Cooperative and decentralized navigation of autonomous underwater gliders using predictive control

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Abstract: Coordinated fleet of Autonomous Underwater Gliders (AUGs) can provide significant benefit to a number of marine applications including ocean sampling, mapping, surveillance and communication. Traditional techniques for navigating underwater vehicles have been designed for single-vehicle operations and do not scale well to multi-vehicle operations and missions. In this paper a navigation system for a fleet of AUGs is developed based on Networked Decentralized Model Predictive Control (ND-MPC). The proposed approach coordinates a group of point-mass mobile agents to achieve a desired formation, while avoiding collisions between themselves. In order to obtain collision free paths, the approach integrates the required collision avoidance constraints. The fleet localization is performed by sensor fusion using adaptive extended Kalman filtering. The free collision and convergence properties are verified through simulations results. The proposed approach can be generalized to formation of heterogeneous autonomous agents.

Keywords: Predictive Control, Decentralized Systems, Co-operative Control, Robot Navigation.

1. INTRODUCTION

During the last years several researchers have focused their attention on the Autonomous Underwater Gliders (AUGs). The AUG is an underwater vehicle which exploits gravity and buoyancy forces to produce vertical lift which can be converted into longitudinal/lateral motion by the use of fixed wings. This kind of vehicle has a great energy efficiency and can operate for thousands of kilometers without maintenance. Moreover it has low maintenance costs and is almost noiseless. For these reasons AUGs have found application in several fields: marine exploration, patrol, mine detection and many others. In order to perform many of these tasks, however, a set of gliders in formation is to be preferred. For instance in case of marine exploration the use of several agents in formation allows to cover larger areas, thus reducing navigation times. The formation problem is indeed one of the most challenging for these kind of vehicles since they are highly nonlinear, underactuated and the feasible control actions are constrained. The coordination control of multiple unmanned vehicles is analyzed by many authors. For example in (Kaminer et al. (2007)) the proposed solution is based on L1 adaptive output feedback controllers.

In order to minimize control complexity and allow easy reconfiguration of formation, decentralized solution are usually preferred to the centralized ones. Different decentralized solutions to the formation control of multiple autonomous vehicles have been developed (Keviczky et al. (2006); Dunbar and Murray (2006); Fang and Antsaklis (2006)). In Balderud et al. (2006, 2007) operational robustness is addressed for applications where clusters of autonomous vehicles are deployed for achieving a shared common objective. In order to improve autonomy and performance of the decentralized control architecture, different cooperative solutions based on communication exchange have been proposed in literature. A decentralized robust Model Predictive Control (MPC) algorithm for multi-vehicle trajectory optimization is presented by Kuwata et al. (2006). A decentralized MPC for formation control have been developed, analyzed and compared with the centralized solution by Vaccarini and Longhi (2007a,b).

In this paper the formation control of a system of multiple autonomous underwater vehicles, in particular gliders, moving according a trajectory is addressed. Only the motion on the horizontal plane is considered for control purposes. The formation control problem is solved by the use of a Nonlinear Decentralized Model Predictive Control technique (Longhi et al. (2008)) which takes into account collision-free, physical and predictive model constraints. The single control agents communicate using a Local Area Network (LAN) in which the position information of the single agent is broadcasted by the main leader, which implements sensor fusion via an Adaptive Extended Kalman Filter (A-EKF), according to a centralized policy.

The proposed approach can be easily adapted to generic unmanned vehicles navigating in formation, for which a kinematic model is provided.
The paper is organized as follows. Section 2 presents the mathematical model of an AUG and its onboard hardware. In Section 3 the sensor fusion algorithm is illustrated. Section 4 describes the approach for solving the formation control problem. Simulation results are presented in Section 5 and concluding remarks of Section 6 end the paper.

2. VEHICLES

2.1 Sensors and Actuators

An underwater glider is powered by an internal blader/ballast which is inflated to vary the buoyancy and providing vertical lift forces. A sliding mass is used for fine adjustments in pitch and roll. Although many different actuation systems are employed by the different manufacturers, all of them are equivalently described by an internal moving mass $\bar{m}$ and a variable internal point mass $m_b$ which correspond to a blader/ballast. By varying the position of $m$ with respect to the Centre of Buoyancy (CB), the pitch and roll movements of the vehicle are controlled. By adjusting $m_b$, buoyancy of the vehicle can be regulated in order to produce vertical displacements and, therefore, horizontal displacements produced by the wings.

