Abstract: This paper shows new possibility of the robust design of simple controllers based on computer analysis of closed loop transient responses achieved over predefined grid of possibly normalized loop parameters by simulation, or by measurement. Evaluation of the collected transients according to chosen performance measures related to monotonicity and time yields the so called Performance Portrait (PP). Based on the PP, controller tuning satisfying desired robust performance may be derived what is illustrated by tuning simple integral (I) controllers for the simplest uncertain 2-parameter plant model. Achieved results that extend the simple tuning proposed by Skogestad (2003) to a broader spectrum of nearly monotonic shapes are compared with the analytical controller tuning based on the double real dominant closed loop pole and then, on an example of a higher order dead-time dominant plant, with the results achieved by the Filtered Smith Predictor. Possible extension to PI control is briefly mentioned.

Keywords: robust control, simulation, animation, interactive systems, integral control, visualization.

1. INTRODUCTION

The simplest approximations of stable Single Input Single Output (SISO) dynamical plants are given by the 2-parameter models. High importance of such simple models for practice may be demonstrated e.g. by the fact that the design of integral controller for such plants with dead time was treated already in the one of the first textbooks on automatic control by Oldenbourg and Sartorius (1944, 1948). Since such controller derived for the simplest plant model may be universally used for controlling broad spectrum of stable plants, Datta and Bhattacharyya (2000) speak about “magic of integral control”. I controllers are appropriate also for systems with large (and possible variable) dead time and measurement noise. Rough and Shamma (2000) e.g. presents use of I-control in combustion engines that are typical both by long delays and noisy measurement. Åström and coworkers (1998), or Skogestad (2003) derived formulas for robust controller tuning that are, in fact, still fixed to a nominal operating point and the situations with plant parameters varying over broader intervals they cover just indirectly by choosing appropriate loop sensitivity. Systematic design of different dead time compensator in the frequency domain was e.g. treated by Normey-Rico & Camacho (2007), Normey-Rico et al. (2009).

In this paper, application of the 2-parameter plant model in robust design of stable SISO plants is supported by the new computer analysis method based on the Performance Portrait (Huba, 2010). The paper is structured as follows. Chapter 2 recalls basic performance measures useful for evaluation of nearly monotonic (MO) setpoint step responses and the application to evaluating disturbance step responses is briefly mentioned. Chapter 4 brings controller design for the simplest uncertain plant model. Chapter 5 shows application of the derived I-controller to controlling higher order stable plant and Chapter 6 brings conclusions.

2. PERFORMANCE MEASURES FOR MONOTONIC RESPONSES

Although the control is today usually accomplished by discretely working computers, here we will neglect the discrete-time aspects and consider signals sampled with a frequency enabling to preserve all their important features. In the time domain the property of stability (ST) and monotonicity (MO) are usually of dominant importance. The Bounded-Input-Bounded-Output and Internal Model Control (IMC) stability require for a bounded plant input \( u(t) \) a bounded plant output \( y(t) \) that for given lower and upper limits can be easily evaluated by numerically testing

\[
-\infty < y_{\min} < y(t) < y_{\max} < \infty ; \quad t \in (0, \infty) \\
-\infty < u_{\min} \leq u(t) \leq U_{\max} < \infty
\]  

Then, suppose, we are already dealing with constrained continuous signals, the main task may be formulated as keeping their transients by appropriate control within limits given by tolerable amplitude or integral deviations from the ideal shapes. In the simplest case we will require the step responses to be monotonic both at the plant input and output.
**Definition 1**

A constrained continuous plant output \( y(t) \) having an initial value \( y_0 = y(0) \) and a final value \( y_{\infty} = y(\infty) \), will be denoted as monotonic (MO) when its all samples fulfill condition

\[
[y(t_2) - y(t_1)] \text{sign}(y_{\infty} - y_0) \geq 0, \quad 0 \leq t_1 < t_2 < \infty
\]  

(2)

The MO plant output may e.g. be motivated by comfort of passengers in traffic control, whereas the MO controller output yields energy savings in actuators, it minimizes their wear, generated noise and vibrations, precision increase in systems with actuator hysteresis and it enables a simple controller design with the saturation avoidance.

