Time Domain Tuning of a Fractional Order PI\(\alpha\) Controller Combined with a Smith Predictor for Automation of Water Distribution in Irrigation Main Channel Pools

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Abstract: This paper proposes a new technique for designing fractional order controllers applied to the automation of main canal pools of a water irrigation systems. As large variation of all the plant parameters is present in such systems, a fractional order PI\(\alpha\) controller combined with a Smith Predictor is designed using time domain specifications. The designed controller is compared with the standard Smith Predictor combined with PI. All these controllers are tuned to fulfill the same time specifications in the case of canal nominal dynamics. In some canals the plant parameters may experience large changes that diminish the performance, and even can unstabilize the closed-loop system. Then the PI\(\alpha\) controller is designed to maximize the robustness to variations in these parameters. Simulated results show the robustness improvements achieved with this controller compared with a conventional PI controller\(^1\).

Keywords: Fractional order control, irrigation main canal pool control, robust control, time domain dynamics, frequency specifications.

1. INTRODUCTION

Water distribution is a worldwide problem that requires efficient use of water. A considerable amount of water in irrigation main canals is wasted because of the lack of an effective control. Nevertheless, automatic control is a powerful tool for improving the efficiency of water distribution in irrigation systems. Therefore, the introduction of automatic control in irrigation main canals has been increasingly considered in recent years (Clemmens, 2006; Litrico et al., 2006).

Designing a control strategy that leads to an efficient controller is difficult, since irrigation canals are complex systems which are distributed over long distances, with significant time delays and dynamics that change with the different regimes of the channel (Malaterre et al., 1998; Rivas-Perez et al., 2003).

The case of study presented in this paper is about the first pool of the Aragon Imperial Main Canal (AIMC) that exhibits large variations in its characteristic parameters.

On the other hand, in recent years, fractional operators have been applied with satisfactory results to model and control processes with complex dynamic behaviour, most of which are distributed parameter processes (Machado, 1997; Podlubny, 1999; Petras, 2002; Chen et al., 2004; Oustaloup et al., 2006). Fractional calculus is the field of mathematics that involves differentiation and integration of a non-integer order. This is a generalization of the standard concepts of differentiation and integration (Oldham et al., 1974; Podlubny, 1999).

Due to the time delay term that irrigation main open canals models presents, a Smith predictor (Smith, 1957) based control scheme was presented in (Deltour et al., 1998) as a suitable technique. This work proposes a Smith predictor scheme combined with a fractional PP\(\alpha\) controller is proposed.

A method, previously developed in (Castillo et al. 2010), that guarantees the fulfillment of the time specifications in the case of the standard closed-loop control for first order plants, has been applied to tune the PP\(\alpha\) controllers combined with a Smith predictor.

Once the nominal time behaviour is ensured, independently of the value of \(\alpha\) (non integer order of the integral action of the PP\(\alpha\) controller), this additional design parameter is used to

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fulfil the strong robustness requirements of the AIMC first pool.

This paper is organized as follows. Section 2 introduces the main irrigation channel pool. Section 3 presents the proposed control scheme and the controller. In this section the time domain and frequency domain tunings are also developed. Section 4 compares the robustness of both controllers, PI and PI'. Finally, Section 5 resumes the main conclusions obtained during the development of this work.

2. IRRIGATION MAIN CHANNEL POOL MODEL

The study presented in this paper is about the first pool of the Aragon Imperial Main Canal. A dynamic model of this pool has to be obtained in order to design its control laws.

Data and results reported here have been obtained from this first pool, known as the Bocal, which is part of the Ebro Hydrographical Confederation (Spain). This canal water supply comes from the Ebro River thanks to the elevation produced by the Pignatelli dam. The Bocal main characteristic parameters are resumed in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Bocal characteristic parameters</th>
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<tr>
<td>pool length</td>
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<tr>
<td>cross section</td>
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<tr>
<td>depth</td>
</tr>
<tr>
<td>width</td>
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<td>design discharge</td>
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This canal pool is operated by using the distant downstream operation method (Malaterre et al., 1998). The downstream end water level is controlled by means of a undershoot gates set located in the ‘Gates House’ at the beginning of the pool. Figure 1 shows an upper view of this pool, including gates house.

![Fig. 1. Upper view of Bocal pool.](image1)

Based on the response to a step-like input signal experiments have been carried out in order to obtain a mathematical model which describes the dynamic behaviour of this irrigation main canal pool.

The resulting experiments output that the irrigation main canal pool dynamics behaviour can be represented by a second order system with a time delay:

\[
G(s) = \frac{Y(s)}{U(s)} = \frac{K}{(T_1s + 1)(T_2s + 1)} e^{-Ls},
\]

where \( K \), \( T_1 \), \( T_2 \) and \( L \) are the static gain, time constants and time delay, respectively.

