Optimised Output Sensitivity Loop Shaping Controller for Ship Rudder Roll Damping

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Abstract: This paper describes the design of a Rudder Roll Damping (RRD) controller based on the linear model of the ship to cope the wave disturbances. A preliminary study of fundamental limitations for linear feedback controllers provides suggestions for the controller design. The RRD is derived using the Internal Model Principle and the optimisation theory. The proposed method directly shapes the output sensitivity function that relates the wave disturbance to ship roll motion. A non-linear constraint optimisation problem is then developed to choose controller parameters in order to make the closed-loop system satisfy specifications and constraints. Numerical results from simulations indicate reasonable roll reduction compared with Minimum Variance Cheap Limiting Optimal Control (MV-CLOC).

Keywords: Marine system navigation, guidance and control; Control architectures in marine systems; Ship Roll Damping

1. INTRODUCTION

The main purpose of the rudder is to control the heading direction of the ship in course-keeping manoeuvres, but it can provide roll motion too, Fossen (1994). The latter assertion is based on the structural fact that the rudder is located aft and also below the centre of gravity of the vessel, thus it can impart both yaw and roll moment. Therefore, it is possible to use the ruder to achieve two different purposes: heading control and roll damping. A RRD system is relatively inexpensive since the device is already present on the ship, even though an up-grade of the rudder machinery could be necessary, and does not require extra space or extra weight on the boat. In addition, the RRD can be combined with other roll damping techniques to achieve higher performances. In any case, a sophisticated control system is necessary to assure good performances under different sailing conditions. An overview of the most important contribution in rudder roll stabiliser is mentioned in Perez (2005) and Perez and Blanke (2010). Ship’s roll stabilising controllers aim at reducing the effect on the roll component of the waves impacting on the ship’s hull. From the point of view of control design, this is seen as the effect of an output disturbance, spread over a certain bandwidth, on the ship’s roll response. As a consequence, it is reasonable to design the controller to directly shape the output sensitivity function under the Internal Model Control (IMC) framework, characterised by a model-based approach that provides explicit relationship between the structure of the controller and that of the plant model, Tzeng et al. (2001). Unfortunately limitations apply on the achievable performances of the closed loop system, due to the Non-Minimum Phase (NMP) of the rudder to roll response. Indeed, under linearity assumptions, the transfer function from rudder angle (or rate) to roll angle (or rate) has a Right-Half Plane (RHP) zero, hence an attempt to damp the roll motion over a certain bandwidth amplifies it over other frequencies, see Doyle et al. (1992). Hearns and Blanke (1998) and later Goodwin et al. (2000) (see also references therein) proposed methods for evaluating the relationship between the desired damping bandwidth, the undesired disturbance amplification and the position of the RHP zero. The result is the estimation of a lower bound for the disturbance amplification outside the range of reduction. Their studies are significant because the output disturbance induced by the waves has energy distributed over a wide range of frequencies.

In this work we present the design phases of a controller for the RRD problem. The controller is derived using the IMC principle, directly shaping the output sensitivity function because it relates the waves disturbance to the ship’s roll motion, following the work in Blanke (2008),
Perez and Blanke (2010), Blanke et al. (2000). The controller is derived solving a non-linear constrained optimisation problem, where the optimisation function quantifies the trade-off between the damping action and the undesired amplification of the output disturbance, while problem constraints are given from specifications for the close loop controlled system. The bound on the allowable disturbance’s amplification is derived from the study of the Poisson integral to which a better approximation is provided with respect to previous results. The controller’s performances, after the optimisation stage, are tested on simulation comparing them with the performances obtained with a Minimum Variance Cheap Limiting Optimal Control approach used as a benchmark, Perez (2005).

The paper is organised as follows. Section 2 resumes the dynamic of ship motion in seaway. Section 3 extends the analysis of the achievable performance for RRD systems. Section 4 shows the controller design following a procedure that combines the sensitivity function loop shaping in the optimisation theory. Section 5 presents the results of the controller testing and draws conclusions of the work.

2. SHIP MOTION

The ship motion models for roll control systems design and testing are usually based on a motion superposition assumption: the sum of two control-induced, \( X^c \), and wave-induced motion, \( X^w \), gives the ship motion variables, \( X \). Therefore the dynamics effects are the sum of a Manoeuvring (calm water assumption) and a seakeeping model for motion in waves, under the assumption of constant forward velocity.

