Abstract: In this paper, we study the performance of a team of classifiers. First, we propose a performance model for a single classifier based on its confusion matrix and prior information. Building upon this model, we propose a supervisor-aided classifier and analyze the performance in comparison to the single classifier. Second, we consider the performance of a team of two classifiers. Since the decisions of the team members need to be fused, we evaluate various logical fusion rules under a team performance model. Then, we propose a supervisor-aided team classifier and analyze its performance.

Keywords: Decision making and cognitive processes, Modeling of human performance, Human operator support, Modeling of HMS, Data-fusion, Perception and sensing

1. INTRODUCTION

The purpose of this work is to assess the performance of a team of classifiers based on the performance of the individual classifier in the team, prior information, and fusion rules that combine the individual classifiers decisions. We define a fusion rule to be synergistic if, under this rule, the performance of a homogeneous team of classifiers (i.e., a team consisting of two classifiers with identical properties) is better than the performance of each classifier operating alone. We show that, while some fusion rules are synergistic, others are not. We also show that, depending on the prior information about the objects to be classified, some fusion rules are preferable to others because of synergistic effects.

The situation we consider is motivated by the Air Force’s Intelligence, Surveillance, and Reconnaissance (ISR) mission where typically a human operator, or a team of operators, examines a stream of images that was sent by Unmanned Aerial Vehicles (UAVs). These images may contain objects of interest where status, whether being a target or not, is unknown to the operators. Based on the examined image, they are to make a classification decision on the object of interest.

The performance of a classifier can be defined in various ways, for instance, the probabilities of misclassification, time required to make a classification decision, and endurance over the mission. In this study of team decisions, we consider the probability of misclassification as the performance measure. Our analysis is based on an assumption that the team operates in a static mode. That is, given an unidentified object, the team may take as long as they need to make their decisions.

1.1 Original Contributions

- We propose a novel single and team classification model that depends on the individual classifier’s confusion matrix and a priori information in a static environment.
- We show that the individual classifier’s decision in the team can be fused by various logical operators and verified that the single classifier is a special case of the fused model.
- We show that there are synergistic fusion rules that improve the team performance compared to the individual performance.
- We demonstrate that a supervised decision based on the individual decision is no worse than the unsupervised case.

1.2 Organization

The organization of the paper is as follows. A literature review on classification and team decision theory is presented in Sec. 2. In Sec. 3, a performance measure for a single classifier is formulated and a supervisory decision scheme is developed. In Sec. 4, the performance measure is extended to a two-classifier team under several fusion rules and a supervisory decision scheme for the team is formulated.
2. BACKGROUND

2.1 Classification in Various Fields

In Gupta and Leu [1989], the problem of static classification is posed and the solution is investigated analytically with two different approaches such as the subset selection and the indifference zone approaches. In Congalton [1991], the author presents a brief review on the available techniques for assessing the accuracy of remotely sensed data and the necessary considerations related to the data such as the classification system, the sampling scheme, the sample size, and spatial autocorrelation.

Novelty detection is the identification of a new or unknown data or signal that a machine learning system is not aware of during training. Novelty detection is similar to the static classification problem, but it focuses on the novelty of the new signal rather than the similarity to the known signal. In Markou and Singh [2003], the state-of-the-art statistical techniques are reviewed.

Pattern recognition is the study of how machines can observe the environment, learn to distinguish patterns of interest from their background, and make sound and reasonable decisions about the categories of the patterns. In Jain et al. [2000], an extensive review survey is presented. In Hester and Casasent [1980], a method for multi-class optical pattern recognition of different perspective views of an object is described.

2.2 Team Decision Theory

The classical team decision problem, which is to “find the best communication system and the best decision rules, given the gross score table, the probabilities of situations, and the cost of communication”, was first formulated in Marschak [1955]. Based on this problem, a static team decision problem is formalized in Radner [1962]. In Ho and Chu [1972], the team decision problem is posed as an optimal control problem. A tutorial on team decision theory and information structures between the teammates is presented in Dandach et al. [2010]. This work considers two fusion rules that combine multiple decision-makers’ opinions to make a final collective decision.