The A-EKF developed in this paper, for localization purposes, needs the following measurements to work:

- absolute position in the horizontal plane (only during the initialization/correction stage)
- heading angle
- relative position among vehicles
- depth

In order to provide the desired information each vehicle should be equipped with an Attitude and Heading Reference System (AHRS), a sonar transponder and a depth gauge. Moreover, the fleet leader must be equipped with a Global Positioning System (GPS), in order to provide the absolute positioning of the formation, whenever it emerges on the sea surface.

The effect of underwater currents can be compensated trough current estimators and relative controllers.

2.2 Communication

The proposed solution is based on coordinated independent agents and on a Local Area Network used for coordinating them. Each agent implements a decentralized MPC policy on the basis of both local and external information acquired by the network.

The underwater communication is performed by means of sonar modems that allows to reach vehicles within a defined radius and are thus used to implement a Local Area Network (LAN). Since the available bandwidth of these modems is relatively tight, the information traffic have to be reduced to the minimum necessary. In order to do this the EKF has been developed such that the required distance measurements are those between each glider and its leader, thus avoiding to know the relative distances among all the gliders and reducing the information flow.

2.3 Formation Vector Model

Let consider a set of $N$ underwater gliders $\{\mathcal{V}_i, i = 1, \ldots, N\}$, that should accomplish to the considered formation keeping task: for each vehicle the position of the leader vehicle with respect to him should be kept equal to a desired value. Assume that at time $t$ a low level controller imposes the desired surge, sway and yaw (angular) speeds $v^i(t)$, $s^i(t)$ and $w^i(t)$ on the horizontal plane by adjusting the position $\mathbf{r}$ of mass $\bar{m}$ and the value of buoyancy mass $m_b$ in this way the high level controller has only the task to define the optimal speeds $v, s, w$ that allow to keep the desired formation within the minimum possible efforts. Assume to sample the continuous-time variables with sampling interval $T_s$ and define the sampled variables $v_k^i \triangleq T_s v^i(kT_s)$, $s_k^i \triangleq T_s s^i(kT_s)$, $w_k^i \triangleq T_s w^i(kT_s)$ that represents finite movements within each sampling interval $T_s$. These movements can also be seen as velocities normalized with respect to the sampling interval $T_s$ and, in the following, they will be referred to as velocities.

Due to physical limits, the positions of mass $\bar{m}$ and the values of mass $m_b$ are constrained to stay within a given range. This implies that surge, sway and angular velocities of the vehicle $v_k^i, s_k^i, w_k^i$ are constrained and their limits depend on lower level controller and on dynamic behaviour of the vehicle. However, for the sake of simplicity, fixed constraints are assumed in the following:

\begin{align}
\mathbf{x} & \leq \mathbf{v}_k^i \leq \mathbf{w}, & \mathbf{x} & \leq \mathbf{s}_k^i \leq \mathbf{w}, & |\mathbf{v}_k^i| & \leq \mathbf{w}, & |\mathbf{s}_k^i| & \leq \mathbf{w}, & |\mathbf{w}_k^i| & \leq \mathbf{w}. \quad (1a) \\
|\Delta v_k^i| & \leq \Delta \mathbf{v}, & |\Delta s_k^i| & \leq \Delta \mathbf{s}, & |\Delta w_k^i| & \leq \Delta \mathbf{w}. \quad (1b)
\end{align}

Defining rotation matrix $\mathbf{T}(0) \triangleq \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, the absolute vehicle configuration on the horizontal plane (see Figure 1) $\mathbf{q}_k \triangleq [q_x^i \, q_y^i \, q_{\theta,k}^i]^T$ is determined by integrating the control action $\mathbf{u}_k^i \triangleq [v_k^i \, s_k^i \, w_k^i]^T$ by the following discrete-time kinematic model:

$$
\mathbf{q}_{k+1} = \mathbf{q}_k + \mathbf{T}^{-1}(q_{\theta,k}^i)\mathbf{u}_k^i. \quad (2)
$$