**ST control** control may further be characterized by performance indices such as IAE defined for a setpoint \( w \) as

\[
IAE = \int_0^\infty |e(x)| dt; e(t) = w(t) - y(t)
\]

(3)

In control engineering it is frequently required to quantify also the binary properties as ST and MO, e.g. by expressing how far the system from the ST, or MO border is. In the frequency domain there are broadly used robust design methods based on assigning stability degree, gain, phase and frequency domain there are broadly used robust design methods. In the time domain the ST and MO signals will be expected to yield lower \( y_{7t0} \), or \( u_{7t0} \) values than responses with higher amplitudes of possible higher-harmonics. Similarly, it is also possible to introduce such quantitative measures in the time domain. The ST or more precisely the instability degree could be indicated by values of the parameters \( Y_{\min}, Y_{\max} \) and \( U_{\min}, U_{\max} \). Similarly, it is also possible to introduce such quantitative measures in the time domain. The ST or more precisely the instability degree could be indicated by values of the parameters \( Y_{\min}, Y_{\max} \) and \( U_{\min}, U_{\max} \) (in 1), or in many different ways – e.g. by achieved IAE values.

**Integral measure for evaluating deviations from strict MO**

\[
\alpha = \int_0^\infty h(t) dt + \int_0^\infty |h(t)| dt
\]

with \( h(t) \) being the closed loop impulse response was denoted as monotonicity index (Åström and Hägglund, 1995; Datta et al., 2000; Skogestad, 2003). Similarly, it is also possible to introduce such quantitative measures in the time domain. The ST or more precisely the instability degree could be indicated by values of the parameters \( Y_{\min}, Y_{\max} \) and \( U_{\min}, U_{\max} \) in (1), or in many different ways – e.g. by achieved IAE values.

Besides of the above integral measures it may be useful to consider amplitude deviations from MO. Due to the finite measurement precision, due to the quantization of digital signals and also due to different technological requirements it is useful to relate MO to an error band specified by the parameters \( \varepsilon_y \), or \( \varepsilon_u \) around the reference MO components of \( y(t) \), or \( u(t) \) signals.

**Definition 2**

A nearly monotonic continuous signal \( y(t) \) with the initial value \( y_0 = y(0) \) and with the final value \( y_{\infty} = y(\infty) \) will be denoted as \( \varepsilon_y \)-monotonic when it fulfills condition

\[
[y(t) - y(t - T)] \text{sign}(y_{\infty} - y_0) \geq -\varepsilon_y, \quad \forall t \in (T, \infty), \forall T > 0
\]

(5)

By introducing notion of \( \varepsilon_y \)-MO signals we aim to allow signals with limited amplitudes of possible higher-harmonics. Thereby, in order not to prolong the time required for testing (5) with any positive \( T \), this has to be chosen in a way capturing sufficient part (e.g. half-period) of the superimposed signal. Number of samples that need to be tested (Huba, 2010) may be decreased due to Theorem 1.

**Theorem 1**

Constrained continuous signal \( y(t) \) having an initial value \( y_0 = y(0) \), a final value \( y_{\infty} = y(\infty) \) and local extreme points \( y_{k1} = y(t_{k1}) \), \( y(t_{k1}) = 0 \); \( i = 1, 2, \ldots \) is nearly or \( \varepsilon_y \)-MO, if all subsequent local extreme points \( y_{k+1} \) fulfill condition

\[
[y_{k+1} - y_{k1}] \text{sign}(y_{\infty} - y_0) \geq -\varepsilon_y, \quad i = 1, 2, 3, \ldots
\]

(6)

Proof: Follows from the fact that for a continuous signal converging to \( y_{\infty} \) the maximal increase in the direction opposite to \( y_{\infty} - y_0 \) in (5) will be constrained by two subsequent extreme points.