3. PERFORMANCE OF A SINGLE CLASSIFIER

Let $X$ be a discrete random variable which denotes the status of an unidentified object such that $X \in \{T, F\}$. Let $A$ be a discrete random variable which denotes the classifier (or operator) decision such that $A \in \{T, F\}$. The conditional probabilities of a classifier making decision given a certain object status are,

\begin{align*}
P(A = T|X = F) &= \alpha, \\
P(A = F|X = T) &= \beta.
\end{align*}

Here, $\alpha$ denotes the false positive rate while $\beta$ denotes the false negative rate of the classifier with $\alpha, \beta \in [0, 0.5]$. Equation (1) provides the confusion matrix of a classifier expressed in conditional probabilities.

Let $P(X = T) = u$ and $P(X = F) = 1 - u$ where $u \in [0, 1]$ denotes the prior information of target population. The probability of misclassification is the sum of probabilities of two faulty outcomes: false positive and false negative:

$$
P_m = P(O = T \wedge X = F) + P(O = F \wedge X = T).$$

Using the product rule yields

$$
P_m = \alpha(1 - u) + \beta u = \alpha + u(\beta - \alpha).$$

Figure 1 illustrates the performance measure with respect to the prior information.

![Fig. 1. Single classifier performance with respect to varying false positive rates ($\alpha$)](image)

The model tells us that for equal false positive and negative rate ($\alpha = \beta = 0.3$), the performance measure is insensitive to the prior information. For larger false negative rate than false positive rates ($\alpha = 0.2$, $\beta = 0.3$), increasing target population linearly increases the probability of misclassification, and vice versa.

**Weighted Performance** Each term in the performance measure can be weighted as,

$$
P_m = \omega_\alpha \alpha(1 - u) + \omega_\beta \beta u,
$$

where $\omega_\alpha, \omega_\beta \in \mathbb{R}$ denote the weighting parameters.

These weighting parameters can be determined based on some external information. For instance, in military operations, the weighting parameters are determined by a policy maker who assesses the potential outcomes of certain instances. Also, the weighting parameters can be exploited for a team of heterogenous classifiers. For example, a misclassification by a novice classifier may be less weighted than that of an expert. In this paper, we assume that $\omega_\alpha = \omega_\beta = 1$.

Footnote 1: Note that “$T$” and “$F$” can be interpreted as “True” and “False”, respectively, or as “Threat” and “Friend”. The subsequent theory does not require choosing an interpretation.
3.1 Supervisory Decisions

Consider a supervisor which is an entity that makes the final decision on the unidentified object property based on the classifier’s suggestions. The supervisory decision is formulated by comparing the posterior probabilities of two hypotheses.

By Bayes’ rule, the posterior probability of \(X = X_0\) conditioned on \(A = A_0\) is

\[
P(X = X_0|A = A_0) = \frac{P(A = A_0|X = X_0)P(X = X_0)}{P(A = A_0)}.
\]

(5)

The posterior probabilities of the four possible outcomes are summarized in Table 1.

| \(X_0\) | \(A_0\) | \(P(X = X_0|A = A_0)\) |
|-------|-------|-----------------|
| \(T\) | \(T\) | \((1 - \beta)u\) |
| \(F\) | \(T\) | \((1 - \beta)u + \alpha(1 - u)\) |
| \(T\) | \(F\) | \(\beta u\) |
| \(F\) | \(F\) | \(\beta u + \alpha(1 - u)\) |

For instance, if the classifier decides \(A = T\), then the supervisor compares the posterior probability of \(P(X = T|A = T)\) and \(P(X = F|A = T)\) from the table, then chooses the most likely hypothesis of \(X\).