Referred to the frame fixed to vehicle $\mathcal{V}_i$, the relative displacement of vehicle $\mathcal{V}_i$, $\mathbf{d}_k^i \triangleq [x_k^i \, y_k^i \, \theta_k^i]^T = \mathbf{T}(q_{\theta,k}^i)(\mathbf{q}_k - \mathbf{q}_k)$ by some manipulation (Vaccarini and Longhi (2007a)) gives the following discrete-time formation vector model:

$$
\mathbf{d}_{k+1}^i = \mathbf{A}_k^i \mathbf{d}_k^i + \mathbf{B}_k^i \mathbf{u}_k^i + \mathbf{E}_k^i \mathbf{u}_k^i, \quad (3)
$$

where:

$$
\mathbf{A}_k^i \triangleq \mathbf{T}(u_k^i), \quad \mathbf{B}_k^i \triangleq -\mathbf{T}(u_k^i), \quad \mathbf{E}_k^i \triangleq \mathbf{T}(u_k^i)\mathbf{T}^{-1}(\theta_k^i). \quad (4)
$$
3. ADAPTIVE EXTENDED KALMAN FILTER

In order to perform formation control the most straightforward approach would be to measure the absolute position of each vehicle and use that information to keep the relative distances at the desired values. This approach, however, can not be used for AUGs as the GPS sensor is able to receive data from the satellites only when it is on the sea surface; it can thus be useful only to provide information about the initial position of each glider (initialization/correction stage of the formation control problem).

Since absolute positions are not directly available, several sensors are employed to derive those measurements. However these data are highly affected by both noise and uncertainties that can lead to unacceptable errors to the position values if not correctly elaborated. To solve this problem an A-EKF is implemented on the main leader and the information on localization are made available to all the gliders by the use of the network. Each control agent has thus the information about position that can be used inside the ND-MPC algorithm. Note that the sensor fusion algorithm is implemented according to a centralized policy, while the formation control algorithm is fully decentralized.

When a glider is below the sea surface it measures at each sample time its depth and the relative distance to its leader. These measurements, together with the control efforts elaborated by the local ND-MPC agent, are sent to the main leader using the LAN. The main leader collects these data and uses them to update the predicted state and obtain a new estimate. This new estimate is used together with the control efforts to elaborate the predicted state at the next sample time, which is sent back to the gliders using the LAN. Finally, the control agent of each single glider uses the predicted state to calculate the new control efforts using the ND-MPC algorithm. Note that the fleet leader periodically emerges, together with the fleet, to be located through its GPS onboard sensor, and thus obtaining the corrected absolute localization of the fleet.

This procedure is repeated again at each sample time, moreover the A-EKF algorithm tries to estimate the noise variance affecting states and measurements during each step using statistical properties of the innovation process.

3.1 Prediction

Consider the kinematic model described by (2) and assume that an additive, zero-mean gaussian white noise is affecting the system.

Defining

\[ L_k = \begin{bmatrix} L_k^1 & 0 & \ldots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & 0 & 0 \\ 0 & \ldots & 0 & L_k^n \end{bmatrix}, \]

where

\[ L_k^i = \begin{bmatrix} L_{2} & 0 & \ldots & 0 \\ T_s \cos \hat{\theta}_{i,k} & -T_s \sin \hat{\theta}_{i,k} \sin \hat{\phi}_{i,k} & -T_s^2 v_{i,k} \cos \hat{\theta}_{i,k} & -T_s^2 v_{i,k} \sin \hat{\phi}_{i,k} \\ T_s \sin \hat{\theta}_{i,k} & T_s \cos \hat{\theta}_{i,k} \cos \hat{\phi}_{i,k} & T_s^2 v_{i,k} \sin \hat{\theta}_{i,k} & -T_s^2 v_{i,k} \cos \hat{\phi}_{i,k} \\ 0 & T_s & T_s^2 v_{i,k} \cos \hat{\thet a}_{i,k} & T_s^2 v_{i,k} \sin \hat{\phi}_{i,k} \end{bmatrix}, \]

the state prediction equation for the entire formation, linearized around the working points \( q_{0,k} = q_{0,k+1} \) and \( u_{0,k} = u_{k+1} = \), is

\[ \dot{q}_{k+1} = \dot{q}_{k} + L_k u_k. \]  