Smooth nearly \( \varepsilon_y \)-, or \( \varepsilon_u \)-MO signals will be expected to yield lower \( u_{7t0} \), or \( u_{7t0} \) values than responses with higher amount of higher-harmonics and to be zero just for strictly MO transients. Whereas (5) and (6) characterize amplitudes of peaks superimposed on reference monotonic signal, \( u_{7t0} \) or \( y_{7t0} \) give contribution of all possible peaks to the overall integral deviation (proportional to the number and amplitudes of peaks) and so they are important in identifying permanent oscillations. By comparing amplitude and integral measures one gets information about the total number of peaks.

In evaluating disturbance step response one has to note that immediately after a disturbance step the plant output starts to rise (fall) and the controller needs some time to balance its effect and to reverse output to move back to the reference value. So, the evaluation of MO output increase (decrease) may start just after its turnover. In general, MO areas for disturbance response are different from those corresponding to the setpoint response and the controller design has to compromise these differences. Next, we are going to deal with exceptional situation when both tunings are the same.
3. THE SIMPLEST INTEGRAL CONTROL

3.1 Control task

For a stepwise constant reference signal \( w \in (0,1) \) and for the uncertain plant with parameters specified over intervals
\[
S(s) = K e^{-T_d s} ; \quad K \in \{ K_{\text{min}}, K_{\text{max}} \}; \quad T_d \in \{ T_{d,\text{min}}, T_{d,\text{max}} \}
\]
(7a)
whereby e.g.
\[
K_{\text{min}} = 1; \quad K_{\text{max}} = 2; \quad T_{d,\text{min}} = 1; \quad T_{d,\text{max}} = 2;
\]
(7b)
the task is to design robust controller that would guarantee the deviation defined e.g. as 2% of the reference value \( w_{\text{max}} = 1 \).

The plant model (7) may be identified by evaluating the average residence time
\[
A_0 = K T_d ; \quad A_0 = \int_0^\infty \{ y(x)-y(t) \} dt
\]
by the step responses (Åström and Hägglund, 1995), or by a general input signal according to Ingemundarson (2000).

In the simplest case, to set the output of the plant (7) to the reference value \( w \), the static feedforward control \( u = w / K_0 \) based on an estimate of the plant gain \( K_0 \) may be used. For plants with an output disturbance the static feedforward control may be extended by "Automatic Reset" based on the Disturbance Observer (DO, Fig.1) inspired by the IMC (Morari and Zafiriou, 1989). For a input disturbances the DO based on the inverse plant model (Ohnishi et al., 1996) may be used. In order to achieve after a setpoint step continuous response, both schemes will be extended by prefilter.

Definition 3

As admissible inputs we will denote piecewise constant setpoint and disturbance values that enable to maintain required steady states by admissible control values
\[
u = (w - v_i) / K - v_i \in \{ U_{\text{min}}, U_{\text{max}} \}
\]
(9)
Theorem 2

For strictly MO transients corresponding to admissible inputs both DO based structures are equivalent to the loop with the explicit linear l-controller with the integral gain
\[
K_f = 1 / (K_0 T_f)
\]
(10a)
Thereby, just in the situation with tuning
\[
T_p = T_f
\]
(10b)
the equivalent prefilter (Fig. 1 below) falls out.

Proof: Due to the plant dynamics (7a) containing just dead time (time shift) and gain both the plant input and output and both the setpoint and the disturbance step response are either MO, or not. During MO transitions from initial to final admissible control values (9) the saturation is avoided and may so be omitted from the control schemes in Fig. 1 that after replacing inner loops from the controller outputs and after some modifications gives finally l-controller (10).

3.2 l-controller tuning

Complete system analysis has to consider two possible outputs \( y_0 \) and \( v_1 \) and at least one disturbance (e.g. the input one \( v = v_1 \) ) that yields finally normalized transfer functions
\[
F_{w_0}(p) = \frac{Y_0(p)}{W(p)} = \frac{e^{p\Omega} / \kappa}{A(p)} ; \quad F_{w_1}(p) = \frac{Y_1(p)}{W(p)} = \frac{\Omega / \kappa}{A(p)}
\]
(11)
\[
\Omega = T_d / T_f; \quad \kappa = K_0 / K; \quad p = T_d s \quad A(p) = p e^{p\Omega} + \Omega / \kappa
\]
Fig. 1 Static feedforward control extended by prefilter and DO for the output disturbance (above), for the input disturbance (middle) and the loop with equivalent prefilter, l-controller and process (7) below, \( \delta \) - measurement noise

It is to see that the setpoint response fully depends on the parameter \( q = \Omega / \kappa \). The same also holds for the disturbance response normed by the plant gain \( K \). From \( F_{w_o,0}(0) = 0 \) it follows that the controller has an integral character, i.e. in steady states a constant disturbance will be eliminated.