The supervisory decision rule by maximum likelihood classification is

\[
O_s = \begin{cases} T & \text{if } \frac{P(X = T|A = A_0)}{P(X = F|A = A_0)} > 1, \\ F & \text{if } \frac{P(X = T|A = A_0)}{P(X = F|A = A_0)} \leq 1. \end{cases}
\]

(6)

Let \(f_{A_0} \in [0, \infty)\) denote the ratio of the posterior probabilities such that,

\[
f_T = f_{A=T} = \frac{(1 - \beta)u}{\alpha(1 - u)},
\]

(7a)

\[
f_F = f_{A=F} = \frac{\beta u}{(1 - \alpha)(1 - u)}.
\]

(7b)

Let \(\delta_{O,A}: \mathbb{R} \to \{0, 1\}\) such that

\[
\delta_T(f) = \delta_{O=\tau}(f) = \begin{cases} 1 & \text{if } f > 1, \\ 0 & \text{if } f \leq 1, \end{cases}
\]

(8a)

\[
\delta_F(f) = \delta_{O=\tau}(f) = \begin{cases} 1 & \text{if } f < 1, \\ 0 & \text{if } f > 1. \end{cases}
\]

(8b)

Then, the conditional probabilities of the supervisor decision given an operator decision are,

\[
P(O_s = T|A = T) = \delta_T(f_T),
\]

(9a)

\[
P(O_s = T|A = F) = \delta_T(f_F),
\]

(9b)

\[
P(O_s = F|A = T) = \delta_T(f_T).
\]

(9c)

\[
P(O_s = F|A = F) = \delta_T(f_F).
\]

(9d)

Based on these probabilities, we assess the performance of a supervisory classifier.

**Performance of Supervisory Decisions**

Assessing the probability of misclassification yields

\[
P_{ms} = P(O_s = T \land X = F) + P(O_s = F \land X = T) = P(O_s = T \land X = F|A = T)P(A = T) + P(O_s = T \land X = F|A = F)P(A = F) + P(O_s = F \land X = T|A = T)P(A = T) + P(O_s = F \land X = T|A = F)P(A = F),
\]

(10)

by the theorem of total probability. Assuming that the supervisory decision is unbiased, we can relax the expression by conditional independence, i.e., \(P(O_s = O_{a0} \land X = X_0|A = A_0) = P(O_s = O_{a0}|A = A_0) \cdot P(X = X_0|A = A_0)\).

Substituting Eq. (9) yields,

\[
P_{ms} = \delta_T(f_T)\alpha(1 - u) + \delta_T(f_F)(1 - \alpha)(1 - u) + \delta_T(f_T)(1 - \beta)u + \delta_{NT}(f_T)\beta u.
\]

(11)

Figure 2 shows the performance measures comparison for a classifier with \(\alpha = \beta = 0.3\). It is noted that there is a region in \(u\) where the supervisory classifier performs better than the unsupervised one. Also, the overall performance of the supervisory classifier is no worse than the unsupervised one regardless of \(u\).

![Fig. 2. Performance measure for a single classifier with and without supervisor](image)

4. PERFORMANCE OF A TWO-CLASSIFIER TEAM

In a similar way to the single classifier case, here we define the performance of a two-classifier team.

Let \(\Theta\) denotes a team of classifiers where \(\Theta \in \{A, B\}\) with \(A\) and \(B\) each representing an individual classifier decision such that \(A, B \in \{T, F\}\). The confusion matrix in conditional probability form for each individual classifier is defined as,

\[
P(\Theta = T|X = F) = \alpha_\Theta,
\]

(12a)

\[
P(\Theta = F|X = F) = 1 - \alpha_\Theta,
\]

(12b)

\[
P(\Theta = F|X = T) = \beta_\Theta,
\]

(12c)

\[
P(\Theta = T|X = T) = 1 - \beta_\Theta,
\]

(12d)

\[
\emptyset = \{T, F\}.
\]

(12e)

The false positive and false negative rates for each individual classifier are defined similarly as in Eq. (1).

