A Supervised Robust Predictive Multi-Controller for Large Operating Conditions of an Open-Channel System

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Abstract: In this paper, a method for designing and for choosing parameters of a supervised robust predictive multi-controller strategy (SRPMC) is proposed. The SRPMC strategy is based on a multi-model representation taking into account the large operating conditions of the real process. In this application to an open-channel system, multiple discrete models were identified for representing the overall range of discharge. The number of models is given by an algorithm initialized by the choice of only one parameter. The SRPMC strategy is compared with a PID control strategy showing its effectiveness and better performances.

Keywords: Multi-model, multi-controller, predictive control, robustness, switched control.

1. INTRODUCTION

The problem of controlling processes that operate within a wide range of operating points requires the decomposition into a set of operating regimes to which a set of local models is associated. Based on this model representation, considering local, global, linear and nonlinear models, Foss et al. proposed and discussed the performances of five predictive controllers with the same performance criterion and constraints, equal control input parameterization and identical optimization algorithms for a batch fermentation reactor [8]. The decomposition into operating regimes is a critical part of the modeling because the applied process knowledge is needed to get a sound model structure. The multimodeling problem is addressed herein. The number of models necessary to cover large operating conditions is discussed.

The difficulties involved by hydraulic systems are nonlinearity, delays and uncontrolled perturbations. In order to compensate the time delay and to overcome the system nonlinearity, Generalized Predictive Control (GPC) has been investigated by Akouz et al. [1] showing that the technique is rather suitable to the regulation of a three-reach canal. Predictive control, based on the output prediction over a long time horizon and the minimization of a quadratic cost function is slightly sensitive to the magnitude and variation of the delay. As shown by Rutz et al. [15], the performance of GPC decreases with variations in the operation conditions. For systems with parametric disturbances, Ramond et al. [14] develop a direct version of adaptive generalized predictive controller. Di Palma and Magni [7] present a study to determine the better model structure in order to make prediction for nonlinear model predictive control. It is proposed a multi-model structure that full use the available information with a correct choice of the regressors. An alternative approach based on a multi-controller synthesis is studied herein.

Sawadogo et al. [16] decomposed a canal into pools separated by gates where the flow depends on both upstream and downstream water level. The proposed decentralized predictive control scheme takes into account explicitly the interactions among the pools. The upstream gate is controlled to maintain the downstream water level as close as possible to a setpoint value in presence of external perturbations. The predictive controller performance was evaluated in realistic conditions and compared to four controllers proposed in the literature showing its effectiveness. Gomez et al. [9] proposed a discrete-time decentralized control scheme for irrigation canals made of a series of pools connected by active gates. The predictive controller is formulated based on a hydrologic time marching process describing the pool behavior. Finally, all the cited papers do not deal with the open-channel control for large operating conditions as discussed in the proposed approach.

In this paper, a method for designing a Supervised Robust Predictive Multi-Controller (SRPMC) is proposed. The SRPMC strategy is expressed in a polynomial form which facilitates the stability and robustness analyses. It is combined to a multi-model representation to take into account the large variations of the real process. An open-channel system is represented by Saint Venant equations and using the usual simplifications, the Hayami transfer is derived for a given operating point. The SRPMC strategy is compared with a PID control strategy presented in [12] showing the better performances.
2. SUPERVISED MULTI-CONTROLLER

A supervised multi-controller structure is required to control a plant with several Operating Modes (OM) [4]. An active supervisory control of OM integrating an additional detection-accommodation loop is presented in 1. OM representation is a common way to establish the link between the plant operator skills and the minimal knowledge for the design of supervision system. This monitoring structure consists of two combined blocks, one for the model-based detection, Operating Mode Detection (OMD), which allows detecting a given process operating mode, and the other one for the control accommodation decision, Control Law Accommodation (CLA), which selects the right controller.