For some tasks, instead of the parameter \( q \) it may be more appropriate to work with its reciprocal value
\[
\tau = 1 / q
\]
(12)
In the case of nominal tuning (\( \kappa = 1 \)) it denotes ratio of the filter time constant \( T_f \) to the dead time \( T_d \).

Optimal tuning. One of the first method for analytical controller tuning (Oldenbourg and Sartorius, 1944) is based
on conditions of the double real dominant pole (DRDP) $A(p_0) = 0 = A'(p_0)$ that should guarantee the fastest MO transients. Such solution gives $p_0 = -1$ and it should occur for $	au = \exp(1) = 2.718$ (13)

In the alternative approach based on the computer analysis the loop behavior is mapped and analyzed over a grid of loop parameters in the plane $(\kappa, \Omega)$. From these data it is then possible to visualize the loop Performance Portrait in Fig. 2, or to derive parameters corresponding to a tolerable overshooting $\varepsilon_y$ shown in Appendix in Tab.1. It is interesting to note that all values $\tau = \tau(\varepsilon_y)$, including e.g. the simple tuning $\tau = 2$ proposed by Skogestad (2003) that corresponds to $100\varepsilon_y = 4.04\%$, are smaller than (13), when e.g. for $\varepsilon_y \to 0 \quad \tau(0) \to 2.703$.

For plant (7) and $\varepsilon_y = 0.02$ (see Tab. 1), MO control requires

$$K_0T_f \geq \tau(0.02)K_{\text{max}}T_d \max = 2.162 \times 2 \times 2 = 8.648$$

Robust tuning means to locate the uncertainty box containing all possible operating points (UB, Fig.2) with vertices

$$UB = \left[ \frac{\kappa_{\text{min}}}{\Omega_{\text{max}}} \right] \left[ \frac{\kappa_{\text{max}}}{\Omega_{\text{min}}} \right]$$

into areas with lower than specified tolerable deviations from MO, i.e. below the line $\Omega = \kappa\dot{\chi}$, specified for $\varepsilon_y = 0.02$ e.g. by the pair $K_0 = 1; \quad T_f = 8.648$. For $K_0T_f = \text{const}$ choice of particular values $K_0$ and $T_f$ does not influence IAE values characterizing all possible transients within a chosen UB. From the radial shape of the borders of MO control in Fig. 2, it follows that critical role is played by the upper left vertex

$$B{(1,1)} = \left[ \frac{\kappa_{\text{min}}}{\Omega_{\text{max}}} \right] = \left( K_0 / K_{\text{max}}, T_{d \max} / T_f \right)$$

By increasing ratio of the upper and the lower limit plant parameters the mean IAE value over the UB will increase. Due to the radial shape of the PP, by shifting UB along chosen line the closed loop properties do not vary.

As an alternative to the experimental design, the analytically derived value $\tau$ (13a) may be used requiring fulfilling

$$K_0T_f \geq \varepsilon \kappa K_{\text{max}} T_{d \max}$$

Critical I-controller tuning: Sustained loop oscillations with period $P_o = 2\pi / \omega$ correspond to the root $s_{\text{crit}} = j\omega$ of the characteristic equation $A(s) = 0$. Critical tuning and the corresponding period of oscillations are

$$\omega = 0 \Rightarrow P_o \to \infty ; \quad \Omega / \kappa = 0 , \quad \text{or} \quad \Omega = 0$$

$$\omega = 2\pi / T_d \Rightarrow P_o = 4T_d ; \quad \Omega / \kappa = \pi / 2$$

In the PP the analytical stability border (Fig. 2, red line) is indicated also by the strong increase of IAE values.