4.1 Fusion Rules

Unlike the case of a single classifier, there can be many ways of assessing the probability of misclassification for a team. By assessing, we mean fusing the classification...
outcomes of the individual classifiers according to some logical rules.

Table 2. Truth table for two-classifier team

<table>
<thead>
<tr>
<th>Classifier A</th>
<th>Classifier B</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Table 2 shows the possible truth table for a two-classifier team. For each entry in the truth table, there can be two outcomes, T or F, which implies that there are 16 possible ways of fusing the outcomes of the classifiers. In this study, we consider the four basic logical operators (conjunction, disjunction, implication, and biconditional) out of the 16 possible fusion rules as the candidate fusion rules as an initial investigation of the approach.

The following shows the formulation of the performance measure for each fusion rule. We assume that the decisions of A and B are conditionally independent given X.

**Conjunction (A \& B)**

\[
P_m = P(A = B = T \land X = F) + P(A = B = F \land X = T) + P(A = T \land B = F \land X = T) + P(A = F \land T \land B = X \land T)
\]

Using the conditional independence assumption, i.e., \( P(A = T \land B = T | X = F) = P(A = T | X = F) \cdot P(B = T | X = F) \),

\[
P_m = P(A = T | X = F)P(B = T | X = F)P(X = F) + P(A = F | X = T)P(B = F | X = T)P(X = T)
\]

\[
\]

Substituting Eq. (12) yields,

\[
P_m = \alpha_{AB} \beta (1 - u) + (\beta_A + \beta_B - \beta_A \beta_B)u = \alpha_{AB} \beta + u(\beta_A + \beta_B - \beta_A \beta_B - \alpha_{AB} \beta_B).
\]

**Disjunction (A \lor B)**

\[
P_m = P(A = B = T \land X = F) + P(A = B = F \land X = T)
\]

\[
+ P(A = T \land B = F \land X = T) + P(A = F \land T \land B = X \land T)
\]

\[
= P(A = T | X = F)P(B = T | X = F)P(X = F) + P(A = F | X = T)P(B = F | X = T)P(X = T)
\]

\[
+ P(A = T | X = T)P(B = F | X = T)P(X = T) + P(A = F | X = T)P(B = T | X = T)P(X = T)
\]

\[
= (\alpha_A + \alpha_B - \alpha_{AB}) \beta (1 - u) + \beta_A \beta_B u
\]

\[
= \alpha_A + \alpha_B - \alpha_{AB} \beta + u(\beta_A \beta_B - \alpha_A - \beta_A + \alpha_{AB} \beta_B).
\]

**Implication (A \Rightarrow B)**

\[
P_m = P(A = B = T \land X = F) + P(A = B = F \land X = F)
\]

\[
+ P(A = T \land B = F \land X = T) + P(A = F \land T \land B = X \land T)
\]

\[
= (1 - \alpha_A + \alpha_{AB}) \beta (1 - u) + (\beta_B - \beta_A \beta_B)u
\]

\[
= 1 - \alpha_A + \alpha_{AB} + u(\beta_B - \beta_A \beta_B - 1 + \alpha_A - \alpha_{AB} \beta_B).
\]

(\( B \Rightarrow A \))

\[
P_m = P(A = B = T \land X = F) + P(A = B = F \land X = F)
\]

\[
+ P(B = T \land A = F \land X = T) + P(B = F \land A = T \land X = F)
\]

\[
= (1 - \alpha_B + \alpha_{AB}) \beta (1 - u) + (\beta_A - \beta_A \beta_B)u
\]

\[
= 1 - \alpha_B + \alpha_{AB} + u(\beta_A - \beta_A \beta_B - 1 + \alpha_B - \alpha_{AB} \beta_B).
\]

