Digital implementation of backstepping controllers via input/Lyapunov matching

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Abstract: Given a continuous-time dynamics described by the cascade connection of two sub-systems in strict-feedback form, and assuming the existence of a backstepping stabilizing controller, one shows the existence of a digital controller ensuring the same Lyapunov performances at the sampling instants. For computational purpose, a software is developed for computing the first terms of the complete solution. A simulated example illustrates the result.

1. INTRODUCTION

Passivity Based Controllers - PBC, encompasses nowadays a wide variety of nonlinear stabilizing designs including adaptive control, damping assignment, energy shaping as well as many other oriented to specific goals. Among the first is the backstepping strategy motivated by the need to remove the relative degree one obstacle to feedback passivation designs. Such a technique was widely developed in a continuous-time context, starting from the contributions of Kokotovic and co-authors (see for example R. Sepulchre, M. Jankovic, P.V. Kokotovic [1997] and references therein). Under suitable assumptions, its basic idea is to apply the feedback passivation strategy on the first part of the system (with fictitious relative degree one) and then to reapply the step-by-step design by augmenting the system up to reach the control variable. At this last step, a stabilizing Lyapunov based feedback is constructed.

Taking in mind the well known fact that passivity is lost under sampling, the implementation of the backstepping controllers according to successive emulations of the continuous-time design is not efficient for increasing sampling periods (specifically for a lower order systems). The object of this work is to set the digital problem at the last step of the backstepping procedure, i.e. one looks for a piecewise constant control which maintains, under sampling, the stabilization performances of the continuous-time backstepping controller. These aspects were recently discussed, for example, in D. Nešic and AR Teel [2006]; L. Burlion, T. Ahmed-Ali, F. Lamnabhi-Lagarrigue [2006]; R. Postoyan, T. Ahmed-Ali, F. Lamnabhi-Lagarrigue [2009]; T.C. Lee and Z. P. Jiang [2006]; A. Rabeh, F. Ikhounane, F. Giri [1999].

The present paper proposes the digital implementation of backstepping controllers in the context of Lyapunov matching (D. Nešic and L. Gruene [2005]), or more general, of matching a target closed-loop behavior under piecewise constant control (S. Monaco and D. Normand-Cyrot [2001]).

To simplify its presentation, the paper deals with a strategy involving the first two steps by considering the cascade connection of two nonlinear continuous-time dynamics. The method is directly generalized to the cascade connection of several dynamics satisfying the strict-feedback form. The applicability of the method is discussed through the presentation of a software computing and simulating an approximated sampled-data solution.

The paper is organized as follows: section 2 recalls basic aspects of a continuous-time backstepping design; the proposed sampled-data strategy is presented in section 3. In section 5 a software for the conception and simulation digital controllers is described. A mechanical example illustrates the sampled-data controller performances and the functionalities of the software in section 6.

2. CONTINUOUS-TIME BACKSTEPPING

In this paper, referring to systems in strict-feedback form, we consider the connection of two sub-systems as follows

$$\dot{\eta} = f(\eta) + g(\eta)\xi$$
$$\dot{\xi} = f_{d}(\eta, \xi) + g_{d}(\eta, \xi)u_c$$

(1)
(2)

where the states $\eta$ and $\xi$ are in $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively and the control vector $u_c \in \mathbb{R}^m$; $f, g$ and $f_{d}, g_{d}$ are complete and smooth vector fields of appropriate dimensions.

The following result describes the stabilizing controller through backstepping approach. The constructive aspects are recalled when they are instrumental for the digital implementation.

Proposition 2.1. (R. Sepulchre, M. Jankovic, P.V. Kokotovic [1997]) - Continuous-time backstepping - Consider the system (1)-(2), and suppose the existence of $\phi(\eta)$ with $\phi(0) = 0$ and a Lyapunov function $W(\eta)$ such that

$$\frac{\partial W}{\partial \eta}(f(\eta) + g(\eta)\phi(\eta)) < 0, \forall \eta \in \mathbb{R}^n \setminus \{0\}.$$
Then, if \( g^{-1}_a(\eta, \xi) \) exists for all \((\eta, \xi)\), the state feedback control law
\[
uc = g_a(\eta, \xi)^{-1}\left(\phi - \frac{\partial W}{\partial \eta}g(\eta) - f_a(\eta, \xi) - K_\xi(\xi - \phi)\right)
\] (4)
with \( \phi = \frac{\partial}{\partial \eta}(f(\eta) + g(\eta)\xi) \) asymptotically stabilizes the origin of (1)-(2), with
\[
V(\eta, \xi) = W(\eta) + \frac{1}{2}(\xi - \phi(\eta))^2
\] (5)
as a Lyapunov function.