By considering single uncertain parameter $T_o$, the UB (15) reduces to a vertical Uncertainty Line Segment, ULS. Similarly, by considering single uncertain parameter $K$ the UB reduces to a vertical ULS.

4. EXAMPLE

In the illustrative example we will show robust design and the corresponding robust performance achievable by using the simplest possible I-controller and then compare these results with the much more complex Filtered Smith Predictor (FSP, Nornery-Rico and Camacho, 2007; Example 6.1).

![Fig. 2 PP of plant (7) with I-controller: $\varepsilon_y$ - MO condition may be satisfied by infinitely many products $T_fK_0 = \text{const}$ defining in the plane $(\kappa, \Omega)$ lines; Uncertainty boxes (15); white point corresponds to (13a) and $\kappa = 1$; IAE levels and $\varepsilon_y$ - MO areas with colors from red to dark blue for $\varepsilon_y = \{10^{-2},10^{-4},10^{-3},10^{-2},2.10^{-2},5.10^{-2},10^{-1}\}$](image-url)

The uncertain plant to be controlled is

$$F(s) = \frac{K_p e^{-L_s}}{(s+1)(0.5s+1)(0.25s+1)(0.125s+1)}; \quad K_p \in \{0.8,1.2\}; \quad L \in \{9,12\}$$

In the cited work, the FSP using primary PI-controller

$$C(s) = K_c \left( 1 + 1/T_c s \right)$$

was tuned using standard robust approach in the frequency domain based on a nominal plant and norm bounded multiplicative uncertainty. As the nominal model an approximation of the original plant by the FOPDT one with

$$F_n(s) = \frac{K_n e^{-L_n s}}{1 + T_n s}; \quad K_n = 1; \quad T_n = 1.5; \quad L_n = 10.5$$

was used. Robust stability was proven for $K_c = 1$; $T_c = T_n$ and $T_f = L_n / 2$, but, this method is not able to guarantee higher requirements on MO transients, expressed e.g. by the amplitude related deviations, or TV$_{\dot{y}}$ values as it is evident from the PP in Fig. 3 above. So, it does not enable to design controller for more advanced applications. From Fig. 3 below it is to see that intuitive modifications are questionable - even the 20 times larger filter time constant $T_f = 10L_n$ does not reasonably improve the considered loop performance for larger plant gains – the FSP needs to be fully retuned.

7471
poles to the closed loop response will be small thus their elimination will contribute with a small increment in the speed of the transients”. Model (7a), (23) used for tuning of the I-controller fully respects this statement – whereas the model (22) used by authors of this statement not. The PP in Fig. 3 fully confirms also another statement of above authors (pp. 145) “the effect of dead-time error is not symmetric”, just the method they have used does not allow dealing effectively with this problem.

Simple I-controller yields indeed higher IAE values than the much more complex FSP. However, up to now no method for achieving higher performance requirements in tuning FSP was available. Having this fact in mind, several authors developed interactive tools to fight with this problem by the “trial and error” method (Guzmán et al., 2008; Normey-Rico et al., 2009). Using the PP method the FSP may be redesigned to respect also this problem in a direct way.

Fig. 3 PP of the plant (20) with the FSP controller based on (21-22) with $T_f = L_o/2$ above and $T_f = 10L_o$ below. MO areas with colors from brown to dark blue corresponding to $\varepsilon_y = \{0,10^{-5},10^{-4},10^{-3},10^{-2},2.10^{-2},5.10^{-2},10^{-1}\}$

Tuning of the I-controller does not require exact knowledge of (20), just the gain and the average residence time (8), when for the maximal dead time $L$ one gets by simple identification

$$T_{d_{max}} = L_{max} + 0.5 + 0.25 + 0.125 = 13.875$$  \hspace{1cm} (23)

