemes have similar coordination performance. On the contrary, the distributed P-FFCG seems to have a slower response to changed conditions. Nevertheless, the related performance, especially during equilibria, are quite good if compared to centralized algorithms. The reason of this slight performance degradation is associated to the conservativeness introduced by P-FFCG and can be observed in Figures 5-6. There, the constrained signals related to the P-FFCG supervision scheme do not approach the constraint boundaries. This, on the contrary, happens under the centralized
6. CONCLUSIONS

In this paper, distributed FFCG schemes have been developed for the supervision of dynamically coupled interconnected linear systems subject to local and global constraints and used for solving constrained coordination problems in networked water distribution systems.

A parallel distributed strategy has been proposed and its feasibility and stability properties analyzed in some details which have demonstrated in the final example their effectiveness also when contrasted with centralized solutions for the same problem. The presented results are encouraging and stimulate further research on the topic.

REFERENCES


Table 3. CPU Time (seconds per step): it is related, in the distributed case to single agent and in the centralized case to the unique supervisor in charge. Exchanged information: in the distributed case it is the amount of information exchanged by an agent with the rest of the network (it is the same for each agent in this example), in the centralized case it is the information received and transmitted by the unique CG or FFCG device.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>CG</th>
<th>FFCG</th>
<th>P-FFCG</th>
</tr>
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<td>RX/TX Data</td>
<td>0.61</td>
<td>0.43</td>
<td>0.18</td>
</tr>
<tr>
<td>ms</td>
<td></td>
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</tr>
</tbody>
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Fig. 6. Applied Voltages