Individual Exhaust gas mass flow estimation using a periodic observer Design

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Abstract: In this paper, we propose a method to estimate the individual exhaust mass flow rate for a gasoline IC engine. In order to achieve this goal a periodic observer for a class of non linear models in the discrete Takagi-Sugeno form is designed. The adopted framework to prove the stability of the observer is based on the Lyapunov theory and use linear matrix inequality (LMI) formalism. Some simulation and experimental results are provided to show the efficiency of the proposed observer.

Keywords: Engine management, periodic structure, fuzzy control.

1. INTRODUCTION

The high demand for consumption, emission and drivability on modern internal combustion engines involves the use of advanced control concepts based on variables coming from each individual cylinder. Unfortunately, many of the variables required to develop an accurate control are not measured on conventional engine mainly due to the price of the sensors for series production [Jeffery & al 2006, Shenton 2008]. The cylinder by cylinder estimation and control (or individual cylinder estimation and control) problems is an interesting and challenging task for the engine control community and it represents a good alternative to achieve some goals related to the consumption and pollution emissions constraints [Morall & al 1993, Ashhab & al 2000 a, Chauvin & al 2007].

In this paper, we focus on the estimation of the individual exhaust mass flow which is very important parameter to achieve a good air-fuel ratio estimation. Indeed, in automotive engine control, the amount of fuel to be injected is often determined from the mean estimated air flow entering into the cylinders (FeedForward term) and the exhaust gas oxygen sensor (Feedback term) [Guzzela & al 1997, Grizzle & al 1994]. In order to upgrade the mean value estimation, in the literature, various methods are proposed to obtain the in-cylinder mass air flow rate for an internal combustion engine [Chevalier & al 2000, Stotsky & al 2002, Ashhab & al 2000, Shiao & al 1996, Kerkeni & al 2010] and also the cylinder by cylinder AFR [Mooral & al 1993, Chauvin & al 2006, Kerkeni & al 2008, Suzuki & al 2006].

The individual exhaust air-fuel ratio is related to the mass of air and to the total mass of gas in the exhaust pipe and also depends on the pressure and the temperature at the exhaust manifold. In order to obtain an accurate estimation of the individual air fuel ratio, a good estimation of the instantaneous exhaust gas flow coming out from each cylinder have to be obtained. The consequence is then dual: development of an accurate control of the fuel injection quantity for each cylinder [Bienvenuti & al 2003], and outperform the conversion efficiency of the 3-way catalyst (TWC) [Turin & al 1993]. The main goal of the paper is to estimate the individual exhaust gas mass flow of each cylinder. The model associated to this problem is nonlinear. Moreover, the SI engine, seen from the cylinders, is cyclic by nature due to the four stroke functioning. In order to take into account the nonlinearities and the cyclic behavior, the original development proposed in this paper is to use a polytopic approach based on a discrete Takagi-Sugeno (TS) [Takagi & Sugeno 1985] periodic observer to estimate the individual exhaust gas mass flow with respect to the cylinder firing sequence. The proposed observer is designed in the crank angle domain. The stability of the estimation error is proved using the Lyapunov theory which results in linear matrix inequalities (LMI) stability conditions [Boyd & al 1994].

The paper is organized as follows. First, the nonlinear model used to represent the four cylinder engine exhaust gas dynamics is presented. Then, section 3 is dedicated to the design of the discrete TS periodic observer which allows estimating the individual exhaust mass flow, where a discrete periodic TS model in the crank angle domain is provided from the nonlinear model and stability conditions are designed. In section 4, simulation and experimental results are described to show the efficiency of the method. Finally, some conclusions and further works are given in the last section.

2. MODEL OF THE EXHAUST MASS FLOW

This section presents the nonlinear model of the exhaust mass flow which will be used to estimate the exhaust gas coming out from each cylinder.
2.1 Dynamic of the exhaust manifold pressure

The model considered is based on the dynamic of the exhaust manifold pressure which is given by the following expression [Heywood 1988]:

\[
\frac{dP_{\text{exh}}}{dt} = \frac{R T_{\text{exh}}(t)}{V_{\text{exh}}} \left( \sum_{i=1}^{n_{\text{cyl}}} D_{\text{exh}}(t) - D_{\text{in}}(t) \right)
\]

(1)

where \( P_{\text{exh}} \) is the exhaust manifold pressure, \( T_{\text{exh}} \) is the exhaust manifold temperature, \( V_{\text{exh}} \) the exhaust manifold volume, \( R \) the perfect gas constant, \( D_{\text{in}} \) the gas mass flow rate at the input of the three way converter (TWC) system, \( D_{\text{exh}} \) the exhaust gas mass flow rate coming out from the \( i^{\text{th}} \) cylinder and \( n_{\text{cyl}} \) the number of the cylinders.