**Biconditional (A \Leftrightarrow B)**

\[
P_m = P(A = B = T \land X = F) + P(A = B = F \land X = F)
\]

\[
+ P(A = T \land B = F \land X = T) + P(A = F \land T \land B = X \land T)
\]

\[
= (1 - \alpha_A - \alpha_B + 2 \alpha_{AB}) \beta (1 - u) + (\beta_A + \beta_B - 2 \beta_A \beta_B)u
\]

\[
= 1 - \alpha_A - \alpha_B + 2 \alpha_{AB}
\]

\[
+ u(\beta_A + \beta_B - 2 \beta_A \beta_B - 1 + \alpha_A + \alpha_B - 2 \alpha_{AB} \beta_B).
\]

4.2 Aggregated Team Performance

The team performance under such fusion rules can be expressed in the following aggregated form (let subscript \( T \) denote “Team”).

\[
P_m = \alpha_T (1 - u) + \beta_T u
\]

Table 3 summarizes the aggregated false positive and false negative rates.

<table>
<thead>
<tr>
<th>F. R.</th>
<th>( \alpha_T )</th>
<th>( \beta_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A &amp; B</td>
<td>( \alpha_{AB} \beta )</td>
<td>( \beta_A + \beta_B - \beta_A \beta_B )</td>
</tr>
<tr>
<td>A \lor B</td>
<td>( \alpha_A + \alpha_B - \alpha_{AB} \beta )</td>
<td>( \beta_A \beta_B )</td>
</tr>
<tr>
<td>A \Rightarrow B</td>
<td>( 1 - \alpha_A + \alpha_{AB} \beta )</td>
<td>( \beta_A - \beta_A \beta_B )</td>
</tr>
<tr>
<td>B \Rightarrow A</td>
<td>( 1 - \alpha_B + \alpha_{AB} \beta )</td>
<td>( \beta_B - \beta_A \beta_B )</td>
</tr>
</tbody>
</table>

**Definition 1. Homogeneous Team**

A homogeneous team is a team such that all of the team members have the same false positive and false negative rates, i.e.,

\[
\alpha_A = \bar{\alpha}, \beta_A = \bar{\beta}, O \in \{A, B\}
\]

where \( \bar{\alpha}, \bar{\beta} \in [0, 1] \).

**Definition 2. Heterogeneous Team**

If a team is not homogeneous, then it is heterogeneous.

Figure 3 illustrates the performance measure of different fusion rules with respect to the prior information for a homogeneous team. Figure 4 shows the performance measure of different fusion rules with respect to the prior information for a heterogeneous team. Compared to a single classifier case, two-homogeneous-classifier teams with conjunction, disjunction, and implication fusion rules are synergistic when \( u \) is appropriately chosen. On the other hand, biconditional fusion is non-synergistic.
Two classifiers with various fusion rules

\[ f_{T,T} = f_{A=T,B=T} = \frac{(1 - \beta_A)(1 - \beta_B)u}{\alpha_A \alpha_B (1 - u)}, \]  
(22a)

\[ f_{T,F} = f_{A=T,B=F} = \frac{(1 - \beta_A)\beta_B u}{\alpha_A (1 - \alpha_B)(1 - u)}, \]  
(22b)

\[ f_{F,T} = f_{A=F,B=T} = \frac{(1 - \beta_B)\beta_A u}{\alpha_B (1 - \alpha_A)(1 - u)}, \]  
(22c)

\[ f_{F,F} = f_{A=F,B=F} = \frac{\beta_A \beta_B u}{(1 - \alpha_A)(1 - \alpha_B)(1 - u)}. \]  
(22d)

Using Eq. (8), we can define the conditional probabilities of a supervisor decision given the team decisions. Table 5 summarizes the probabilities.

| \( O_s \) | \( A_0 \) | \( B_0 \) | \( P(O_s = O_{a0}|A = A_0 \land B = B_0) \) |
|---|---|---|---|
| \( T \) | \( T \) | \( T \) | \( \delta_T(f_{T,T}) \) |
| \( F \) | \( T \) | \( T \) | \( \delta_T(f_{T,F}) \) |
| \( T \) | \( F \) | \( F \) | \( \delta_F(f_{F,T}) \) |
| \( T \) | \( F \) | \( T \) | \( \delta_F(f_{F,F}) \) |
| \( F \) | \( T \) | \( F \) | \( \delta_F(f_{F,F}) \) |
| \( F \) | \( F \) | \( F \) | \( \delta_F(f_{F,F}) \) |