Intuitively the notion of operating mode is linked to the tracking objective and also to its closed-loop performance. These modes induce a partition of the process model \( G \) of the plant \( P \) into a finite class of linear models \( G = \{G_1, G_2, \ldots, G_g\} \), where the \( i \)th linear model of the plant \( P \) is denoted \( G_i \) and \( g = \text{card}(G) \), to which is associated a controller family. The corresponding designed controller achieves the best performance of the closed-loop \( (G_i, C_i) \). In order to determine when and to which controller one should switch, a detection method is detailed in the following. The detector consists of three functions:

- the simulation of the models \( G = \{G_i, i \in I\} \) controlled by the signal \( u \), which is the output of the active controller;
- the residue evaluation for each output model according to the fixed criterion;
- the function mode isolation based on the detection rule.

For each \( i \in I \), at the time \( kT_d \) where \( T_d \) is the monitoring period, the multi-model output recursive square error criterion \( J(kT_d) = [J_1(kT_d), J_2(kT_d), \ldots, J_n(kT_d)] \) is computed with the recursive formula:

\[
J_i(kT_d) = J_i(kT_d - 1) + \frac{1}{N} (\varepsilon_i^2(kT_d) - \varepsilon_i^2(kT_d - N)).
\]

where \( N \) is the size of the sliding window, \( \varepsilon_i(u, k) = y(k-T_n) - y_i(k-T_n) \) is the \( i \)th identification error, \( y \) the process output and \( y_i \) the \( i \)th estimate of the process output, with \( k > N \).

The couple \( (D, t_d) \) defines the detection test which describes each OM detected \( D(kT_d) \) and the detection time \( t_d(kT_d) \).

The detection rule is computed on-line by:

\[
D(kT_d) = \{P = G_m, m = \arg \min J_i(kT_d)\}, \quad 1 \leq i \leq g.
\]

At each monitoring period, a minimization of the criterion given by (1) is carried out to activate the controller corresponding to the model with the smallest index. At the starting time, it is assumed that \( D(0) = G_1 \). The detection time is defined by:

\[
t_d(kT_d) = \{kT_d, D(kT_d) \neq D(kT_d - 1)\}.
\]

For a correct signal to noise ratio a good choice of \( N \) and \( T_d \) allows a good tuning of the multi-model based detector.

The accommodation block selects the adequate controller according to the detection vector \( D \) and to the supervision set-point \( \Sigma \). In this paper, the supervision set-point \( \Sigma \) consists in tracking the output of a single-input single-output system to a constant reference input signal.

Fig. 1. SRPMC structure for 4 operating modes.

The accommodation vector \( \alpha \) is a piecewise continuous switching signal which represents the series of the successive accommodated controllers. For the time \( kT_d \), the activated controller is \( C_h(kT_d) \), where the accommodation information vector \( \alpha \) is expressed by:

\[
\alpha(kT_d) = \{l, [D(kT_d) = G_l] \wedge (J_l < \pi_l)\}.
\]

If the performance condition \( J_l < \pi_l \) is not satisfied, an emergency shutdown procedure is activated and a maintenance operation is carried out on the damaged area of the system.

3. ROBUST PREDICTIVE POLYNOMIAL SYNTHESIS

The GPC strategy proposed in [6] and also presented in [3], is related to the minimization of a linear quadratic criterion involving future inputs and outputs in a receding horizon sense. [11] proposed the introduction of polynomials \( H_R \) and \( H_S \) for improving the robustness of the regulated system. The proposed approach is inspired by PSMR (Partial State Model Reference) which is applied to the model of type ARIMAX (Auto-Regressive Integrated Moving-Average eXogenous input) that could permit the obtention of a robust controller but this approach is not a predictive controller synthesis method. Single-input single-output time invariant systems described by discrete models sampled at the time \( kT_e \) where \( T_e \) is the control period defined as a multiple integer of the detection period \( T_d \), i.e., \( k = kT_e \). For systems with delay, the ARIMAX can be expressed by [11]:

\[
\tilde{A}(q^{-1})y(k) = B_d(q^{-1})\Delta u(k) + C(q^{-1})v(k),
\]

where

\[
\tilde{A}(q^{-1}) = H_S(q^{-1})A(q^{-1}), \quad \Delta u(k) = H_S(q^{-1})u(k), \quad H_S(q^{-1}) = I - q^{-1} \text{ to force an integral action}, \quad A(q^{-1}) = 1 + a_1q^{-1} + \ldots + a_nq^{-n}.
\]

The polynomial \( B_d \) is expressed according to the delay \( d \):

\[
B_d(q^{-1}) = b_1q^{-(d+1)} + b_2q^{-(d+2)} + \ldots + b_nq^{-(d+n)}.
\]

The accommodation \( \Delta u(k) \) is expressed by (1), and \( v(k) \) is an uncorrelated random sequence. \( \tilde{A}(q^{-1}) \) and \( B_d(q^{-1}) \) are coprime polynomials and the product \( C(q^{-1})v(k) \) is a moving average form.