The mass flow rate \( D_{\text{in}} \) is described based on the following adiabatic orifice flow [Heywood 1988]:

\[
D_{\text{in}}(t) = C_{d,\text{exh}} \cdot A_{\text{exh}} \left( \frac{P_{\text{exh}}(t)}{\sqrt{R T_{\text{exh}}(t)}} \right) \cdot d\left(P_{\text{exh}}, P_{\text{atm}}\right)
\]

(2)

where the function \( A_{\text{exh}} \) with the discharge coefficient \( C_{d,\text{exh}} \) expresses the geometric flow characteristics. The differential pressure function \( d\left(P_{\text{exh}}, P_{\text{atm}}\right) \) is defined as:

\[
d\left(P_{\text{exh}}, P_{\text{atm}}\right) = \begin{cases} 
\left( P_{\text{atm}} \right)^{\gamma} & \text{if } P_{\text{atm}} > P_{\text{exh}} \\
\frac{\sqrt{\gamma + 1}}{\sqrt{\gamma - 1}} \left( 1 - \left( P_{\text{atm}} \right)^{\gamma-1} \right) & \text{if } P_{\text{atm}} \leq P_{\text{exh}} 
\end{cases}
\]

(3)

with the constant \( \gamma = 1.4 \) and the pressure ratio \( P_{\text{atm}} = \frac{P_{\text{exh}}}{P_{\text{exh}}} \) between the atmospheric pressure and the exhaust manifold pressure, and the critic ratio \( P_{\text{exh}} = (2/\gamma + 1)^{\frac{1}{\gamma}} \). The exhaust mass flow rate \( D_{\text{exh}}(t) \) from cylinder number \( i \) is consider as the function [Chauvin & al 2006] of the form:

\[
D_{\text{exh}}(t) = \alpha_i(t) \cdot d_i\left(\theta(t)\right)
\]

(4)

where \( d_i\left(\theta(t)\right) \) is a shape function depending on the crankshaft angle value \( \theta(t) \) obtained as an interpolation of a large number of available data, and which can be represented for each cylinder on Fig. 1. The function \( \alpha_i(t) \) is supposed to be constant for the cylinder number \( i \) on each engine cycle, i.e., each 720° for the crank angle.

2.2 Dynamic of the exhaust pressure sensor

The observer is based on the measurement of the exhaust manifold pressure provided by a piezoelectric pressure sensor situated in the middle of the exhaust manifold. The dynamic of this sensor is chosen as a first order dynamic:

\[
\tau_{\text{exh}} \frac{dP_{\text{exh}}(t)}{dt} + P_{\text{exh}}(t) = P_{\text{in}}(t)
\]

(5)

where \( P_{\text{exh}}(t) \) is the measured exhaust manifold pressure, and \( \tau_{\text{exh}} \) the time constant of the pressure sensor.

![Fig. 1. The functions \( d_i\left(\theta(t)\right) \) for each cylinder](image)

2.3 Non linear continuous model

To summarize, the equations (1) to (5) are used to design a continuous state space model to derive an observer. Considering the state vector \( X(t) = \text{col}\{P_{\text{exh}}(t), P_{\text{exh}}(t)\} \), the state space model is given by:

\[
\dot{X}(t) = \begin{bmatrix} -C_{d,\text{exh}} A_{\text{exh}} \frac{\sqrt{R T_{\text{exh}}(t)}}{V_{\text{exh}}} & 0 \\
-1/\tau_{\text{exh}} & -1/\tau_{\text{exh}} 
\end{bmatrix} X(t)
\]

\[
0 \begin{bmatrix} RT_{\text{exh}}(t) \sum_{i=1}^{n_{\text{cyl}}} \alpha_i(t) \cdot d_i\left(\theta(t)\right) 
\end{bmatrix}
\]

(6)

and the output equation:

\[
y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} X(t)
\]

(7)

In order to take into account the periodic behaviour of the functions \( d_i\left(\theta(t)\right) \) the problem will be transposed in the crank angle domain.