**Proof 1.** Considering \( \phi(\eta) \) as a fictitious control for the first \( \eta \)-dynamics, (3) assumes that the fictitious state feedback \( \xi = \phi(\eta) \) asymptotically stabilizes the dynamics (1) at the origin. Then, setting \( y = \xi - \phi(\eta) \), (1)-(2) is rewritten as
\[
\eta = f(\eta) + g(\eta)(\phi(\eta) + y)
\] (6)
\[
y = f_a(\eta, \xi) - \frac{\partial \phi}{\partial \eta} \eta + g_a(\eta, \xi)u_c,
\] (7)
so describing the second part as an \( \eta \)-y dynamics. Setting \( V \) as in (5) and
\[
u_c = g_a^{-1}(\eta, \xi)\left(-f_a(\eta, \xi) - y + \frac{\partial \phi}{\partial \eta}(f(\eta) + g(\eta)\xi) + v\right)
\] (8)
one achieves \( \nu \rightarrow y \) passivity with storage function \( V \). Then, setting \( v = -K_\eta y \) with \( K_\eta > 0 \), one gets
\[
V = \frac{\partial W}{\partial \eta}(f(\eta) + g(\eta)\phi) - K_\eta y^2 < 0
\] (9)
because of (3) and asymptotic stabilization at the origin follows.

Some comments are helpful for the digital design: The controller \( u_c \) achieves two goals: it renders the link \( \nu \rightarrow y \) passive and improves damping through negative gain output feedback. Moreover, substituting at the first step \( \xi = \phi \) with \( \xi = \phi + v_0 \), the condition (3) guarantees passivity of the link \( v_0 \rightarrow y_0 \) with
\[
y_0 := \frac{\partial W}{\partial \eta}g(\eta).
\]
and storage \( W \). The condition (3) can be relaxed to negative semi-definiteness, provided the output \( y \) of the closed-loop system (1)-(2) under (8) is zero state detectable (ZSD) from the input \( \nu \) into the output \( y \). It holds if and only if (1) is ZSD for the input \( v_0 \) to \( y_0 \) and for \( \xi = \phi \).

These interpretations through passivation of the backstepping procedure make clear that the lost of passivity under sampling should destroy the stabilizing performances of the digital implementation of \( u_c \) through emulation. In fact, setting \( u_c = \text{Cst} = u_0 \), the negativity of \( V \) is no more guaranteed. Several digital strategies can be proposed. Starting from (1)-(2) and recalling that backstepping has been introduced to remove the relative degree one obstacle to passivation, one notes that, under sampling the dynamics of \( \eta \) has relative degree one. It results that passivation through piecewise constant control of the discrete-time equivalent should probably be discussed. However, this sets another difficulty linked to the fact that passivity in discrete-time relies to systems with direct input-output link only.

To simplify the digital design, our approach starts with the transformed system (6)-(7) and develop digital strategies to achieve negativity of the first difference
\[
V(\eta_{k+1}, \xi_{k+1}) - V(\eta_k, \xi_k).
\] (10)

Following the ideas of S. Monaco and D. Normand-Cyrot [2001] and of F. Tiefensee, S. Monaco, D. Normand-Cyrot [2009], a digital controller, ensuring the negativity of (10) through reproduction at the sampling instants the continuous-time negativity of \( V \), is described in the next section.

### 3. SAMPLED-DATA DESIGN

Let \( \delta = [\eta^T \ y]^T \) be the state of the transformed system (6)-(7) rewritten as
\[
\dot{x} = f_c(x) + g_c(x)u_c
\] (11)
with
\[
f_c = \left[ f(\eta) + g(\eta)(\phi(\eta) + y) \right]
\]
and \( g_c = \left[ 0 \ g_a(\eta, y + \phi) \right] \).