2.1 Manoeuvring model

Using the notation reported in Perez (2005) and applying the Newtonian mechanics, the rigid body equations of motions in 6 DOF for a marine vessel can be conveniently written as:

\[
M_{RRD} \ddot{\eta} + M_{RB} \nu + C_{RB} \nu = \tau_n + \tau_c + \tau_p
\]

\[
\eta = J(\eta) \nu
\]

where \( \eta \triangleq [n, e, d, \phi, \theta, \psi]^T \) and \( \nu \triangleq [u, v, w, p, q, r]^T \) are the generalised position orientation vector and linear angular velocity vector, respectively described in an inertial and in a body fixed frame. \( J(\eta) \) relates one frame to another. Terms \( \tau_n, \tau_c \) and \( \tau_p \) represent hydrodynamics, control and propulsion forces respectively. \( M_{RB} \) is the generalised mass matrix and \( C_{RB} \) is the Coriolis and centripetal acceleration matrix. From (1), the roll motion dynamics have been studied considering a 3-DOF model expressed in a b body-fixed frame, thus taking into account couplings in sway-roll-yaw and a constant surge velocity, as in Blanke and Christensen (1993). The model in (1) can be then rewritten as a state-space system as in (2), where the state is given by \( x = [x, p, r, \phi, \psi] \) denoting sway velocity, roll rate, yaw rate, roll and yaw angle respectively, \( M \) is a matrix comprising mass, inertia and added mass and added inertia, \( f(x) \) express hydrodynamics and hydrostatic forces. \( I \) is a unit matrix. Details are found in Blanke and Christensen (1993).

\[
\dot{x} = \begin{bmatrix}
M^{-1} & 0 \\
0 & I
\end{bmatrix} f(x) + \begin{bmatrix}
M^{-1} \\
0
\end{bmatrix} \tau_c + \tau_p
\]

2.2 Seakeeping model

A roll motion of a ship in a seaway is mainly affected by the waves, which are random both in time and space. Therefore, mathematical models for ocean waves and their effects on the ship motion are depicted in a stochastic framework, Perez and Blanke (2002), and the observed sea surface, at a certain location and for short periods of time, is considered a realization of a stationary and homogeneous, zero mean Gaussian stochastic process, that is characterised by a narrow-band Power Spectral Density (PSD). The analysis techniques proposed in the literature are developed in the frequency domain. The key element for this methods are the Motion Response Amplitude Operators (Motion-RAO), which describe the ship motion due to external wave excitation on the ship hull as function of the excitation frequency, ship forward speed and the wave direction relative to the ship. The motion RAO, the PSD of the waves and the transformation to encounter frequency define the energy content of the ship motion induced by waves.

3. FUNDAMENTAL LIMITATIONS IN RUDDER ROLL STABILISERS

Under linear superposition assumption, the performances of a RRD can be measured by means of the output sensitivity function, the ship response to wave-induced disturbances in roll is defined as:

\[
S(s) \triangleq \phi_{ol}(s)/\phi_{cl}(s) = (1 + C(s) \cdot G(s))^{-1}
\]

where \( \phi_{cl} \) and \( \phi_{ol} \) are respectively the closed-loop and open-loop roll angle. In particular \( \phi_{ol} \) is the ship response to wave disturbances. The desired performance for the RRD is to reduce \( S(j\omega) \) in the frequency range where the wave-induced motion has the most energy. The rudder to roll transfer function presented in Eq. (3) presents a zero in the Right-Half Plane. Thus there is a Non-Minimum Phase (NMP) dynamics in ship response to rudder command, which means the systems has an inverse response, Perez (2005).