**Performance of Supervisory Decisions** We assess the performance measure of a supervised team with respect to the fusion rules. Let \( O_f \in \{T,F\} \) denote the fusion rule decision. By the product rule, the probability of misclassification for a two-classifier team with a supervisor is

\[ P_{m_s} = P(O_s = T \land X = F)P(O_f = T)P(O_f = F) \]
\[ + P(O_s = T \land X = F)P(O_f = F)P(O_f = F) \]
\[ + P(O_s = F \land X = T)P(O_f = T)P(O_f = T) \]
\[ + P(O_s = F \land X = T)P(O_f = F)P(O_f = F). \]  
(23)

Assuming that \( O_s \) and \( X \) are conditionally independent given \( O_f \), we get

\[ P_{m_s} = P(O_s = T|O_f = T)P(X = F \land O_f = T) \]
\[ + P(O_s = T|O_f = F)P(X = F \land O_f = F) \]
\[ + P(O_s = F|O_f = T)P(X = T \land O_f = T) \]
\[ + P(O_s = F|O_f = F)P(X = T \land O_f = F). \]  
(24)

Due to page restrictions, we provide an outline for assessing Eq. (24) for the biconditional fusion rule as an example. For biconditional rule, the outcome of the fusion rule \( O_f \) is equivalent to the followings:

\[ O_f = T \Leftrightarrow (A = T \land B = T) \lor (A = F \land B = F), \]  
(25)

\[ O_f = F \Leftrightarrow (A = T \land B = F) \lor (A = F \land B = T). \]  
(26)

For mutually exclusive events \( a \) and \( b \), it can be shown that

\[ P(O_s|a \lor b) = \frac{P(O_s|a)P(a) + P(O_s|b)P(b)}{P(a) + P(b)} \]  
(27)

and

\[ P(X \land (a \lor b)) = P(X \land a) + P(X \land b) \]  
(28)

are true. Using Eq. (25)-(28), together with Table 4 and 5, we can evaluate the probability of misclassification in Eq. (24).

Figure 5 compares the performance of fusion rules for a homogeneous team with \( \alpha_O = \beta_O = 0.5, O \in \{A, B\} \).
The probability of misclassification for the supervised team decision is always less than or equal to the probability of misclassification for the unsupervised team decision for the four fusion rules and of $u$. Also, the probability of misclassification for the supervised team decision is always less than or equal to the probability of misclassification for the single classifier decision. Therefore, the fusion rules with supervisory control are always synergistic over the unsupervised single classifier.

However, there are supervised fusion rules that perform worse than unsupervised fusion rules in some region of $u$. Figure 6 shows supervised team performance for $\alpha_0 = \beta_0 = 0.3$, $\Theta \in \{A, B\}$. For instance, the supervised team with conjunction rule does worse than the unsupervised team with conjunction rule for $u \in [0.55, 0.8]$. The implication of these results is that there exists a performance-optimal fusion rule for a classifier team that varies by the population variable $u$. Investigation of the optimal fusion rule is left as future work.

5. CONCLUSION

In this paper, we studied the performance of a classifier team under several fusion rules. It was shown numerically that the supervised decisions for a single classifier are no worse than the unsupervised decisions regardless of the prior information. Moreover, we showed that there are synergistic fusion rules for unsupervised and supervised team decisions compared to a single classifier. The study suggests that depending on the level of prior information, there is a performance-optimal fusion rule for the team.

In the future, we expect to investigate the following directions: 1. investigation of the optimal fusion rule among the 16 possible combinations under varying prior information, 2. formulation of an approach when the prior information of the population is unavailable.

REFERENCES