To improve controller robustness when faced to high frequency perturbations, an augmented model is then considered by introducing the term \( H_R(q^{-1}) = 1 + q^{-1} \) in the model equation:

\[
\tilde{A}(q^{-1})y(k) = \hat{B}_d(q^{-1})\tilde{u}(k - d - 1) + C(q^{-1})v(k),
\]

with:

\[
\tilde{u}(k - d - 1) = \Delta u(k - d - 1) \frac{H_R(q^{-1})}{H_R(q^{-1})}.
\]
and \( \tilde{B}_d(q^{-1}) \) and expressed by: \( \tilde{B}_d(q^{-1}) = b_1q^{-d} + b_2q^{-(d+1)} + \ldots + b_nq^{-(d+n)} \), where \( b_1 = b_1 \), \( b_1 = b_1 + b_2 \) and \( b_n = b_n \).

The control \( \tilde{u}(k) \) is then determined by minimizing the following linear quadratic cost function:

\[
 J = \sum_{j=N_1}^{N_2} [y(k + j) - y_d(k + j)]^2 + \lambda \sum_{j=1}^{N_u} [\tilde{u}(k + j - N_1)]^2,
\]

where \( \tilde{u}(k + j) = 0 \) for \( j \geq N_u \). \( y_d(k + j) \) is the reference trajectory to follow which is available \( j \) steps in advance. \( N_1 \) is the initial horizon, \( N_u \) and \( N_2 \) are the input and the output prediction horizon respectively, \( \lambda \) is the control weighting factor.

To minimize this cost function, the estimation of the future output values, \( y(k + j) \), is needed. Thus, the polynomial identities (10) are used.

\[
\begin{align*}
C(q^{-1}) &= \hat{A}(q^{-1})E_j(q^{-1}) + q^{-j}F_j(q^{-1}), \\
\hat{B}_d(q^{-1})E_j(q^{-1}) &= C(q^{-1})G_{j-d}(q^{-1}) + q^{-j-1}H_{j-d}(q^{-1}),
\end{align*}
\]

where \( C(q^{-1}) = 1 + c_1q^{-1} + \ldots + c_{nC}q^{-nc} \), with \( nC = n \hat{A} \), \( E_j(q^{-1}) = 1 + e_1q^{-1} + \ldots + e_{nE_j}q^{-nE_j} \), with \( nE_j = j - 1 \), \( F_j(q^{-1}) = f_0 + f_1q^{-1} + \ldots + f_{nf}\), \( q^{-nf} \), with \( nF_j = \max(nc - j, nA) \), \( G_{j-d} = g_0^{-d} + g_1^{-d}q^{-1} + \ldots + g_{nG_{j-d}}^{-d}q^{-nG_{j-d}} \), with \( nG_{j-d} = j - d - 1 \), and \( H_{j-d} = h_0^{-d} + h_1^{-d}q^{-1} + \ldots + h_{nH_{j-d}}^{-d}q^{-nH_{j-d}} \), with \( nH_{j-d} = \max(nc, nB_d) + d - 1 \).

The solutions of the polynomial identity can be determined by a recursive algorithm as described in [11], but they will not be detailed here. Combining the previous equations with the model equation (5), the output predictor can be shown to be given by [11]:

\[
\hat{y}(k + j) = G_{j-d}(q^{-1})\tilde{u}(k + j - d - 1) + \hat{y}_0(k + j/k).
\]

Notice that the term \( \hat{y}_0(k + j/k) \) depends on the available information up to the instant \( k \) and is given by:

\[
C(q^{-1})\hat{y}_0(k + j/k) = H_{j-d}(q^{-1})\tilde{u}(k - 1) + F_j(q^{-1})y_g(k).
\]