Assume \( u_c = \text{Cst} = u_0 \) over each interval of length \( \delta \), then the sampled-data equivalent of (6)-(7) is described by the difference expansion
\[
x_{k+1} = F^{\delta}(x_k, u_k) = \delta^{l_k+1}u_{k}\xi_k + x_k
\] (12)
where \( \delta^{l_k+1}u_{k}\xi_k + x_k \) indicates the Lie series operator associated with the closed-loop dynamics \( f_c + u_{k}\xi_k \) ; given a function \( h(x) \), \( L_y h(x) = (x)^T \frac{\partial h}{\partial x} \) denotes the Lie derivative of \( h \) in the direction \( f \) and \( 1 \) denotes the identity operator.

Given \( u_c \) as in (4), the Lyapunov function \( V(\eta, \xi) \) with negative time-derivative set as in (9), one computes through integration:
\[
V(x_k) = V(x_{k+1}) - \int_{t_k}^{t_{k+1}} \delta V(x_\tau) \mathrm{d}\tau,
\] (13)
where \( x_k \) indicates the closed loop continuous-time \( x \)-dynamics under \( u_c \) as in (4).

The equality (13) describes the continuous-time target difference. The sampled-data redesign consists in computing a piecewise constant control law \( u_k \) in order to match (13) at the sampled-instants. That is, \( u_k \) must verify the equality:
\[
V(\delta^{l_k+1}u_{k}\xi_k + x_k) - V(x_k) = \int_{t_k}^{t_{k+1}} \delta V(x_\tau) \mathrm{d}\tau
\] (14)
where \( x_k = x_\tau(t = k\delta) \). Hence the digital control \( u_k \) reproduces, at the sampling instants, the Lyapunov performances of the continuous-time closed loop system and achieves asymptotic stabilization of the sampled system for small enough sampling periods. A solution to the input/Lyapunov matching problem has been proposed in F. Tiefensee, S. Monaco, D. Normand-Cyrot [2010] for the stabilization of a SMIB (single-machine infinity bus system) in the IDA-PBC (Interconnection and Damping Assignment - Passivity Based Controller) context. Presently such a strategy is specialized to the backstepping context.

**Theorem 3.1.** Consider the system (1)-(2), and suppose the existence of a continuous-time controller \( u_c \), computed as in (4), such that (5) is verified. Then there exists a digital feedback \( u_k = u^\delta \) which ensures the same Lyapunov evolution under sampling and guarantees the asymptotic stability of the sampled-data equivalent system (12) for a small enough sampling period. The solution is computed solving the equality (14) up to any order of the approximations in \( \delta \). It takes the form:
\[
u_k = u^\delta = u_0 + \sum_{i \geq 1} \frac{\delta^i}{(i+1)!} u_{di}.
\] (15)
Proof 2. The proof of this Theorem is worked out showing that there exists a solution to (14) the form of asymptotic developments in powers of δ (15). Considering the Lie series commutation Theorem one has

\[ V(\delta \dot{f}(x_{k}, \delta u_{k})) = e^{\delta \dot{f}(x_{k}, \delta u_{k})} V(x_{k}), \]

for a fixed couple \((x_{k}, u_{k})\), (14) can be rewritten as the formal series equality:

\[ \delta Q_{k}(\delta, u_{k}) = e^{\delta \dot{f}(x_{k}, \delta u_{k})} V - e^{\delta \dot{f}(x_{k}, \delta u_{k})} V(x_{k}) = 0. \]

Setting \( u_{d0} = u_{c}(x)|_{x_{k}} \) and \( \delta = 0 \):

\[ Q_{k}(0, u_{d0}) = (L_{f} + u_{d0}L_{g}) V(x_{k}) - (L_{f} + u_{c}(x) L_{g}) V(x_{k}) = 0. \]

Since \( L_{g} V(x) = g_{a}(\eta, y + \phi) y \neq 0 \) for all \( y \neq 0 \) with \( g_{a} \) invertible one has

\[ \frac{\partial Q_{k}(\delta, u_{k})}{\partial u_{k}} \bigg|_{\delta = 0} = g_{a}(\eta, y + \phi) y|_{x_{k}} \neq 0. \]

By the implicit function Theorem (J. Lee [2006]), a solution \( u_{c} \) to (14) exists in a neighborhood of \((\delta, u_{k}) = (0, u_{d0})\).