3.2 Update

Consider the measurement equation described by

\[ z_{k+1} = G(q_{k+1}) + V_{k+1}, \]

where \( q_{k+1} \) and \( z_{k+1} \) represent the state vector and the measurement vector at time \((k+1)\), \( V_{k+1} \) is the noise vector affecting measurements which is white, zero-mean gaussian and with a covariance matrix \( R_{k+1} \) which depends on the sensor used.

Assume that vector \( G \) describes the formation geometry, i.e.

\[ G = [l_1, l_2, \ldots, l_n]^T, \]

where \( l_i \) indicates that \( V \) has \( l_i \) as its leader. \( l_i \) is set to 0 since the main leader has the virtual vehicle \( V^0 \) as leader. With this notation the measurement equation can be written as

\[ G(q_{k+1}) = \begin{bmatrix} q_{l_1,k+1,1}^2 & q_{l_1,k+1,1} & \ldots & q_{l_1,k+1,n} \\ q_{l_2,k+1,1}^2 & q_{l_2,k+1,1} & \ldots & q_{l_2,k+1,n} \\ \vdots & \vdots & \ddots & \vdots \\ q_{l_n,k+1,1}^2 & q_{l_n,k+1,1} & \ldots & q_{l_n,k+1,n} \end{bmatrix}^T, \]

with

\[ q_{l_i,k+1} = \sqrt{(q_{l_i,k+1,1}^2 + q_{l_i,k+1,1}^2) + (q_{l_i,k+1,1}^2 - q_{l_i,k+1,1}^2)^2}, \]

\( i = 2, \ldots, n \).

Linearizing the measurement equation around the working point \( q_{k+1} = q_{k+1} \) and defining

\[ A_{d,k} \triangleq \begin{bmatrix} A_{d,k}^1 & 0 & \ldots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & 0 & 0 \\ 0 & \ldots & 0 & A_{d,k}^{n} \end{bmatrix}, \]

with \( A_{d,k}^i \triangleq \begin{bmatrix} 1 & 0 & -T_s v_{i,k} \sin \hat{\theta}_{i,k} & -T_s \hat{\theta}_{i,k} \cos \hat{\phi}_{i,k} \\ 0 & 1 & T_s v_{i,k} \sin \hat{\theta}_{i,k} & -T_s \hat{\theta}_{i,k} \sin \hat{\phi}_{i,k} \\ 0 & 0 & 1 & 0 \end{bmatrix} \)

the update equation can be described by

\[ q_{k+1} = q_{k+1} + K_{k+1} \left[ \Delta \theta_{k+1} - G(q_{k+1}) \right] \]

(7)

The gain matrix and prediction matrix are defined as

\[ K_{k+1} = P_{k+1} C_{k+1}^{-1} \]

(8)

\[ P_{k+1} = [I - K_{k+1} C_{k+1}] P_{k+1} \]

(9)

\[ P_{k+1} = A_{d,k} P_{k+1} A_{d,k}^T + Q_{d,k} \]

(10)

where \( Q_{d,k} \) is the covariance matrix of noise affecting the states and \( C_{k+1} \) is defined by

\[ C_{k+1} = \frac{\partial G(q(k+1))}{\partial q(k+1)} \bigg|_{q(k+1) = q_{k+1}}. \]

3.3 Adaptive Estimate of \( Q_{d,k} \) and \( R_{k+1} \)

The adaptive procedure presented in this paper can be used whenever matrices \( Q_{d,k} \) and \( R_{k+1} \) have the form...
where $\sigma_{\eta,i}$ and $\sigma_{\eta}$ are the parameters to be adapted. Equation (12) is valid whenever the kinematic model describes the state dynamics with statistically independent errors. Equation (13) is valid whenever the measurements are statistically independent.