The optimal control \( \tilde{u}_{opt} \) obtained by the minimization of the criterion \( J \) with respect to \( \tilde{u}(k) \) is then given by:

\[
\tilde{u}_{opt} = -\sum_{j=N_1}^{N_2} m_{1j}(\hat{y}_0(k + j/k) - y_d(k + j))
\]

where \( m_{1j} \) are the components of the first row vector of the matrix \((G^T G + \lambda I_{N_u})^{-1} G^T \) and where:

\[
G = \begin{pmatrix}
    g_{N_1}^{N_1} & \ldots & g_{0}^{N_1} & 0 & \ldots & 0 \\
    g_{N_1-1}^{N_1} & \ldots & g_{1}^{N_1} & g_{0}^{N_1} & \ldots & 0 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
    g_{N_2}^{N_2} & \ldots & g_{N_2-1}^{N_2} & g_{N_2}^{N_2} & \ldots & g_{0}^{N_2} \\
    g_{N_2-1}^{N_2} & \ldots & g_{N_2-2}^{N_2} & \ldots & \vdots & \vdots \\
    \end{pmatrix}
\]

with \( g_i^j \) are the coefficients of the matrix \( G \). The coefficients of the first column of \( G \) are the same that those obtained from the step response of the transfer \( \frac{q^{-1}B_d(q^{-1})}{A(q^{-1})} \).

In the two degrees of freedom controller, denoted \( RST, p_u \) and \( p_p \) represent respectively the perturbations at the control input, at the system output and \( b \) is the measurement noise.

The RST controller equation is written:

\[
S(q^{-1})u(k) = -R(q^{-1})y_g(k) + T(q)y_d(k).
\]

By combining the equation (13) with the equation (12) and taking into account the expression of \( \tilde{u}(k) \) given in (8), a RST controller is then obtained and has the following structure:

\[
S(q^{-1})u(k) = T_{GPC}(q)y_d(k) - R(q^{-1})y_g(k),
\]

with:

\[
S(q^{-1}) = \left[ C(q^{-1}) + q^{-1}\sum_{j=N_1}^{N_2} m_{1j}H_j(q^{-1}) \right] H_S(q^{-1}),
\]

\[
R(q^{-1}) = \left[ \sum_{j=N_1}^{N_2} m_{1j}F_j(q^{-1}) \right] H_R(q^{-1}),
\]

\[
T_{GPC}(q) = \left[ C(q^{-1})\sum_{j=N_1}^{N_2} m_{1j}q^j \right] H_G(q^{-1}).
\]

The polynomial \( T_{GPC}(q) \) reflect the predictive charateristic of the proposed control and implies the knowledge of the reference signal \( y_d(k) \) in advance. For realizability reasons, the reference \( y_d(k) \) is considered known \( N_2 \) steps in advance.

The RST structure is used here to analyze the closed loop poles. The stability of these closed loop poles depends on the choice of the parameters \( N_1, N_u, N_2 \) and \( \lambda \). The closed loop poles are given by:

\[
A(q^{-1})S(q^{-1}) + B_d(q^{-1})R(q^{-1}) = P(q^{-1}).
\]

Notice that the terms \( H_2 \) and \( H_3 \) were included in the polynomials \( S \) and \( R \). Generally, the prediction horizon \( N_1 \) is chosen such as the product \( N_1T_c \) is equal to the pure delay of the system, i.e. \( N_1 \) is chosen equal to \( d + 1 \). The command prediction horizon \( N_u \) is very often set to 1 for simple applications [11]. \( N_u \) set matrix dimensions and \( N_u = 1 \) simplifies greatly the computations. Furthermore, \( N_u \) has a weak influence on the system stability. The choice of the parameters is often restricted to \( N_2 \) and \( \lambda \) resulting in a good compromise between stability and time response. To the best of the authors knowledge, the coupling between the robust RST design described in [11] and the choice of \( N_2 \) and \( \lambda \), was not used before in predictive control. This is one of the main issues proposed in this paper. The stability and the robustness of the predictive control written as a RST controller can be studied by means of the modulus and delay margins.