For a δ small enough, there exists a controller

\[ u_{d}^{\delta} := \gamma_{3}(\delta), \]

with

\[ \gamma_{3}(\delta) = Q_{k}^{-1}(\delta), \quad \gamma_{3}(0) = u_{d0} \]

such that

\[ Q_{k}(\delta(x_{k}), \gamma_{3}(\delta)) = 0. \]

The Lyapunov stability of the sampled-data system is proven showing that, since \((L_{f} + u_{d0}L_{g}) V(x) < 0 \) and \( u_{d}^{\delta} \) satisfies (14), one verifies at the sampling instants

\[ V(x_{k+1}) - V(x_{k}) = \int_{k}^{k+1} \delta (L_{f}(\cdot) + u_{c}(\cdot) L_{g}(\cdot)) V(x(\tau)) d\tau < 0 \]

(16)

The first terms of \( u_{d}^{\delta} \) are computed as

\[ u_{d0} = u_{c}|_{x_{k}}, \quad u_{d1} = \frac{\partial u_{c}}{\partial \eta} (f + g^{x} \gamma_{2})|_{x_{k}}, \quad u_{d2} = \frac{\partial u_{c}}{\partial \xi} (f + g^{x} u_{c})|_{x_{k}}. \]

The term \( u_{d2} \) is computed to satisfy the equality

\[ u_{d2} L_{g} V|_{x_{k}} = u_{c}|_{x_{k}} L_{g} V|_{x_{k}} + \frac{u_{d1}}{2} ad_{f, g, \xi} V|_{x_{k}}. \]

We note that in the multi-input/multi-output (MIMO) case the uniqueness of the solution is not ensured.

The approximate controller \( u_{d0}^{\delta} = u_{d0} + \delta u_{d1} + \frac{\delta^{2}}{2} u_{d2} \) ensures the equality (14) up to \( \delta^{3} \) (error in \( \delta^{2} \)) and guarantees the practical stability (D. Nešić, AR Teel, P.V. Kokotovic [1999]) of the closed-loop sampled equivalent system for a small enough sampling period, since the Lyapunov function decreases at least up to an error in \( O(\delta^{3}) \). Such a solution adds specific terms to the emulated control; by increasing the order of the controller, the domain of attraction of the equilibrium point is enlarged; as a consequence longer sampling periods can be considered and a more accurate convergence is observed.

4. HIGHER ORDER STRICT-FEEDBACK SYSTEMS

The method generalizes for input affine systems in strict-feedback form with \( x \in \mathbb{R}^{n+1} \).
• the simulation of meaningful tests to find suitable degree of approximations.

The software architecture drives several modules for different goals: input - output matching, Lyapunov matching, Lyapunov redesign by backstepping, single and multi-rate implementations. Each module has the same two level architecture:

• **User level** - the software interface, where the data are specified and the results are displayed.
• **Code level** - represented by the design algorithms and by the run-time behavior.

The backstepping module of the SimNL Sys application is specified in the next paragraph.

### 5.1 System model specification - backstepping module

The module is developed for nonlinear systems as in (1,2), \( W(\eta), f(\eta), g(\eta), f_k(\eta, \xi) \) and \( g_k(\eta, \xi) \), the gain \( K \), and the initial conditions \( x(0) \) are specified at the user level. Some tests are performed on the input data to sketch the dimensions, the equilibriums and the specific assumptions.

The tests are followed by the continuous-time design. \( \phi(\eta) \) is computed such that \( \frac{dW}{d\eta} = -K_0 L_0 W \) and the continuous-time controller \( u_c(\eta, \xi) \) is computed as in (4).

The following step is dedicated to the design of an approximate sampled-data feedback law as in (15), ensuring the reproduction of the Lyapunov function \( V \) of the augmented system up to \( \delta^3 \) (error in \( O(\delta^3) \)) at most. To design \( u_{d2} \), the software computes a pseudo-inverse \( \gamma^+ \) for \( L_{gk} V \), such that \( L_{gk} V \gamma^+ = 1 \) and set

\[
\gamma^+ = \frac{L_{gk} V^T}{(L_{gk} V)(L_{gk} V)^T}
\]

Setting,

\[
u_{d2} = u_c + \frac{1}{2} (u_{d1}^T L_{f_k} L_{gk} V - L_{gk} L_{f_k} V u_{d1}) \gamma^+.\]

### 5.2 Simulation

The software provides simulations for three classes of nonlinear dynamical behaviors: continuous-time, discrete-time and sampled-data. For continuous-time simulations, the integration step \( d \) is specified by the user, the system (2) is integrated by a functional method, as specified in (12), or by the numerical method Dormand-Prince 5 with fixed step \( d \). For sampled-data simulations the continuous-time system is controlled by a piecewise constant control with sampling period \( \delta \) defined by the user. A natural condition for a meaningful sampled-data simulation is \( \delta >> d \). A discrete-time system is simulated imposing \( d = \delta \).