Following Hearns and Blanke (1998) and Goodwin et al. (2000) the limitations imposed by the presence of a RHP zero in (3) can be studied using the Poisson integral:

\[
\int_{-\infty}^{+\infty} \log |S(j\omega)| \frac{q_1}{q_1^2 + \omega^2} d\omega = \pi \cdot \log \prod_{i=1}^{N_p} \frac{|p_i + q_1|}{|p_i - q_1|}
\]
Fig. 1. Comparison of the Poisson Integral using the approach in Goodwin et al. (2000) (black dashed line) and the one proposed in this work, which add the area under the black solid line triangle centred in $\omega_s$ where $q_1$ is the RHP zero, $N_p$ is the number of unstable poles and $p_i$ is the $i$th unstable pole. The term $w(q_1, \omega) = \frac{q_1}{\omega - q_1}$ is called the Poisson kernel, which represents a weight relative to the position of the RHP zero. Assuming a linear controller without unstable pole-zero cancellation, the open-loop system is stable, therefore (5) is equal to zero. The integral expresses a weighted area balance of the integrand function. This means that if the output disturbance is to be attenuated in a range of frequencies $\omega \in (\omega_1, \omega_2)$, then there must be an amplification for frequencies outside that range. This trade-off is concentrated in a limited bandwidth depending on the Poisson kernel and the RHP zero. In order to find an estimate of $M_2$ $\triangleq \|S(j\omega)\|_{\infty}$, which represents the maximum amplification outside the damping bandwidth, the right hand side of (5) can be overestimated. Therefore a general approximation (Hearns and Blanken (1998)) of the area shaped from the sensitivity function can be considered. The approximation is composed by three rectangles reported in Fig. 1. The controller is to be designed to achieve the following performances:

$$|S(j\omega)| \leq M_1 < 1 \quad \forall \omega \in (\omega_1, \omega_2)$$

$$|S(j\omega)| = 1 \quad \omega \in (0, \omega_1) \cup (\omega_1, \infty)$$

The second constraint is derived from the trade-off linking the output and complementary sensitivity $S(s) + T(s) = 1$. A low value of $S(j\omega)$ at high frequency generates a high value of $T(j\omega)$ at high frequencies with a consequent measurement noise amplification. From (6) follows the terms $\int_{-\infty}^{+\infty} \log |S(j\omega)| w(q_1, \omega) d\omega$ and $\int_{-\infty}^{+\infty} \log |S(j\omega)| w(q_1, \omega) d\omega$ are null, because calculated over a zero-amplification region. Therefore we have:

$$\int_{-\infty}^{+\infty} \log |S(j\omega)| w(q_1, \omega) d\omega = \int_{\omega_1}^{+\infty} \log |S(j\omega)| w(q_1, \omega) d\omega + \int_{-\infty}^{\omega_1} \log |S(j\omega)| w(q_1, \omega) d\omega = 0$$

The integral of the Poisson Kernel is defined as:

$$\Theta_{q_1}(\omega_q, \omega_b) \triangleq \int_{\omega_1}^{\omega_b} w(q_1, \omega) d\omega = \left(\tan^{-1} \frac{\omega_b}{q_1} - \tan^{-1} \frac{\omega_q}{q_1}\right)$$

Therefore considering the approximation of the sensitivity function in Fig. 1, and the integral of the Poisson Kernel it is possible to overestimate the right hand side of Eq. (7) as:

$$\Theta_{q_1}(\omega_q, \omega_b) \cdot \log M_2 + \Theta_{q_1}(\omega_q, \omega_b) \cdot \log M_1 + \Theta_{q_2}(\omega_q, \omega_b) \cdot \log M_2 \geq 0$$

Obtaining:

$$c_1 \log M_1 + c_2 \log M_2 \geq 0.$$  (10)

Following Goodwin et al. (2000), $\omega_q = -\infty$ and $\omega_b = +\infty$ led to $c_2 = \pi - c_1$ and $c_1 = \Theta_{q_1}(\omega_q, \omega_b)$

$$M_2 \geq M_1^\frac{c_2}{c_1}$$

(11)

The result of Goodwin et al. (2000), shown in (11), gives a lower bound to the amplification peak $\|S(j\omega)\|_{\infty}$ outside the damping region, but any information on the location of $\|S(j\omega)\|_{\infty}$ cannot be provided. An improvement to the $\|S(j\omega)\|_{\infty}$ can be obtained assuming a control feedback designed to achieve a maximum damping $M_2 \leq M_1 < 1$ at a frequency $\omega_s$. The specifications in (6) hold. The damping region is centred to $\omega_s$. That means the area overestimation embraces the dashed triangular in Fig. 1. The result is given in (12):

