SELECTING THE CONTROL CONFIGURATION FOR A NONLINEAR SYSTEM BY ON-LINE CONTROLLERS’ PARAMETERS OPTIMIZATION

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Abstract: The multiobjective nature of the most real engineering control systems requirements and how to determine the best control strategy for a specific problem motivate the development of optimization-based design frameworks and computer aided control system design methods to search for acceptable designs with low computational load. It is proposed an approach to find in real time the best control strategy by optimizing the controllers’ parameters of three control strategies for the various operating points of a nonlinear system. The proportional-integral-derivative (PID) controller, the coprime factor uncertainty using H-infinity optimization and the mixed sensitivity H-infinity control strategies are used for this purpose. The genetic algorithm is used to tune and optimize the controllers’ parameters, penalizing the individuals of the control strategies that do not satisfy the problem’s given performance, and then defining the best control strategy for each operating point of the system. Designing controllers as presented offers the advantage of achieving the plant best working conditions in real time.

Keywords: PID controllers, H-infinity control, mixed sensitivity problem, genetic algorithms.

1 INTRODUCTION

Evolutionary algorithms together with computer aided control system design have been extensively used to design suitable controllers to meet the desired performance specification of complex systems and other applications. Optimization based methods have shown to be a valuable tool in assisting the control engineer in selecting suitable controller parameters. The application of evolutionary computing techniques, in general, and genetic algorithms (GA’s) in particular has seen wide ranging applications in control systems design (Herreros et al., 2002, Coello et al., 2005, Oduguwa, 2005, Vlachos, 2002).

In this work, the Continuous Stirred Tank Reactors (CSTR), which presents operational problems due to complex open-loop nonlinear behavior in the form of input/output multiplicities, is chosen as the test system. A typical control system for a CSTR would include a set of controllers, PID for example, for each operating point. Gain scheduling is then used to combine these controllers to vary the controller parameters with time and other changes. The problem is not the set-point stability performance while the CSTR is in the steady-state conditions of the particular operating point. Degradation of the controllers’ performance is often observed in the change phases between set points due to demand changes or disturbance signals. Robustness is difficult to be achieved in such cases, but it is a key requirement for improving performance of this system.

There is a trend to design controllers by eliminating industrial processes modeling uncertainties (Jelali, 2006; Ingimundarson and Hägglund, 2005). H-infinity is a very popular control design methodology for achieving robust control by minimizing any transfer function of the plant augmented by model uncertainties (Kozub, 2002; Hoo et al., 2003; Parlaktuna and Ozkan, 2004; Petronilho et al., 2005; Costa and Silva, 2008).

Tomaz and Silva (2004) and Costa and Silva (2008) designed a robust controller using the coprime factor uncertainty H-infinity loop shaping and mixed sensitivity H-infinity, for a linear model of the CSTR. These controllers were then applied to a non-linear system model. The non-linear model was used to simulate all operating conditions. The PID, the coprime factor uncertainty using H-infinity optimization (CFU H-infinity) and the mixed sensitivity H-infinity control strategies were implemented, and the genetic algorithm (GA) run to tune and optimize the controllers’ parameters. This approach aims to find in real time the best control strategy by optimizing the controllers’ parameters of the three control strategies for each operating point of the CSTR. The GA penalizes the individuals of the three control strategies that do not satisfy the problem’s given performance, thus, establishing the best control strategy for each operating point of the CSTR.
2 CONTINUOUS STIRRED TANK REACTOR (CSTR)

The most important unit operation in a chemical process is generally a chemical reactor. Chemical reactions are either exothermic (release energy) or endothermic (require energy input) and therefore require that energy either be removed or added to the reactor for a constant temperature to be maintained. Exothermic reactions are usually studied because of the potential safety problems (rapid increases in temperature) and the possibility of multiple steady-states. The system under consideration is the CSTR that generally presents operational problems due to complex open loop nonlinear behavior in the form of input/output multiplicities (Russo, 1996). These nonlinear characteristics prove the need and the complexity of the control system design. Linear and nonlinear SIMULINK models are available for this process (Fig 1).