The sampled-data or discrete-time simulation algorithm is presented below: The first step is the initialization, the initial conditions are specified and the analytical expressions of the controllers are computed. In the first loop the hold and sampling operations are simulated. The current value of \( u_d(k) \) is assigned depending on the previous measures of the state (or depending on the initial conditions) and thus is applied to the process. The computation of the analytic expression of \( u_d \) is performed for successive degree of approximation. For the second loop, the control input is kept constant over \( \delta \) and the system behavior is integrated for each step \( d \). When \( i = (k + 1)\delta \) the algorithm leaves the second loop and the state measures are updated. The simulation runs up to final time value \( t_f \) that is also specified by the user.

**Remark 5.1.** We underline that the design of the control law \( u^\delta_d \) according to equality (14) is performed off-line. Its evaluation at the sampling instants is performed on-line. In this way the on-line computing time is reduced.

![Fig. 1. E × δ × K_y](image)

(a) Emulated performances, \( u_d = u^\delta_d \)

(b) 2nd order sampled-data controller performances, \( u_d = u^\delta_d \)

### 5.3 Performance indicators

To evaluate the performances of the digital control two performance indicators are defined. The mean square error between the continuous-time and the sampled-time Lyapunov evolutions,

\[
E := \sqrt{\frac{\sum_{k=1}^{n} (V(x(t)))_t=k\delta - V(x(k\delta)))^2}{n}}.
\]

and a relative error by dividing \( E \) by the maximal value of Lyapunov function, i.e. \( E^* = E/\max(V(x)) \).

Another tool of the software visualizes the performances of the system in relation with the approximation degree \( P \) of the control, the sampling period \( \delta \), and the gain of the control \( K_y \). This module offers a 3D simulation comparing the error amplitude \( E \) in terms of \( \delta \) and \( K \) for different fixed values of \( P \).

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6. EXAMPLE

The problem of dynamic positioning of ships which consists in keeping marine vessel vehicles over preferred course and position or following desired path by means of the ship propulsion system is considered. The simplified dynamical model of the ship is given by

\[ \eta = J(\eta) v \]
\[ \tau = M \dot{v} + F \eta + D \dot{v} \]

The control \( \tau = [\tau_1, \tau_2, \tau_3] \) is provided by the thruster system. The state space is given by \( x^T = [\eta^T, v^T] \), where \( \eta^T = [X, Y, \psi]^T \) represents the position and the heading of the ship; \( v = [u, v, \gamma]^T \), the ship-body rates that indicate linear velocity on \( x \) and \( y \), and the angular velocity. Furthermore, \( M \) is the positive symmetric inertia matrix, \( D \) is a damping matrix, and finally \( F \) includes the forces and moments. The rotational matrix \( J(\eta) \) is as usual:

\[ J(\eta) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \end{bmatrix} \]

6.1 Conception in continuous time

Writing (19)-(20) in the form (1)-(2)\(^1\) Assuming that \( M \) is no singular, so that \( M^{-1} \) exists, one gets from (4):

\[ u_c = F \eta + D \dot{v} + M (\dot{\phi} + \phi) \]

as continuous-time stabilizing controller, where

\[ a = -P \eta J(\eta) - K_\eta (v - \phi) \]

Since:

\[ V = \frac{1}{2} (\eta^T P \eta + (v - \phi(\eta))^T (u \xi - \phi(\eta))) \]

considering \( \phi = -K_\phi J \eta \) one gets

\[ V = -K_\phi \eta^T P J^T P \eta - K_\eta (v - \phi(\eta))^T (u - \phi(\eta)) < 0 \]

and the negativity of \( V \) is ensured by \( K_\eta > 0 \) and \( K_\phi > 0 \).