When choosing $K_0 = 1$, the filter time constant may be determined according to (14) as $T_{f} = \tau(c_y)K_{max}T_{d_{max}}/K_{0}$, whereby the values $\tau(c_y)$ for 2% and 10% were taken from Tab. 1. From the PP in Fig. 4 it is obvious that for the output $y_1$, the deviations achieved in the critical corner exactly match the expectations, so that no corrections are necessary. Since the $\varepsilon_y -$ MO conditions are nearly matched also by the output $y_0$, it means that any output corresponding to some distribution of dynamical terms in (20) among the feedback and the feedforward path would match the required specification. Explanation for this (may be surprising result), when the extremely simple model gives precise results, may be taken from the same source as the above example (Normey-Rico and Camacho, 2007, pp. 174): “…when the dead-time is dominant, the contribution of the open loop

Fig. 4 PP of the I-controller with $T_f$ tuned to guarantee lower than 2% ($\varepsilon_y = 0.02$, above) and lower than 10% amplitude deviations from MO ($\varepsilon_y = 0.1$, below). MO areas with colors from brown to dark blue corresponding to $\varepsilon_y = \{0,10^{-5},10^{-4},10^{-3},10^{-2},2.10^{-2},5.10^{-2},10^{-1}\}$
5. CONCLUSIONS

Confrontation of the Performance Portrait method with the results of the robust control design based on expressing the norm-bounded deviations from the nominal model in the frequency domain shows the new method to be superior both in the precision and in spectrum of details they can offer. The I-controller tuning based on the DRDP has been shown to be slightly conservative. Of course, one could deal with the question, if the discrepancy in results is due to the limited precision of numerical computations, or it is expressing influence of infinitely many poles neglected when just the DRDP was considered in deriving the controller. However, for vast majority of engineering tasks the identified results do not play a primary role – more important is broad spectrum of different dynamics modification offered by the PP method. When e.g. extending the plant model (7) by stable time constant $T$ known with a relatively low uncertainty, the tuning parameters from Tab. 1 may immediately be used for controllers canceling $T$ by numerator of the DO filter, i.e. with the equivalent PI controller having the integral time constant $T_I = T$ and with the equivalent prefilter $F_{ep}(s) = \left[1 + \frac{T}{T_I} s\right] \left[1 + \frac{T}{T_I} s\right]^{-1}$.

By being much more flexible end detailed than the traditional analytical methods and by offering simple solution to the robust controller tuning the new method allows to visualize any loop property and so dealing with challenging control tasks as e.g. designing Smith predictor for integrating and unstable plants, where, the existing interactive tools (Guzmán et al., 2008; Normey-Rico et al., 2009) bring just limited improvements and modifications of the analytical methods. The new method directly gives solution matching the chosen performance requirements (if it exists), but it also generates new information that can possibly lead to further improvements and modifications of the analytical methods.

REFERENCES


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APPENDIX

Tab. 1 Experimentally determined $IAE_0$ and $IAE_1$ values corresponding to outputs $y_0$ and $y_1$ under $\varepsilon_y$ – MO setpoint step responses of the loop with I-controller and the nonmodelled dynamics approximated by $T_d$. $\varepsilon_y$ represents tolerated deviations from monotonicity; Bold – tuning (13) corresponding to the double real dominant pole (DRDP).

<table>
<thead>
<tr>
<th>$100\varepsilon_y$ [%]</th>
<th>10</th>
<th>5</th>
<th>4.04</th>
<th>2</th>
<th>1</th>
<th>0.1</th>
<th>0.01</th>
<th>0.001</th>
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<tr>
<td>$\tau = \kappa / \Omega$</td>
<td>1.724</td>
<td>1.951</td>
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<td>2.162</td>
<td>2.268</td>
<td>2.481</td>
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<td>2.625</td>
<td>2.703</td>
</tr>
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<td>$q = \Omega / \kappa = 1 / \tau$</td>
<td>0.580</td>
<td>0.515</td>
<td>0.5</td>
<td>0.465</td>
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<td>0.403</td>
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<td>$IAE_0 / T_d$</td>
<td>1.105</td>
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<td>1.17</td>
<td>1.240</td>
<td>1.314</td>
<td>1.486</td>
<td>1.571</td>
<td>1.625</td>
<td>1.703</td>
</tr>
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<td>$IAE_1 / T_d$</td>
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7473