Robustness objectives are linked to the delay margin \( \Delta \tau \), to the modulus margin \( \Delta M \) and the noise-output sensitivity function \( S_{y,b} \), output-perturbation sensitivity function \( S_{yp,b} \) and input-perturbation sensitivity function \( S_{yp,b} \) in the frequency domain. In [11], a method for calibration of the sensitivities functions is proposed. At a given control period \( T_c \), typical values of the modulus margin and of the phase margin used for a robust design are \( \Delta M \geq 0.5 \) and \( \Delta \tau \geq T_c \). [10]
which insure the stability and the robustness properties of the closed loop system. These values make it possible to define the templates for the sensitivities functions [11]. When the sensitivities functions are inside the templates as depicted in Figures 2, 3, the system is robust to disturbances and model uncertainties showing the stability and the robustness of the closed loop to a multiplicative model uncertainty of a supplementary delay of one control sampling period.

\[
S_{yb} = -B_y R \frac{1}{P}, \quad S_{up} = AS \frac{1}{P}, \quad S_{up} = -AR \frac{1}{P}.
\]  
(21)

The tuning of the parameters \(N_1, N_2, N_n\) and \(\lambda\) can be made by combining the analysis of the stability and the robustness of the sensitivities functions. According to the values of these parameters the sensitivities functions will be or not inside the templates.

4. MULTI-MODELING OF OPEN-CHANNEL SYSTEM

In French Pyrenees, the Lunax-Save system was built to supply with water the river Gesse. From a hollow jet valve, the water is drained into an open-channel with a circular section, one kilometer long, before to supply the river. Actuator dynamics are negligible.

Saint Venant equations are accurately used with respect to the following assumptions: unidimensional discharge, the canal slope \(\gamma\) sufficiently weak to do the approximation: \(\sin \gamma \approx z\), vertical accelerations are negligible. The diffusive wave equation (22), a simplified model of Saint Venant equations, is preferred when the two following assumptions are verified: lateral discharges are null and inertia terms are negligible compared to one representing the energy lost by friction.

\[
\frac{\partial Q(x,t)}{\partial t} + C(Q, z, x) \frac{\partial Q(x,t)}{\partial x} - D(Q, x, z) \frac{\partial^2 Q(x,t)}{\partial x^2} = 0,
\]  
(22)

where \(Q [m^3/s]\) is the discharge, \(z [m]\) is the absolute water surface elevation and \(x [m]\) is the reach length, with a celerity \(C [m/s]\) and a diffusion \(D [m^2/s]\):

\[
\begin{align*}
C(Q, z, x) &= \frac{1}{C_{eff}} \left( \frac{\partial \mathcal{L}}{\partial x} - \frac{\partial (\mathcal{L} f_s)}{\partial z} \right), \\
D(Q, z, x) &= \frac{1}{L_{eff} \partial x^2},
\end{align*}
\]  
(23)

where \(\mathcal{L}\) is the water surface width and \(f_s\) is the friction slope. Hayami equation is obtained by linearizing (22) for a reference discharge \(Q_r\):

\[
\frac{\partial q(x,t)}{\partial t} + C_e \frac{\partial q(x,t)}{\partial x} - D_e \frac{\partial^2 q(x,t)}{\partial x^2} = 0,
\]  
(24)

where \(Q = Q_r + q\). The discharge variation \(q\) from the reference discharge \(Q_r\) is drained with a mean speed of constant celerity \(C_e\) and is diffused with a constant diffusion coefficient \(D_e\). As discussed in [13], the Hayami transfer function linking the upstream discharge \(Q_{up}(s)\) to the downstream discharge \(Q_{down}(s)\) for a reach of length \(x\) can be derived from the Hayami equation (24) and is expressed by:

\[
F(x, s) = e^{\frac{\gamma z}{C_e} \left( 1 + \sqrt{1 + 4 \frac{\gamma z}{C_e} s} \right)}.
\]  
(25)