2.4 Crank angle domain transformation

The crank angle domain refers to replacing time by crankshaft angle as the independent variable for the observer design. The main advantage to consider this domain is that it is possible to cope with the periodicity of the model. Moreover, [Yurkovich & al 1997, Chin & al 1986] argued that
In the angle domain. Considering this sample period, the \( \theta \), \( \dot{\theta} \) and \( \ddot{\theta} \) are non linear and \( \omega \) is the period of the model and "mod" stands for the modulo function. All dynamics, except the one for fuel injection, vary less in the crank angle domain; the sampling period is fixed in the crankshaft angle using the following transformation:

\[
\frac{dX(t)}{dt} = \frac{dX(\theta)}{d\theta} \frac{d\theta}{dt} = dX(\theta) \theta
\]

(8)

where \( \theta \) is the engine speed which is always measured on engine management system.

2.5 Periodic non-linear discrete model

For needs of implementation in the embedded engine control, the estimation must be casted in discrete form. A simple Euler method is used to obtain a discrete model from the continuous one. The sample angle \( \theta_i \) is chosen equal to \( 6^\circ \) in the angle domain. Considering this sample period, the functions \( \alpha_i(\theta) \) are supposed to vary only each \( 720^\circ \) can be considered as slow varying variable in the angle domain, so:

\[
\alpha_i(\theta + 1) = \alpha_i(\theta)
\]

(9)

Remark 1: To develop the observer the overlap existing between two consecutive cylinders (cf. Fig. 1.) is neglected.

So, considering the cylinder firing sequence, without loss of generality, a firing order 1-3-4-2 is assumed for a four cylinders engine, then four phases must be considered:

Choosing the augmented state vector:

\[
\bar{X}(\theta) = col \{ P_{eb}, P_{exh}(\theta), \alpha(\theta), \alpha_2(\theta), \alpha_i(\theta), \alpha_4(\theta) \}
\]

if \( 0 < \theta < 180 \) (cylinder 3 in the exhaust phase):

\[
\bar{X}(\theta + 1) = \begin{bmatrix}
    f_1(P_{eb}, T_{eb}) & 0 & 0 & 0 & f_3(d_1, T_{eh}) & 0
    \\
    \text{cst} & 1 - \text{cst} & 0 & 0 & 0 & 0
    \\
    0 & 0 & 1 & 0 & 0 & 0
    \\
    0 & 0 & 0 & 0 & 1 & 0
    \\
    0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \bar{X}(\theta)
\]

(10)

and the nonlinear functions:

\[
f_1(P_{eb}, T_{eb}) = 1 - \frac{\dot{\theta}}{N_c} C_{\text{cst}} A_{\text{eb}} \left( \frac{\sqrt{RT_{	ext{eb}}(t)}}{V_{	ext{eb}}} \right) \cdot d(P_{eb}, P_{exh})
\]

\[
f_3(d_1, T_{eh}) = \left( \frac{RT_{	ext{eh}}(t)}{V_{	ext{eh}}} \right) \left( \frac{\dot{\theta}}{N_c} \right) d_1(\theta(t)), \text{cst} = \frac{\dot{\theta}}{\tau_{\text{max}} N_c}
\]

If \( 180 < \theta < 360 \) (cylinder 4 in exhaust phase) are given by:

\[
\bar{X}(\theta + 1) = \begin{bmatrix}
    f_1(P_{eb}, T_{eb}) & 0 & 0 & 0 & 0 & f_2(d_i, T_{eh})
    \\
    \text{cst} & 1 - \text{cst} & 0 & 0 & 0 & 0
    \\
    0 & 0 & 1 & 0 & 0 & 0
    \\
    0 & 0 & 0 & 1 & 0 & 0
    \\
    0 & 0 & 0 & 0 & 1 & 0
    \\
    0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \bar{X}(\theta)
\]

(11)

And so on for each phase.

3. DESIGN OF THE TS PERIODIC OBSERVER

In this part, the design of the periodic TS observer is presented. As the periodic model obtained in the previous section is nonlinear a way to deal with is to consider a polytopic approach. The first step is the development of a periodic TS model derived from the discrete nonlinear model of the section 2. Then, the design of the associated observer and the stability conditions of the state estimation error are exposed.