Assume that $\gamma_{i,k+1} = z_{i,k+1} - C_i(\hat{q}_{k+1}|k)$: similarly to the linear case, it represents the components of the innovation process at time $k+1$. Using the procedure described in Jazwinski (1970) we can write

$$\hat{\sigma}^2_{\eta,i,k+1} = \left(C_{i,k+1}Q_{d,k}C_{i,k+1}^T\right)^{-1}\max\left\{\gamma^2_{i,k+1} - \hat{\sigma}^2_{\eta,i,k+1}ight\}$$

and

$$\hat{\sigma}^2_{\nu,i,k+1} = \max\left\{\gamma^2_{i,k+1} - \left[C_{i,k+1}A_{d,k}P_{h,k}A_{d,k}^TC_{i,k+1}^T + C_{i,k+1}^2\eta_{i,k}Q_{k}C_{i,k+1}\right]0\right\},$$

where $C_i$ is the $i$-th row of matrix $C$ and $\hat{\sigma}_{\eta,i}$ and $\hat{\sigma}_{\nu,i}$ represent the smoothed estimates calculated as the average value among the last $l$ values (to be chosen a priori), i.e.

$$\hat{\sigma}^2_{\eta,i}(k) = \frac{1}{(l_{\eta} + 1)p}\sum_{j=0}^{l_{\eta}}\sum_{i=1}^{p}\hat{\sigma}^2_{\eta,i}(k - j),$$

and

$$\hat{\sigma}^2_{\nu,i}(k + 1) = \frac{1}{(l_{\nu} + 1)l_{\nu}}\sum_{j=0}^{l_{\nu}}\hat{\sigma}^2_{\nu,i}(k + 1 - j).$$

4. FORMATION CONTROL

The navigation system of the considered fleet is based on the well known cascaded leader-follower approach (Vaccarini and Longhi (2007a,b)), where

- the reference trajectory $\mathcal{T}^*$ is generated by a virtual reference vehicle $V^0$ which moves according to the considered unicycle model;
- each vehicle $V^j$ follows one and only one leader $V^i$, $j \neq i$; $V^1$ follows virtual vehicle $V^0$ which exactly tracks the reference trajectory $\mathcal{T}^*$;
- each vehicle $V^j$ should keep the reference formation pattern $d^j$, from its leader $V^i$.

In the proposed navigation system, each vehicle $V^j$ is equipped with an independent control agent $A^j$ which collects local and remote information (i.e. the position vector provided by the A-EKF) and iteratively performs a nonlinear optimization for computing the local control action. As previously stated, each vehicle $V^j$ tracks a leader $V^i$ with a defined displacement. The set of all displacements defines the formation (Figure 2).

In the implementation of the proposed navigation system the following assumptions are made:

- Each control agent $A^i$ communicates with its neighboring agents by a Local Area Network (LAN) only once within a sampling interval.
- The communication network introduces a delay $\tau = 1$.

![Fig. 2. The considered leader-follower architecture.](image-url)
condition:
\[
\begin{aligned}
&\left(\hat{d}^{C,i}_{k+p|k}(u^*_{k-1}, \hat{u}^i_{k+p-1|k}) - d^{C,i}_{k}\right)^2 + \\
&\mu|\hat{u}^i_{k+p-1|k}|^2 + \sigma|\Delta \hat{u}^i_{k+p-1|k}|^2 \leq r^i_k,
\end{aligned}
\]
where \(r^i_k\) is known at time \(k\) and defined as:
\[
\begin{aligned}
r^i_k \triangleq &\left(\hat{d}^{C,i}_{k} - \hat{d}^{C,i}_0\right)^2 + \mu|\hat{u}^i_{k-1}|^2 + \sigma|\Delta \hat{u}^i_{k-1}|^2 - \\
&\sum_{h=1}^{p} \left[\left(\hat{d}^{C,i}_{k+h-1|k}(u^*_{k-1}) - \hat{d}^{C,i}_k\right)^2 - \\
&\left(\hat{d}^{C,i}_{k+h-1|k}(u^*_{k-1}) - \hat{d}^{C,i}_k\right)^2\right].
\end{aligned}
\]
Under these assumptions the set \(\mathcal{A}\) of control agents \(\mathcal{A}^i\), \(i = 1, \ldots, N\), guarantees the local stability of the equilibrium point \(\bar{d} \triangleq (\hat{d}^{C,i}_0)^T, \ldots, (\hat{d}^{C,N}_0)^T\) for the whole closed-loop system.