![Fig. 1. SIMULINK model of the CSTR.](image)

The differential equations (1) and (2) describing the system in terms of dimensionless variables (Russo, 1996) are:

\[
\begin{align*}
\frac{d}{d\tau} x_1 &= -\phi x_1 k(x_2) + q(x_{1f} - x_1) \\
\frac{d}{d\tau} x_2 &= \beta \phi x_1 k(x_2) - (q + \delta)x_2 + \delta m + q x_{2f} \\
k(x_2) &= \exp\left(\frac{x_2}{1 + \frac{x_2}{\gamma}}\right)
\end{align*}
\]

where \(x_1\) is the concentration, \(x_2\) is the temperature (controlled variable), and \(m\) is the temperature of the pack of cooling (manipulated variable). \(K(x_2)\) is a dimensionless function and the parameters values are given in Table 1.

### Table 1: The process parameters values

<table>
<thead>
<tr>
<th>(\beta)</th>
<th>(\phi)</th>
<th>(\delta)</th>
<th>(\gamma)</th>
<th>(q)</th>
<th>(x_{1f})</th>
<th>(x_{2f})</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0</td>
<td>0.072</td>
<td>0.3</td>
<td>20.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The chemical process presents three operating points, where the second point is unstable, and \(m\) is set 1.0 for all cases (Table 2).

### Table 2: Operating points

<table>
<thead>
<tr>
<th>(x_{1s})</th>
<th>(x_{2s})</th>
<th>(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8560</td>
<td>0.5528</td>
<td>0.2354</td>
</tr>
<tr>
<td>0.8859</td>
<td>2.7517</td>
<td>4.7050</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The objective to be addressed by the GA is the steady-state error \((e_s)\) less than 1%. The system is also required to meet the following design constraints:

i - overshoot \((M_o)\) not exceeding 15%,
ii - the smallest possible settling time \(t_s\),
iii - the smallest possible rising time \(t_r\).

3 PID CONTROLLER

Most industrial processes are controlled by PID controllers. To implement such a controller, three parameters should be determined: proportional gain \((K_p)\), which has the effect on reducing the steady-state error \((e_s)\); the integral time \((T_i)\), which has the effect of eliminating the \(e_s\), but it may worsen the transient response; derivative time \((T_d)\), which improves the stability, transient response and reduces the overshoot. So far, great effort has been devoted to develop methods to reduce the time spent on optimizing the choice of controller parameters, such as in Ko and Edgar, (2004); Chen and Seborg, (2004) and Herreros et al., (2002). For a wide range of practical processes, those tuning approaches work quite well, but sometimes it is laborious and time-consuming, particularly for nonlinear processes. Therefore, those methods usually need manual retuning before being used to control industrial processes.

4 LOOP-SHAPING H-INFINITY DESIGN

A plant \(G\) can be represented by a normalized left coprime factorization (Glover and McFarlane, 1989):

\[
G = M 'N
\]

The perturbed model of the \(G\) can be represented as:

\[
G_p = (M + \Delta_m)'(N + \Delta_n)
\]

where \(\Delta_m\) and \(\Delta_n\) are stable unknown transfer functions which represent the uncertainty in the nominal plant \(G\). For the perturbed feedback system in figure 2, it is desired to find a controller \(K\) that stabilizes a set of perturbed plants \(G_p\) defined by:

\[
G_p = (M + \Delta_m)'(N + \Delta_n)\|A_x \Delta_m\| \leq \varepsilon
\]

where \(\varepsilon > 0\) is the stability margin and \(\|\cdot\|_\infty\) is the H-infinity norm.
The controller $K$ is the control solution of the robust stability property, found by minimizing equation 7 (McFarlane and Glover 1990).

$$\gamma^+ = \left\{ \begin{array}{l} \gamma \leq 1/(1-GK)^{-1} M \end{array} \right. \leq \frac{1}{\epsilon} \quad (7)$$

Fig. 2. H-infinity robust stabilization problem.