The moment matching method detailed in [13] is used to identify (25) to a second order transfer function with delay:

\[
F(s) = \frac{e^{-\tau s}}{1 + w_1 s + w_2 s^2},
\]  
(26)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameters</th>
<th>Value</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_e)</td>
<td>(z)</td>
<td>(\xi)</td>
<td>(w_1)</td>
<td>(w_2)</td>
<td>(\tau)</td>
</tr>
<tr>
<td>0.75</td>
<td>0.455</td>
<td>1.9</td>
<td>91.9</td>
<td>245</td>
<td>17360</td>
</tr>
<tr>
<td>1.5</td>
<td>0.646</td>
<td>2.3</td>
<td>163.9</td>
<td>234</td>
<td>10950</td>
</tr>
<tr>
<td>2.8</td>
<td>0.929</td>
<td>2.8</td>
<td>300.6</td>
<td>222</td>
<td>12310</td>
</tr>
<tr>
<td>1.5</td>
<td>1.243</td>
<td>5.5</td>
<td>980.6</td>
<td>199</td>
<td>8396</td>
</tr>
</tbody>
</table>

where the transfer parameters [2] can be written as:

\[
\begin{align*}
\begin{cases}
w_1 = (-\theta + \sqrt{\Lambda})^{1/3} + (-\theta - \sqrt{\Lambda})^{1/3}, \\
\frac{w_2}{C_e} = \frac{D_e}{(1 - 3)} \left( 1 - \frac{D_e}{w_1 C_e^2} \right),
\end{cases}
\end{align*}
\]  
(27)

with \(\theta = \frac{6x D_2^2}{C_e^5}\) and \(\Lambda = \frac{4x^2 D_3^3}{C_e^6} \left( \frac{9D_e}{C_e} - 2x \right)\).

According to the sign of \(\Lambda\), two following cases can be distinguished considering \(C_N = \frac{C_e^2}{C_e^2}\):

If \(C_N \leq 1\), the reach is short. The coefficient \(w_1\) becomes negative leading to an unstable transfer function. The order of the model to identify must be decreased.

If \(C_N > 1\), the reach is long and \(w_1\) is expressed by:

\[
\begin{align*}
&\begin{cases}
w_1 = 2\rho^{1/3} \cos(\phi/3), \quad \rho = \sqrt{\theta^2 + |\Lambda|}, \quad \phi = \pi + \arctan \frac{\gamma}{\sqrt{|\Lambda|}},
\end{cases}
\end{align*}
\]  
(28)

\[
\begin{align*}
\mathcal{L} &= 2\sqrt{2Rz - \frac{z^2}{2}}, \\
\mathcal{P} &= \mathcal{R} = \mathcal{R}_t = \mathcal{R}_o, \\
\mathcal{S} &= \frac{1}{2} \left( \theta - \sin \theta \right).
\end{align*}
\]  
(29)

Table 1. Parameters identified analytically versus considered discharges.

For an open-channel system with a circular profile, coefficients \(C_e\) and \(D_e\) defined by (23) are finally calculated as:

\[
\begin{align*}
C_e &= \frac{Q_e}{L^3} \left( \frac{2}{\mathcal{R}_t} - \mathcal{R}_o \right),
\end{align*}
\]  
(20)

\[
\begin{align*}
\mathcal{P} &= \frac{5P_w}{S} \left( \frac{C_e}{2} + 4\mathcal{R}_t - 2z^2 \right) - 8R),
\end{align*}
\]  
(21)

The gallery dynamics was modelled by a multi-modeling approach on the operating range discharge \(Q \in [0.5; 5] m^3/s\). Each model is a second order with delay (26), and the values of the corresponding parameters \(w_1\), \(w_2\) and \(\tau\) must be calculated. The parameter values are determined (see Tab.1) for a selected discharge \(Q_e\), according to the geometrical data of the gallery, i.e. radius \(\mathcal{R} = 0.9 m\), length \(x = 946.65 m\), slope \(\gamma = 0.0026\) rad and Manning coefficient \(n = 0.015\).

The four continuous models \(F_i(s)\) identified according to the multi-modelling method (see Tab.2) were discretized at monitoring period \(T_d = 12 s\), giving four discretized models \(G_i\). The monitoring period \(T_d\) was chosen with respect to the inequality:
Table 2. Continuous transfer functions according to the discharge area.