3.1 Development of periodic TS model

In order to take into account the nonlinearity of the model [Taniguchi & al 2001] and to preserve the periodicity of the engine, a periodic Takagi-Sugeno (TS) form is adopted which is written as [Kerkeni & al 2009] as:

\[
x(\theta + 1) = \sum_{i=1}^{N_c} h_i(z(\theta)) \left[ A_{x(i)}^{(c)} x(\theta) + B_{u(i)}^{(c)} u(\theta) \right]
\]

\[
y(\theta) = \sum_{i=1}^{N_c} h_i(z(\theta)) c_{x(i)}^{(c)} x(\theta) + c_{u(i)}^{(c)} u(\theta)
\]

(12)

with \( A_{x(i)}^{(c)} \), \( B_{u(i)}^{(c)} \) and \( c_{x(i)}^{(c)} \) are p-periodic matrices, \( c = \theta \mod p \), \( p \) is the period of the model and “mod” stands for the modulo function. The vectors \( x(\theta) \), \( z(\theta) \), \( u(\theta) \) and \( y(\theta) \) are respectively the state, premise, control and output vectors. \( z(\theta) \) is assumed to be measurable. \( r \) is the number of linear sub-models (or rules) and \( h_i(z(\theta)) \) are non linear functions satisfying the convex sum property \( h_i(z(\theta)) \geq 0 \), \( \sum_{i=1}^{N_c} h_i(z(\theta)) = 1 \). A way to derive such TS models is to use the so-called sector nonlinearity approach [Taniguchi & al 2001].

In our case, two nonlinear functions \( f_i(P_{eb}, T_{eb}) \) and \( f_z(\sum d_i, T_{eh}) \) are considered. They are bounded such that:

\[ i \in \{1, 2\}, f_i(\cdot) \leq \bar{f}_i. \]
Remark 2: As $\sum_{i=l} d_i$ is chosen in the nonlinear function $f_i(\theta)$, there will be a small deviation between the TS model and the nonlinear one during the overlaps.

From (12), a four rules periodic TS representation of the nonlinear model on the four phases ((10), (11),...) is given by:

$$\tilde{X}(\theta + 1) = \sum_{i = 1}^{4} h_i(\theta) \cdot A_i^{(l)} \tilde{X}(\theta)$$

$$Y = C\tilde{X}(\theta)$$

(13)

As an example, the matrices for $0 \leq \theta < 180^{\circ}$ are given by:

$$A_1^{(l)} = \begin{bmatrix}
T_1 & 0 & 0 & 0 & T_2 & 0 \\
1 - c & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}$$

$$A_2^{(l)} = \begin{bmatrix}
T_1 & 0 & 0 & 0 & T_2 & 0 \\
1 - c & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}$$

so on for each of the four period.

### 3.2 Design of the discrete TS periodic observer

A periodic fuzzy observer of the model (12) can be considered as:

$$\dot{\hat{\theta}}(\theta + 1) = A^{(l)}(\theta) \hat{\theta}(\theta) + B^{(l)}(\theta) u(\theta) + S^{(l)}(\theta) K^{(l)}(\theta) (y(\theta) - \hat{y}(\theta))$$

$$\dot{\hat{y}}(\theta) = C^{(l)}(\theta) \hat{\theta}(\theta)$$

(14)

The dynamic of the prediction error becomes:

$$\ddot{\hat{\theta}}(\theta + 1) = \left( A^{(l)}(\theta) - S^{(l)}(\theta) K^{(l)}(\theta) C^{(l)}(\theta) \right) \hat{\theta}(t) = \tilde{A}^{(l)}(\theta) \hat{\theta}(t)$$

(15)

The next Theorem is based on the use of a quadratic periodic Lyapunov function [Bolzern & al 1988]:

$$V_\epsilon(\hat{\theta}) = \hat{\theta}^T(\theta) P^{(l)}(\theta) \hat{\theta}(\theta)$$

(16)

Consider the following quantity:

$$\gamma^{(l)} = \left[ -P^{(l)(\text{mod} m)} \quad * \right] \tilde{A}^{(l)} \Theta^{(l)(\text{mod} m)} = 0$$

(17)

with:

$$\tilde{A}^{(l)} = S^{(l)(\text{mod} m)} A^{(l)(\text{mod} m)} - K^{(l)(\text{mod} m)} C^{(l)(\text{mod} m)}$$

$$\Theta^{(l)(\text{mod} m)} = -S^{(l)(\text{mod} m)} - \left( S^{(l)(\text{mod} m)} \right)^T P^{(l)(\text{mod} m)}$$

for all $l \in \{0, ..., p - 1\}$ and $P^{(p)} = P^{(0)}$.