The maximum stability margin is achieved when the lowest value of $\gamma$ is found by:

$$\gamma_{\text{min}} = \frac{1}{\gamma_{\text{max}}} = (1 + \rho(XZ))^2 \quad (8)$$

where $\rho$ is the maximum magnitude of the eigenvalues. $X$ and $Z$ are the unique positive definite solutions to the algebraic Riccati equations:

$$\left( A - BS'D' \right) Z + Z \left( A - BS'D' \right)' - Z C R' C Z = 0 \quad (9)$$

$$\left( A - KS'D' \right) X + X \left( A - BS'D' \right)' - X B S' B' X + C' R' C = 0 \quad (10)$$

where $R = I + D D'$

$$S = I + D' D \quad (11)$$

and $A$, $B$, $C$, $D$ are the state space representation of $G$.

The state-space equations of the controller $K$ are generated solving the algebraic Riccati equations. The solution to the H-infinity optimization problem in general is obtained by reducing $\gamma$ iteratively until the minimum is reached within a given tolerance. This process yields a controller which is sufficiently close to being optimal.

Because robust stabilization does not include any reference to performance requirements, this controller gives no guarantee of the system’s performance, but rather robust stability. Prior to robust stabilization, performance specifications are translated into frequency domain, and the plant is pre- and post-compensated by weighting functions (Fig. 3) to shape the open-loop singular values (McFarlane and Glover, 1990 and 1992).

The performance specifications required for the closed-loop system are thus included in the design.

The shaped plant $G_s$, is given by:

$$G_s = W_s G W_t \quad (13)$$

where $W_t$ and $W_s$ are the pre- and post-compensators respectively, and is robust stabilized against coprime factor uncertainty. The final feedback controller is the combination of $K$ and the two weighting functions:

$$K = W_t K W_s \quad (14)$$

A difficulty that sometimes arises with H-infinity control is the selection of weights such that the H-infinity optimal controller provides a good trade-off between conflicting objectives.

5 MIXED SENSITIVITY H-INFINITY DESIGN

Assuming that we have a regulation problem in which we want to reject a disturbance $D$ entering at the plant output and that the measurement noise is relatively insignificant (Fig. 4). For this problem it makes sense to shape the closed-loop transfer functions $S = (I+GK)^{-1}$ between $D$ and the output, and $K S$ the transfer function between $D$ and the control signals. It is important to include $K S$ as a mechanism for limiting the size and bandwidth of the controller, and hence the control energy used.

$$W_1 K W_2 T$$

Fig. 3. The shaped plant and controller

A cost function is minimized to eliminate the process disturbances and make the system more robust. To solve the optimization problem the $H_{\infty}$ norm is applied to the sensitivity functions augmented by the pre-compensators to limit the maximum values of these functions, and determine the stabilizing controller $K$ that minimizes the cost function given in (15):

$$\begin{bmatrix} W_s S \\ W_s T \\ W_s KS \end{bmatrix} \quad (15)$$

Regarding Eq. (15), the disturbance $D$ is typically a low frequency signal. Therefore it will be successfully rejected if the maximum singular value of $S$ is made small over the same frequency range.
low frequencies, finding a stabilizing controller that minimizes $\|y, S\|_\infty$, where $W_1(s)$ is a scalar low pass filter with the bandwidth equal to that of the disturbance. A high pass filter $W_2(s)$ with a crossover frequency approximately equal to the desired closed-loop bandwidth is used to limit the control inputs in high frequencies. The matrix $W_3(s)$ is chosen to weight the input signal and not directly to shape $S$ or $K_S$. The main aim of $W_3(s)$ is to force equally good tracking of each of the primary signals.

The state-space equations of the controller $K$ are obtained by solving the algebraic Riccati equations. Figure 4 shows the sensitivity functions minimization for the input function tracking.

6 THE GENETIC ALGORITHM

The genetic algorithm is a versatile evolutionary optimization algorithm that usually works with a population of points or solutions instead of single point search and evolves this population towards overall improvement by encouraging reproduction and sharing of good attributes between the various members of the population. No derivative information is required.