<table>
<thead>
<tr>
<th>[Q_{min}; Q_{max}]</th>
<th>F(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0, 1]</td>
<td>\frac{1 + 0.245s^{1.173607} + 0.0019s^{-2}}{1 + 0.245s^{1.173607} + 0.0019s^{-2}}</td>
</tr>
<tr>
<td>[1, 2]</td>
<td>\frac{1 + 0.245s^{1.173607} + 0.0019s^{-2}}{1 + 0.245s^{1.173607} + 0.0019s^{-2}}</td>
</tr>
<tr>
<td>[2.3, 6]</td>
<td>\frac{1 + 0.245s^{1.173607} + 0.0019s^{-2}}{1 + 0.245s^{1.173607} + 0.0019s^{-2}}</td>
</tr>
<tr>
<td>[3.6, 5]</td>
<td>\frac{1 + 0.245s^{1.173607} + 0.0019s^{-2}}{1 + 0.245s^{1.173607} + 0.0019s^{-2}}</td>
</tr>
</tbody>
</table>

\[ 3T_d \leq \min_{i=1,...,4} (\tau_i). \] For the dam-gallery of Lunax-Save, the minimum delay is 73 s computed for a discharge \( Q = 4.4 \text{ m}^3/\text{s} \). These models were used to detect the current operating mode as described in section 2.

Finally, the discretized models \( G_{i,c} = \frac{b_i(q^{-1})}{a_i(q^{-1})} \) were computed according to the continuous models \( F_i(s) \) with a sampling time corresponding to the control period \( T_c = 5T_d = 60 \text{ s} \). These models were used to determine the RST controllers as described in section 3.

For the interval \([0, 1]\), the discrete transfer function at control period \( T_c \) is given by:

\[
G_{1,c} = q^{-4} \begin{bmatrix}
0.053q^{-1} + 0.083q^{-2} + 0.0019q^{-3} \\
1 - 1.291q^{-1} + 0.423q^{-2}
\end{bmatrix}
\] (30)

5. APPLICATION TO THE DAM-GALLERY OPEN-CHANNEL SYSTEM

The optimal predictive control for the dam-gallery open channel system was implemented in the form presented in Figure 1. The detection vector \( D \) given by the OMD block, is determined according to the sliding window size \( N = 4 \). In the CLA block, the accommodation signal \( \alpha \) is calculated according to \( D \).

The discharge set-point \( q_{M_{1, obj}} \) is issued from CLA block. The control \( u_r \) which corresponds to the upstream discharge, is sent to the nonlinear plant which provides the downstream discharge \( q_{M_{1}} \). The nonlinear plant simulation is carried out by the Simulation of Irrigation Canals (SIC) software, which solves the Saint Venant equations using the Preissmann’s scheme [17]. The gallery was represented with SIC according to the geometrical data of the process. The input of the SIC model of the gallery is the upstream discharge \( u_r \) and the output is the downstream discharge \( q_{M_{1}} \).

Following the robustification procedure proposed by [11], the RST polynomials were computed according to the selection of the optimal output prediction horizon \( N_2 \) with the \( H_R \) polynomial which minimized the effect of the high-frequencies disturbances not shown here.

The increase of the output prediction horizon \( N_2 \) aims at robustifying the controller. However, a great value of prediction horizon is not sufficient so that the sensitivity functions \( S_{yp_{p}} \) and \( S_{yb} \) are inside their respective upper and lower templates. After some tests, output prediction horizons \( N_2 \) are selected for each controller. They provided the best compromise between robustness and fastness. In the following, only the result for one controller among four will be detailed. Then, it was necessary to tune control weighting factor \( \lambda \) to robustify the controller. The robustification results are only presented for the discrete model \( G_{2,c} \). Figures 2, 3 show the influence of the control weighting factor \( \lambda (\lambda = 15 \text{ in dashed line, } \lambda = 30 \text{ in continuous line and } \lambda = 50 \text{ in dash-dot line}) \) respectively on the sensitivity functions \( S_{yp_{p}} \) and \( S_{yb} \) according to a prediction horizon \( N_2 = 10 \). These curves show that the increase of the control weighting factor \( \lambda \) improves the robustness of the controller but slow down the system response. After some tests, control weighting factors \( \lambda \) are selected for each controllers for a sliding window size \( N = 4 \). This choice makes it possible the reaction inside a control period because \( T_c = 5T_d \). These parameters lead to the computation of \( R_i, S_i \) and \( T_i \) polynomials for each \( i^{th} \) OM. The polynomials for the first model are given by (31).