$$\int_{-\infty}^{+\infty} \log |S(j\omega)| w(q_1, \omega) d\omega \leq \int_{\omega_1}^{+\infty} \log M_2 w(q_1, \omega) d\omega + \int_{\omega_2}^{+\infty} \log M_1 \log M_2 w(q_1, \omega) d\omega +$$

$$+ \int_{-\infty}^{\omega_1} \left(\log M_1 - \log M_3 - \log \omega_1 - \log \omega_2\right) w(q_1, \omega_1) d\omega +$$

$$+ \int_{-\infty}^{\omega_1} \left(\log M_1 - \log M_3 - \log \omega_1 - \log \omega_2\right) w(q_1, \omega_2) d\omega +$$

$$+ \int_{-\infty}^{\omega_1} \left(\log M_1 - \log M_3 - \log \omega_1 - \log \omega_2\right) w(q_1, \omega) d\omega$$

(12)

The solution of inequality (12) is given by:

$$0 \leq (\Theta_{q_1}(\omega_q, \omega_1) + \Theta_{q_1}(\omega_2, \omega_b)) \log M_2 +$$

$$+ q_1 \left(\log \omega_2^2 - q_1^2\right) - \log \left(\omega_2^2 + q_1^2\right) \log M_1 - \log M_3 +$$

$$+ \omega_2 \left(\log \omega_2^2 - q_1^2\right) - \log \left(\omega_2^2 + q_1^2\right) \log M_1 - \log M_3 +$$

$$+ \omega_1 \log M_1 - \log M_1 \log \Theta_{q_1}(\omega_1, \omega_1) +$$

$$+ \omega_1 \log M_1 - \log M_3 \log \Theta_{q_1}(\omega_1, \omega_2)$$

One groups the terms dependent on the minimum damping ($M_1$) and on the maximum damping ($M_3$) in (13). Then, choosing $\omega_q = -\infty$ and $\omega_b = +\infty$, the lower bound $M_2$ on the output sensitivity positive amplification is given by:

$$M_2 \geq M_1^\frac{d_1}{\Theta_{q_1}(\omega_1, \omega_b)}$$

$$M_3$$

(14)

Equations (11) and (14) can be used to evaluate whether a controller can be designed to fulfil specified constraints on the location of the maximum damping frequency $\omega_s$, the attenuation bandwidth window $[\omega_1, \omega_2]$, the minimum damping required in the damping window and the value of the maximum damping. The comparison between the overestimation of $M_2$ $\|S(j\omega)\|_{\infty}$ in (11) and (14) is shown in Fig. 2. As function of the lower limit of the attenuation bandwidth $\omega_1$, with $\omega_s$ equal to the natural roll roll frequency $\omega_n$. The location of the RHP zero is at $\omega = 0.1794[rad/s]$. The over estimation of the approach proposed in this paper gives higher values than the approach of Goodwin et al. (2000). The gap between the
Lower Bound of the Disturbance Amplification as function of the lower limit of the damping region

Fig. 2. Comparison between (11) (red dashed line) and (14) (blue solid line) for the lower bound estimation of \( \omega \). The set of specifications for the closed-loop system is defined as:

\[ \omega = \omega_n, \quad M_1 = 0.5 \quad \text{and} \quad M_3 = 0.3 \]

two values grows with the spreading of the damping region, meaning the importance of the information on the maximum damping to overestimate the Poisson Integral. The contribution of this study gives the designer a tool to establish whether the damping region can be extended to lower limits. The results presented in Fig. 2 shows that any linear controller cannot assure a damping region extended to low frequencies, close to the RHP zero. As an example with a lower frequency bound of \( \omega_1 = 0.5 \text{rad/s} \), a required damping \( M_1 = 0.5 \) for \( \omega \in [\omega_1, \omega_2] \) and \( M_3 = 0.3 \) at the natural roll frequency, a disturbance amplification of at least 110% is expected, if waves induced motion has a significant energy for \( \omega \not\in (\omega_1, \omega_2) \).

4. CONTROLLER DESIGN AND OPTIMISATION

Results of Section 3 clearly show how the closed loop performances are affected from trade-off, when the disturbance is not known exactly in terms of energy content. Therefore specifications for a RRD should require sensitivity reduction over a range of frequency possibly where the disturbance is likely to be. At the same time it is required to avoid excessive amplifications, in case of changes in the sea conditions or vessel speed or heading, the energy content can be concentrated outside the frequency range of reduction. This is likely to happen when sailing in quartering seas (\( 0^\circ < \chi < 90^\circ \)), because the transformation from wave to encounter frequencies can map significant energy below \( \omega_1 \), defined in (6).