The basic mechanisms behind a GA are the use of a parallel search where solutions in one search iteration or generation undergo a number of transformations in order to achieve better solutions in the next generation. Fit individuals are expected to emerge from the search where the optimal solution found is represented by the fittest individual obtained. A GA searches from a population of search points, not a single one. This helps to reduce the possibility of the search being trapped in local minima within the search space. In its search mechanisms, it uses probabilistic transmission rules.

These features allow GA’s to work in difficult multi-modal spaces characterized by disjoint feasible domains. The main cost of using a GA is that of function evaluation. Increasing the efficiency of the GA will mean reducing the number of trials (the number of generations multiplied by the number of individuals per population) needed to arrive at optimal or near-optimal solutions. Pre-mature convergence and entrapment in regions of local optima have to be avoided.

A GA starts by creating a random initial population of potential solutions, which is composed of a set of fixed-length individuals. The individuals of this population are then encoded and concatenated into strings. In each generation, the population evolves by using the main genetic operators in GA’s: selection, crossover and mutation. Individuals are chosen based on their fitness measure to act as parents of the offspring which will constitute the new generation. A new population will be created by proportionally reproducing fit individuals from the current population according to their fitness and inserting them in the new population.

7 CONTROL STRATEGIES FRAMEWORK

The GA is used to search for the optimal PID parameters and for the weighting functions that shape the open-loop plant to give the robust stabilization features whilst satisfying performance requirements of design optimization [12]. To evaluate the controller’s performance, the simulation of the non-linear model of the CSTR is taken as the objective function. The controller block alternates between the PID, the CFU H-infinity and the MS H-infinity control strategies.

The chromosome for the PID controller is divided in three sections, each one corresponding to one parameter of this controller (Fig. 5-a). For the CF H-infinity strategy, the chromosome is divided in four sections. The first three sections give the $W_1$ (Equation 16). Matrix $W_2$ is a diagonal matrix of two constants, thus the fourth section gives $k_2 = [k_2(1) k_2(2)]$ (Fig. 5-b). The chromosome for the (MS H-infinity) is divided in 5 sections: the first two are for the low pass filter pole ($P_b$) and zero ($Z_b$); the third and fourth are for the high pass filter pole ($P_a$) and zero ($Z_a$) respectively, and the last section gives the values of the weighting function $W_3$ (Fig. 5-c).

$$K_c = \frac{(s + z)}{(s + p)}$$  \hspace{1cm} (16)

(a) $K_c$, $K_i$, $K_d$

(b) $K_c$, $Z_a$, $P_a$, $K_a$

(c) $P_b$, $Z_b$, $P_b$, $Z_b$, $K_b$

Fig. 5: Chromosome structure: (a) PID; (b) CF H-infinity and (c) MS H-infinity.

Linearized models in state space for the three operating points of the CSTR were obtained to determine the state space $H_{\infty}$ controllers using the Riccati equations for a particular weighting function set. These designs are then inserted into the non-linear SIMULINK CSTR model controller block. The simulation of the model allows for evaluation of all the required performance criteria (Section 2) aggregated in a single objective function for optimization purposes. For all the design examples given here, CSTR performance is evaluated corresponding to a step response between the various operating
points. The complete controller design involves finding controller parameters covering the various operating points within the temperature range of the CSTR.

8 THE GENETIC ALGORITHM IMPLEMENTATION

Due to the parallel nature of GA, the three problems of designing the weighting functions for the two H-infinity controllers’ strategies and a set of PID parameters run concurrently. The controllers’ parameters were represented in a Gray-coded bit string chromosome structure, and the genetic algorithm parameters were set the same for the three cases as: 32 alleles for each variable to be optimized, 20 individuals, 10 generations and mutation rate of 1%. Step inputs between the operating points were applied to evaluate the nonlinear model.

The three implementations are the same in the way the design problem is proposed. Each problem has its own population of solutions. Each population experiences selection and crossover driven by the same cost function. To achieve a consistent overall design, each population presents its proposed solutions to perform the CSTR simulations. Results of the simulations are used by each separate GA concurrently to scan for its relevant outputs and drive its own GA operators. The three design frameworks employ the GA toolbox of MATLAB™.