6.2 Sampled-data design

A second order approximate digital solution for (19),(20) is given by:

\[ u_{d2}^\delta = u_{d0} + \frac{\delta}{2} u_{d1} + \frac{\delta^2}{6} u_{d2} \]

where in the present example:

\[ u_{d0} = u_c \]
\[ u_{d1} = F \eta + D \dot{v} + M (\dot{\phi} + \phi(\eta)) \]
\[ u_{d2} \]

is chosen such that following equality holds true:

\[ (v - \phi(\eta))^T u_{d2} = (v - \phi(\eta))^T u_c - \frac{1}{2} u_{d1} (M^{-1} N + M^{-1} \phi) + \frac{1}{2} (N + \phi + (v - \phi)^T (M^{-1} D + \frac{\partial \phi}{\partial \eta} J(\eta))) M^{-1} u_{d1} \]

with \( u_c = F \eta + D \dot{v} + M (\dot{\phi} + \phi(\eta)) \).

Setting,

\[ \gamma^+ = \frac{1}{(v - \phi)^T M^{-2}(v - \phi)} M^{-1} (v - \phi) \]

then

\[ u_{d2} = u_c - \frac{1}{2} u_{d1} (M^{-1} N + M^{-1} \phi) \gamma^+ + \frac{1}{2} (N + \phi + (v - \phi)^T (M^{-1} D + \frac{\partial \phi}{\partial \eta} J(\eta))) M^{-1} u_{d1} \gamma^+ \]

Some more details can be found in L. Catenaro [2010].

6.3 Results

To help the user in the choice of the gain \( K_\eta \) and the amplitude of the sampling period, the software performs automatically several simulations to compare the \( u_{d0}-\)emulated and the \( u_{d2}^\delta \) performances. These simulations are worked out considering the error \( E \), in (18), for each strategy. Figures 1(a) and 1(b) represent the error \( E \) on the Lyapunov evolution for \( u_{d0} \) and \( u_{d2}^\delta \) with \( K_\eta \in [1,15] \) and \( \delta \in [0.01,0.5] \). The blue scale indicates a low error level while the red scale indicates a high error level (white zones represent unacceptable error level). As the intuition suggests, sampled-data controllers achieve good Lyapunov response for small values of \( K_\eta \) and \( \delta \), and the error on \( V \) increases for high \( K_\eta \) and \( \delta \) values. Comparing figures 1(a) and 1(b) one concludes that \( u_{d2}^\delta \) notably enables us to increase the range choice of \( K_\eta \) and \( \delta \) for an error on \( V \) low with respect to the emulated control.

Two different simulated cases are considered. In the first one, the control gain and the sampling period are chosen as \( [K_\eta, \delta] = [5,0.2] \) such that both, \( u_{d0} \) and \( u_{d2}^\delta \), work well. Figures 2(a)-(2b) depict the controller performances in these conditions. Even if the emulated strategy ensures an acceptable Lyapunov matching, some oscillations appear on the state trajectory. The corrective terms in \( u_{d2}^\delta \) improve the Lyapunov reproduction and ensure the ship position stabilization without oscillation and with a trajectory close to the continuous-time closed-loop one. In the second simulated case depicted in figures 3(a)-(3b), one chooses \( [K_\eta, \delta] = [5,0.3] \). The emulated control does not ensure Lyapunov matching and thus does not achieve asymptotic stability, while \( u_{d2}^\delta \) still ensures good performances.

7. CONCLUSIONS

This paper illustrates the performances of input-output matching like strategies in the context of backstepping stabilization of systems in strict-feedforward form. This implementation of backstepping stabilizing strategies starts from the transformed dynamics and set the digital problem at the last step of the procedure. However, as backstepping strongly relies to passivity properties and passivation under state feedback, a more direct digital scheme should consider the system before transformations to work out passivation under digital feedback. Work is progressing in this direction making reference to some new passivity concepts introduced and discussed in S. Monaco and D. Normand-Cyrot and F. Tiefensee [2008], S. Monaco, D. Normand-Cyrot, F. Tiefensee [2009, 2011], F. Tiefensee, S. Monaco, D. Normand-Cyrot [2010] to deal with the possible preservation of some passivity criteria under sampling.
Fig. 2. Closed-loop systems performances for $\delta = 20ms$ and $K_y = 5$.

Fig. 3. Closed-loop systems response for $\delta = 30ms$ and $K_y = 5$.

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