Table 2 resumes the identified parameters of the plant (1) under nominal discharge regime of the pool.

<table>
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<th>Table 2. Nominal parameters of G(s)</th>
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<tr>
<td>( K_0 )</td>
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<tr>
<td>( T_{10} )</td>
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<tr>
<td>( T_{20} )</td>
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<tr>
<td>( L_0 )</td>
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</table>

On the other hand, a change in the pool discharge regime means a wide variation of dynamical parameters \( K, T_1, T_2 \) and \( L \) in the following ranges:

\[
0.0125 \leq K(t) \leq 0.125 \\
500 \leq T_1(t) \leq 15000 \\
10 \leq T_2(t) \leq 3000 \\
200 \leq L(t) \leq 600
\]

3. CONTROL SCHEME

3.1 Control objectives

Firstly, the nominal time response provided by the controller must fulfil the following restrictive goals in terms of the settling time in the ±5% band, \( t_s \), overshoot, \( M_p \) and steady state error, \( e_{ss} \):

\[
t_s \approx 1500 s \\
M_p \approx 5\% \\
e_{ss} = 0
\]
Secondly, the controller must ensure a stable controlled system along the parameters variation ranges (2).

3.2 Smith predictor

One of the main advantages of a Smith predictor based control scheme (Smith, 1957) is the suppression of the time delay term of the closed loop transfer function denominator. Previous works (Feliu et al., 2009) demonstrated that the robustness to parameters variations can be improved using this control strategy in comparison to the conventional unity feedback one.

Expressing (1) as:

\[ G(s) = G'(s)e^{-Ls}, \]

Figure 3 represents the diagram block of the Smith predictor, supposing that \( G'(s) \) and \( L_s \) correspond to the estimated plant. Notice that that input to the process (gate opening) is \( u(t) \).

\[ M(s) = \frac{Y(s)}{Y'(s)} = \frac{R(s)G'(s)}{1 + R(s)G'(s)} e^{-Ls}. \]

Thus, the scheme of Smith predictor shown in Fig. 3 can be represented as a unity feedback scheme using \( G'(s) \), plus a time delay term (Fig. 4).

\[ R'(s) = K_p' + \frac{K_i'}{s}, \]

\[ R''(s) = K_p'' + \frac{K_i''}{s^\alpha}, \]

where \( K_p \) and \( K_i \) are the proportional and integral gains of the controllers and \( \alpha \) the non-integer order of the integral action in the case of the fractional controller. Expression (6) is a particular case of (7) when \( \alpha=1 \). Hereinafter, the superindex \( ' \) and \( '' \) mean the integer and fractional case.

3.4 Frequency Domain Tuning

Due to the integral action of both controllers if the controlled system is stable, a zero steady state error is ensured. In the case of PI controller, a frequency tuning can be proposed by means of the gain crossover frequency, \( \omega_c \), and the phase margin, \( \phi_m \), in order to control the settling time and the overshoot of the time response. Under the assumption of a predictor perfect tuning, the complex equation that allows obtaining the controller parameters from \( \omega_c \) and \( \phi_m \) can be expressed as:

\[ R(j\omega_c)G'(j\omega_c) = e^{-j(\pi-\phi_m)}. \]

Equation (8) provides the controller parameters that guarantee the fulfillment of the frequency specifications as function of the nominal parameters of the plant, \( K_0, T_{10} \) and \( T_{20} \).

The tuning equations for the PI controller are:

\[ K_p' = \frac{\omega_c^2 T_{10} T_{20}}{K_0} \cos \phi_m + \frac{\omega_c (T_{10} + T_{20})}{K_0} \sin \phi_m \]

and

\[ K_i' = \frac{\omega_c}{K_0} \left[ (1 - \omega_c^2 T_{10} T_{20}) \sin \phi_m + \omega_c (T_{10} + T_{20}) \cos \phi_m \right]. \]

The fractional PI controller parameters also depend of \( \alpha \) and yield as:

\[ K_p'' = \frac{\omega_c^2 T_{10} T_{20}}{K_0} \left[ \cos \phi_m + \frac{\sin \phi_m}{\tan \left( \frac{\pi}{2} \alpha \right)} \right] + \omega_c (T_{10} + T_{20}) \sin \phi_m \]

and

\[ K_i'' = \frac{\omega_c}{K_0} \left[ (1 - \omega_c^2 T_{10} T_{20}) \sin \phi_m + \omega_c (T_{10} + T_{20}) \cos \phi_m \right]. \]

so we can use \( \alpha \) to increase the robustness performance of the \( PI'' \) controller in comparison to the \( PI \) controller.
In order to obtain the time requirements (3) the following frequency specifications have been set:

\[ \omega_c = 1.717 \cdot 10^{-3} \text{ rad } \text{s} \quad \text{and} \quad \phi_m = 70.5^\circ \]  

(13)

The resulting PI controller is:

\[ R'(s) = 32.2569 + \frac{0.05544}{s} \]  

(14)

Figure 5 shows the time response that (13) provides using a sample time \( h = 60 \) s.