Optimization is carried out using a single aggregating objective with a simple penalty function for the constraints. This function gives the adaptation level of each possible solution (individual) to the problem; the better the solution the smaller the penalty. The optimization for steady state reduction can be described by:

Minimize: steady state error + constraint violation penalties

- overshoot penalty:
  If higher than maximum value:
    penalty = 800 * [exp(actual value) + penalty]
  Else: penalty = zero

- stabilization time penalty:
  If higher than maximum value:
    penalty = 25 * [exp(actual value) + penalty]
  Else: penalty = zero

- rise time penalty:
  If higher than maximum value:
    penalty = exp(actual value) + penalty
  Else: penalty = zero

The control response is measured for a step input in temperature corresponding to changes between the various operating points. The performance of the best PID and the best of both H-infinity controllers are compared for each operating point. The best among them is chosen to control the plant and then define the control strategy for that operating point. Using three populations of solutions increases the requirements to compute the GA’s operators for ranking, selection, crossover and mutation. The set up of these parameters were found to be a compromise in designing the control systems and the computational cost involved in the design process, since the parameters optimization happens during the step response between two operating points.

9 RESULTS

Three design frameworks employing the GA were applied to design PID and the two H-infinity control strategies. The GA parameters were set the same for the three cases. All the design specifications were achieved. The convergence graphs (Fig. 6-a,b,c) show that improved performance was achieved for the MS H-infinity control strategy that outperformed the other two for all scenarios at the various operating points.
The performance of the two not chosen control strategies for step responses from 0 to the first operating point (Fig. 7), indicates the responses are slower achieving the steady state and shows similar transient behavior. For the PID controller, the response presented slightly higher error than for the others.

![Fig. 7. Step responses for the three strategies from 0 to the first operating point](image)

Figure 8 shows the step response for all the three controllers corresponding to changes from the first to the second operating point. In this case, the PID controller showed slightly better behavior than the coprime factor uncertainty H-infinity optimization.

![Fig. 8. Step responses for the three strategies from the first to the second operating point](image)

For a step input from the second to the third operating point (Fig. 9), it can be seen that the model controlled by the CFU H-infinity strategy presented higher overshoot and thus, was more penalized and was worse classified. All the design specifications: settling time, rise time, overshoot and steady state error were better for the plant controlled by the mixed sensitivity H-infinity strategy.

![Fig. 9. Step responses for the three strategies from the second to the third operating point](image)

The three control strategies perform well in terms of the design specification, and the differences are small. The one that presents even slightly better performance defines the control strategy to be applied for the operating point in question.

Table 3 shows the GA running time to determine the controller parameters for each operating point. It can be seen that the MS H-infinity parameters were obtained in shorter time than the others for all the operating points. Since this is an on-line design procedure, running time is an important issue to be considered. Performance robustness, the main goal, was better achieved by mixed sensitivity design strategy even in shorter time as expected.

<table>
<thead>
<tr>
<th>Table 3: GA running time of each control strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Point</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>PID</td>
</tr>
<tr>
<td>CFU H-infinity</td>
</tr>
<tr>
<td>MS H-infinity</td>
</tr>
</tbody>
</table>

Figure 10 also shows the controlled variable step responses between the different operating points in ascending and descending orders.

![Fig. 10. Step responses between set points](image)
10 CONCLUDING REMARKS

The genetic algorithm was combined in this work with the PID, the coprime factor uncertainty using H-infinity and mixed sensitivity H-infinity control design strategies for real time design procedure of a control system for the CSTR. PID controllers are easy to implement and very applicable to control many industrial processes. The use of the H-infinity controller leads to increased robustness.

The presented approach aims to select a control strategy in real time by optimizing the controllers’ parameters using genetic algorithm. It is inherently simple to set up and implement for a variety of control strategies design scenarios.

Good results indicate the viability of this approach for such applications. Designing these controllers on-line offers the advantage of achieving the plant best working conditions in real time.

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