\[
R_1 = 0.8954 - 0.246q^{-1} - 0.7672q^{-2} + 0.3743q^{-3},
S_1 = 1 - 0.8307q^{-1} + 0.0255q^{-2} + 0.0194q^{-3} + 0.0141q^{-4} - 0.0376q^{-5} - 0.1155q^{-6} - 0.0737q^{-7} - 0.0016q^{-8},
T_1 = 0.0015q^{5} + 0.0089q^{4} + 0.0241q^{3} + 0.0431q^{2} + 0.0611q^{1} + 0.0762q^{0} + 0.0145q^{-1}. \]

(31)

Due to the process dynamics, there is significant bump in the control \( u_r \). AntiWindup Bumpless Transfer (AWBT) compensators \( W_i = \frac{1}{1+\omega_i q^{-1}} \) have been introduced as proposed in [4]. After several trials, \( \omega_i = -0.99, i = 1, ..., 4 \) give good performances.
A method to design a supervised robust predictive multi-controller was described. The stability and robustness analyses were facilitated by the polynomial expression of the multi-controller. In the application of the open channel system, multiple discrete models were identified for representing the overall range of discharge. An algorithm was presented to determine the number of models necessary to cover all the range of operating conditions. From the identified discrete models, a bank of robust predictive controllers was designed. The supervised predictive multi-controller makes it possible the tracking of objectives in large operating conditions of a real process. The SRPMC strategy was compared with a PI strategy showing its effectiveness and better performances.

6. CONCLUSION

REFERENCES


Fig. 4. SRPMC results (a) output perturbations $p_n$, (b) input perturbations $p_i$, (c) accommodation signal $\alpha$, (d) setpoint $q_{M1, obj}$ (continuous line) and output $q_{M1}$ (dashed line), (e) effective control $u_r$.

In Figure 4.d, the set-point $q_{M1, obj}$ is presented in continuous line. The downstream discharge measurement $q_{M1}$ is subjected to disturbances with noise of 0.15 m$^3$/s from $t = 50$ min, of 0.3 m$^3$/s from $t = 130$ min, and of $-0.4$ m$^3$/s from $t = 200$ min (see Figure 4.a). The upstream discharge is subjected to step disturbances with a noise of 0.15 m$^3$/s from $t = 40$ min (see Figure 4.b). The noise has a variance of 0.01. The accommodation signal $\alpha$ is displayed in Figure 4.c. The effective control $u_r$ is depicted in Figure 4.e. In Figure 4.d, the measured downstream discharge $q_{M1}$ is displayed in dashed line. The optimal predictive control is robust in presence of the disturbances and the downstream discharge is close to the setpoint.

In order to evaluate the performance of the SRPMC strategy, a PI controller was designed according to the recent method presented in [12]. A fixed filtered PI controller with the following structure was used:

$$R_{PI}(s) = K_p\left[1 + \frac{1}{T_i s}\right] \frac{1}{\left(\frac{s}{\omega_n} \right)^2 + 2\xi \frac{s}{\omega_n} + 1},$$

with $\omega_n = \frac{1}{\sqrt{T_i T_d}}$, $\xi = 0.5$ for a robust design. $T_i$ was chosen according to Lîtrico’s method as $T_i = 2T_d$, where $T_d$ is the largest time delay model of the plant. In this application $T_d = T_{d1} = 252$s. $K_p$ is determined from the Nicholls plot of each model after obtaining a gain margin of about $9dB$ for the nominal model (the third model) but taking into account that for the first model the minimal gain margin is at least $2dB$ (in this case $4.5dB$). Notice that for the “worst” model the phase margin is about 71.5°.

The PI and SRPMC performances were compared on the same scenario. The PI strategy is robust to the noise and the disturbances but is slower than the SRPMC strategy. A criterion of volume in lack or in excess was chosen to show the effectiveness of the strategies. Over a period of 6 hours, the SRPMC strategy provides a volume in lack or in excess of 2290 m$^3$ and the PI strategy a volume of 6775 m$^3$. Thus a volume of 4485 m$^3$ is spared by using the SRPMC strategy.