The collision-free property, instead, is guaranteed as long as the following constraints are satisfied at each sample time \(k\):
\[
|N(A^i_k d^i_k + B^i_k u^i_k)| \geq d + \sqrt{2}\max(|l|, |s|, |v|, |r|),
\]
with \(l = 1, \ldots, n\) and \(l \neq i\).

5. SIMULATION RESULTS

The developed strategy have been tested on the formation control of an underwater glider fleet composed by \(N = 5\) vehicles, with starting and objective formations described in figure 3.

![Fig. 3. Starting and Final Formations.](image)

The virtual leader had to follow an “S” trajectory.

The physical constraints used in simulation are:
- \(0.04\, \text{m/s}^{-1} \leq v_k \leq 0.4\, \text{m/s}^{-1}\), \(|\Delta v_k| \leq 0.02\, \text{m/s}^{-2}\),
- \(-0.03\, \text{m/s}^{-1} \leq s_k \leq 0.03\, \text{m/s}^{-1}\), \(|\Delta s_k| \leq 0.01\, \text{m/s}^{-2}\),
- \(-0.04\, \text{rad/s}^{-1} \leq \omega_k \leq 0.04\, \text{rad/s}^{-2}\), \(|\Delta \omega_k| \leq 0.005\, \text{rad/s}^{-2}\).

The parameters are:
- minimum safe distance \(d = 3\, \text{m}\),
- cost function weights: \(\rho_x = 10\), \(\rho_y = 10\), \(\rho_\theta = 200\), \(\mu = 0.5\), \(\alpha = 1\) and \(\eta = 0.4\),
- prediction horizon \(p = 3\).

The total number of simulation steps is \(K = 1000\).

As it can be seen in Figure 4 the main leader follows the virtual leader and the formation quickly assume the desired geometry. The transient phase for this kind of configuration is short. A similar result can be achieved changing the reference trajectory and the formation pattern.

![Fig. 4. Trajectories followed by the five gliders. Each vehicle is identified by a color. The simulation is frozen at sample times \(k = 0\), \(k = 500\) and \(k = 1000\).](image)

More complex patterns and trajectories require a greater transient to achieve steady state. During this time the vehicle positions differ from the desired position, as it can be seen in Figure 5, which shows the \(x^i_k\), \(y^i_k\) and \(\theta^i_k\) variables representing lateral, longitudinal and angular relative distances between glider \(V^i\) and glider \(V^j\) at time \(k\), namely the components of the displacement vector \(d^i_{k,j}\). The steady state error, however, is almost zero for all the agents.

Many other simulations, not reported here for the sake of brevity, have been performed and the results confirm the effectiveness of the proposed approach.

Finally, the performance of the ND-MPC with measurements provided by the described A-EKF has been compared to:
- a ND-MPC with A-EKF which can access not only to the relative distances between leaders and followers, but also to the relative distances of all the gliders in formation;
- a ND-MPC in which the state is supposed to be accessible.
A decentralized MPC strategy for the formation control of underwater glider fleets has been proposed. The use of MPC allows to take into account physical constraints, collision-free constraints and behaviour predictions. Moreover the A-EKF allow the use of AHRS, sonar, GPS and depth gauge to perform sensor fusion and provide good position estimates. These values can be sent on the communication network and can be used by each decentralized control agent to perform control. The decentralization improves autonomy and reliability and MPC provides good control performances. Simulations have shown the effectiveness of the developed approach.

In order to complete the architecture, the dynamic controller of the glider has to be developed. For the real-time implementation of the control strategy, optimization of the control algorithm is required together with the implementation of the LAN protocol. The A-EKF algorithm should be realized with a decentralized approach: in this way a change in the number of working gliders would not require a new configuration of the control problem. This approach could also be easily integrated in a diagnostic system with fault tolerant capabilities (Longhi et al. (2008)).

**REFERENCES**