4.1 Control Design

For RRD, the control problem to be solved can be formulated as:

- \( O \): the objective is to decrease the roll motion induced on the ship hull by the waves using the rudder as actuator.
- \( SP \): the set of specifications for the closed-loop system to be satisfied

\[ \text{CL} : \text{the control law belongs to the class of closed loop control systems. It is a function of the reference roll angle and the measured roll angle.} \]

As reported in (4) the actual achievable damping is determined by the magnitude of the sensitivity function \( S(s) \), which depends on the RHP zero and the ship dynamics. A way to set specifications on the closed loop system is to shape a desired sensitivity function, that represents an ideal objective defined by means of \( SP \). The latter, similarly to (6), are:

1) \( |S_{\text{des}}(j\omega)| < M_1 < 1 \quad \omega \in [\omega_1, \omega_2] \)
2) \( |S_{\text{des}}(j\omega)| = 0 \quad \omega \not\in [\omega_1, \omega_2] \)
3) \( \min |S_{\text{des}}(j\omega)| = M_3 < M_1 < 1 \quad \omega = \omega_s \)

Using a fixed controller for RRD, the attenuation bandwidth has been chosen according to the sea and travelling conditions (e.g., sailing speed). The most of the energy content of the roll motion is located around the roll natural frequency; therefore an usual choice for \( \omega_s \) is the natural roll frequency. Thus this condition is not always verified, therefore specifications are given for a large damping bandwidth. From (15), a possible form for the desired sensitivity function is:

\[ S_{\text{des}} = \frac{s^2 - 2\xi_s \omega_s s + 1}{(1 + \frac{\omega_s^2}{\omega_n^2})(1 + \frac{s}{\omega_n})} \]

The parameters \( \omega_s, \xi_s \) and \( \beta_s \) must be chosen. If \( \omega_s = \omega_\phi \), the attenuation bandwidth has the negative peak in \( \omega_\phi \) and the damping coefficient \( \xi_s \) can be computed imposing the second constraint in (15). Considering a decoupled feedback loop the Internal Model Principle can be used to obtain the desired specifications in (15), as in Tseng et al. (2001). Using the rudder as command input the controller \( C_{\delta \phi} \), required to achieve the desired sensitivity function in (16), is:

\[ C_{\delta \phi} = (S_{\text{des}}^{-1} - 1) \cdot G_{\delta \phi}^{-1} \]

Unfortunately there are unavoidable consequences due to the presence of a RHP zero in (3). Indeed, it is not possible to achieve the perfect control if the internal stability of the system is required, because the process inversion makes the controller \( C_{\delta \phi} \) unstable. A stable controller can be obtained from (17) replacing the RHP zero in \( G_{\delta \phi} \) with its Left-Half Plane mirror, using an all-pass term, in order to maintain the DC gain of the controller in (17). At a high frequency pole \( \tau_h \) has been added to obtain a proper transfer function:

\[ C_{\delta \phi}(s) = \frac{\delta(s)}{-\phi(s)} = \frac{s_{\text{des}} - 1}{G_{\delta \phi}} \cdot \frac{s - q_1}{s + q_1} \cdot \frac{1}{1 + \tau_h s} \]

Equation (18) is not realisable because of the zero on the imaginary axis. Considering the roll rate as control the controller becomes strictly proper and realisable:

\[ C_{\delta \phi}(s) = \frac{\delta(s)}{-\phi(s)} = \frac{\delta(s)}{-s\phi(s)} = \frac{C_{\delta \phi}(s)}{s} \]

4.2 Control Optimisation

The controller form in (19), similar to the one introduced by Blanke et al. (2000), is useful because changing \( \beta_s, \xi_s \) and \( \omega_s \) in (16) allows the controller in (19) to adapt to different sea conditions. Moreover a new constraint for limiting the amplification peaks should be defined and
Fig. 3. Bode Diagram of the sensitivity function during the optimisation process. The minimum of the optimisation is given for the red dashed line. The desired sensitivity is shown as a black dashed line. The circle line shows the waterbed effects when the controller (24) is tuned up with an attenuation bandwidth centred around 0.7 rad/s.