Note that the PI controller fulfils perfectly the desired time specifications (3), therefore the assumption of \( G_e'(s) = G'(s) \) and \( L_e = L_0 \) allows to use (8) as tuning equation.

Figure 6 represents the time responses that a \( P^{\alpha} \) controller combined with Smith predictor provides for different values of \( \alpha \), \( \alpha \in [0.8, 1.2] \). The non-integer integral action has been implemented using the Grünwald-Letnikov approximation and a sample time, \( h = 60 \) s.

3.5 Time Domain Tuning

In a previous work (Castillo et al, 2010) a new time domain tuning method was proposed. This method consists on modifying the original frequency specifications for the \( P^{\alpha} \) in function of the value of \( \alpha \). Figure 8 shows the schematic process of this new time domain tuning method and the conventional frequency method.

Figure 7 represents the dependence of settling time and overshoot in function of \( \alpha \) value.

The results shows that the \( P^{\alpha} \) time response strongly depends of the value of \( \alpha \) though all these controllers achieve the same two frequency specifications (\( \omega_c \) and \( \phi_m \)).

In our case of study, only the PI controller fulfils the time domain specifications (3).

Fig. 5. PI combined with Smith predictor time response.

Fig. 6. \( P^{\alpha} \) combined with Smith predictor time response (\( \alpha \in [0.8, 1.2] \)).

Fig. 7. Dependence on \( \alpha \) of the time response characteristic parameters \( t_s \) and \( M_p \) (frequency domain tuning).

Fig. 8. Frequency-domain tuning method vs. new time-domain tuning method.
The method presented at (Castillo et al, 2010) consists on selecting a new couple of frequency specifications, $\omega_c^*$ and $\phi_m^*$, which guarantees the fulfilment of the time response requirements, $t_s$ y $M_p$. The selection process is carried out by means of the minimization of a time dependant functional.

The particular frequency modification law obtained in order to fulfil (3) in this case is:

$$\begin{align*}
\omega_c^* &= 0.02328 \alpha^2 - 0.06429 \alpha + 0.05879 - 0.01618 \\
\phi_m^* &= 2.63268 \alpha^2 - 6.09549 \alpha + 5.15484 - 0.465373 \\
\end{align*}$$

$\forall \alpha \in [0.8, 1.2]$. Out of this range no solution that fulfils the time domain specification has been found.

Figure 9 represents the time response that the $PI^\alpha$ controller combined with Smith predictor provides for different values of $\alpha$, $\alpha \in [0.8, 1.2]$, using (15).

Fig. 9. $PI^\alpha$ combined with Smith predictor time response by time domain tuning ($\alpha \in [0.8, 1.2]$).

Figure 10 represents the dependence of settling time and overshoot in function of $\alpha$ value.

Thus, the $PI$ controller combined with Smith predictor can not fulfil simultaneously the nominal time response requirements (3) and the robustness range (2) and only the $PI^\alpha$ controllers with $\alpha \in [0.85, 0.95]$ can guarantee both requirement sets.

The best $PI^\alpha$ controller that fulfils (2) and (3) is:

$$R^f(s) = 22.0641 + \frac{0.1069}{s^{0.90}}, \quad (16)$$

which keeps the same robustness to $L$ and $T_1$ changes than $PI$ controller, increases a 10% the robustness to $K$ changes, and has no limit to changes in $T_2$.

In order to illustrate the robustness performance of both controllers, Fig. 11 shows the time response of both controllers in each parameter limit value (2).

5. CONCLUSIONS

In this paper a $PI$ and $PI^\alpha$ controller combined with a Smith predictor have been compared.
A method, previously developed in Castillo et al. 2010, that guarantees the fulfilment of the time specifications in the case of the standard closed-loop control for first order plants, has been applied to tune $PI^\alpha$ controllers combined with a Smith predictor structure in the case of first order plus delay plants.

A family of $PI^\alpha$ controllers that provides the same time response than the $PI$ controller under nominal condition has been found.

The additional parameter $\alpha$ has been chosen in order to maximize the robustness to plant parameters changes.

The comparative results show that the $PI$ controller combined with a Smith predictor can not fulfil simultaneously the nominal time response requirements (3) and the robustness range (2).

The best $PI^\alpha$ controller has been chosen in order to maximize the robustness range.

![Graphs showing time response of PI and PI^alpha controllers combined with Smith predictor](image)

Fig. 11. $PI$ and $PI^\alpha$ controllers combined with Smith predictor: time response in the limit allowed limit of $K, T_1, T_2$ and $L$.

REFERENCES