Theorem [Kerkeni & al 2009]: The prediction error (15) is globally asymptotically $p$-stable if there exits matrices $P^{(l)} > 0$, $S^{(l)}$ and $K^{(l)}$, for $l \in \{0, ..., p - 1\}$ and $i \in \{1, ..., r\}$ such that the following LMI conditions (18) and (19) hold for all $l \in \{0, ..., p - 1\}$ and $Y^{(l)}$ defined in (17):

$$Y^{(l)} < 0, i \in \{1, ..., r\}$$

$$\frac{2}{r - 1} \left( Y^{(l)} + Y^{(l)} \right) < 0, i, j \in \{1, ..., r\}, i \neq j$$

(19)

In the particular case of the some dynamics of the combustion engine, the periodicity of the parameters ($p = 720$) is very large compared to the sample angle of the model ($\theta = 6^\circ$), the LMI problem (17) associated with the observation of the model can become quickly infeasible. In order to reduce the conservatism of stability conditions, other results are provided in [Kerkeni & al 2009].

### 4. EXPERIMENTAL RESULTS

In this section, simulation and experimental results are provided. The engine used in the experiments is an inline four-cylinder four stroke gasoline engine. Considering the observer (14) and the model (13) and the application of the Theorem gives the observer gains.

As the individual exhaust gas mass flow rate is not measured in the test bench a simulation example is first used where all the state variables are available. The results are given on Fig. 2 to 6. The results obtained in simulation are of course really promising as all the state variable are reaching the associated real state. The estimation of the $\alpha_i(\theta)$ is given Fig. 6, all the other vary the same way.

Experimental results (Fig. 7 to 9) are obtained on the test bed of the LAMM, the main features of the engine are listed in table 1. The estimation of $\alpha_i(\theta)$ and $\alpha_i(\theta)$ are given Fig. 9 and follow the same evolution as the one of the exhaust manifold pressure (Fig. 7) and the results are going in the good way.

![Fig. 2. Estimated exhaust manifold pressure](image-url)
Fig. 3. Error on % on the estimation of exhaust pressure

Fig. 4. Estimated of exhaust mass flow rate of cylinder 3

Fig. 5. Error on the estimation of exhaust mass flow rate of cylinder 3 in %

Fig. 6. Estimation of $\alpha_\theta$

Table 1

<table>
<thead>
<tr>
<th>Type</th>
<th>4 Cylinders in line PEUGEOT TUJP (NFZ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bore/Stroke</td>
<td>78.5/82 mm</td>
</tr>
<tr>
<td>Displacement</td>
<td>1587 cm3</td>
</tr>
<tr>
<td>Compression ratio</td>
<td>9.6:1</td>
</tr>
<tr>
<td>Maximum Power</td>
<td>65 KW at 5600 rpm</td>
</tr>
<tr>
<td>Maximum Torque</td>
<td>135 Nm at 3000 rpm</td>
</tr>
<tr>
<td>Fuel Metering System</td>
<td>Multi-point electronic injection Bosch MP 5.1</td>
</tr>
<tr>
<td>Distribution</td>
<td>DOHC 2 Valves per cylinder</td>
</tr>
<tr>
<td>Valve Timing (at 0.7 mm lift)</td>
<td>IVO 4° ATDC, IVC 46° ABDC, EVO 42° BBDC, EVC 0°</td>
</tr>
</tbody>
</table>

Fig. 7. Estimation of exhaust pressure measured (red) and estimated (blue)
Fig. 8. Error on % on the estimation of measured pressure

Fig. 9. Estimation of $\alpha_1(\theta)$, $\alpha_3(\theta)$ and their difference

6. CONCLUSIONS

In this work, a new method to estimate the exhaust gas flow rate coming out from each cylinder has been presented. This goal was achieved using a periodic non linear discrete observer in the Takagi-Sugeno form. Future work will expand the model by incorporating the cylinder pressure model. This work will lead to the development of an observer and controller structure for the air fuel ratio of each cylinder.

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