parameters chosen accordingly. Therefore an optimisation problem should be set up, which choose controller parameters minimising a cost function. Cost function must penalise all the controllers leading to a sensitivity function further from the ideal characteristics defined in (16), hence an expression for the index to be minimised is given by:

\[ J = \| S_{des}(j\omega) - S(j\omega) \|_2 \]  \hspace{1cm} (20)

where the sensitivity function is calculated applying the controller expression to \( G_{\phi\delta} \) defined in Eq. (3). Being the specifications and the cost function defined relative to the response in roll angle, (18) has been considered for optimisation and then (19) has been used to obtain a roll rate feedback controller. Then specifications imposed by (15) are included in the optimisation problem along with a constraint on the maximum amplification peak outside the damping region:

\[ c_1: \min |S(j\omega)| = M_3 < M_1 < 1 \hspace{1cm} \omega = \omega_s \]
\[ c_2: \arg \min |S(j\omega)| \in [\omega_3 - \Delta\omega \hspace{0.2cm} \omega_s + \Delta\omega] \]
\[ c_3: |S(2j\omega)| < M_1 < 1 \hspace{1cm} \omega \in [\omega_1, \omega_2] \]
\[ c_4: |S(j\omega)|_{\infty} \leq M_{2,\text{sup}} \hspace{1cm} \omega \not\in [\omega_1, \omega_2] \]  \hspace{1cm} (21)

Referring to (18) and (19), the controller expression has been changed, considering:

(1) Change in the Controller Gain
(2) Change of the Controller Dominant Pole
(3) Addition of a Lead Net

The controller gain is given by:

\[ K_g = (\beta_s + \beta_s^{-1} - 2\xi_s\omega_s)/(K_{\text{roll}}) \]  \hspace{1cm} (22)

Therefore lowering \( K_g \) contributes to lower both positive and negative peaks of the sensitivity function \( S(s) \). The controller dominant pole is \( q = q_1 \) in (18), namely the Left-Half Plane mirror of the RHP zero. Bringing \( q \) close to imaginary axis, contributes to reduce both positive and negative peaks of the sensitivity function. High values of the couple \((K_g, q)\) can lead to unstable control systems, because the RHP zero dynamics at the denominator of \( S(s) \) becomes dominant. Still the cost function of the optimisation process tends to avoid this case, because of the much higher values of the cost function. The addition of a lead net to the controller is justified because an increase of the magnitude in the open-loop control system, brings directly a reduction of \( S(s) \). The RRD problem combining the three actions together can be solved. Defining the vector of controller parameters \( x \), the controller and the sensitivity function expressions are:

\[ C_s(s) = C_{\delta\phi}(s) \cdot x_1 \cdot \frac{1 + \frac{s}{q_1}}{1 + \frac{s}{\omega_s}} \cdot \frac{1 + x_3s}{1 + \frac{x_3s}{\omega_s}} \]
\[ C_{\delta\phi}(s) = C_s(s)/s \]
\[ S_k(s) = \frac{1}{1 + C_s(s)G_{\phi\delta}(s)} \]  \hspace{1cm} (23)

then, from (20) and (23), the non-linear programming problem statement is:

\[ \min_{x} J = \min_{x} \| (S_{des}(j\omega) - S_k(j\omega, x)) \|_2 \quad \text{subject to constraints in (21)} \]  \hspace{1cm} (24)

The solution of (24) is called the Optimised-RRD (OPT-RRD) and that is the optimal solution in terms of minimising the cost function according to the constraints in (21). Figure 3 shows the optimisation process of the sensitivity function \( S(s) \). The minima gives the results for the optimised RRD. The optimisation algorithm uses a Sequential Quadratic Programming (SQP) method. In this method, a Quadratic Programming (QP) sub-problem was solved at each iteration. The optimisation for the non linear programming problem guarantees that the result is a local minimum and the solution depends on the basin of attraction that contains the initial point. An investigation on the existence of a global minima for (24) has been tested against a large set of initial values in the parameter space. In order to avoid saturations on the hydraulic machinery, a modified cost function was chosen to weight the control effort in the optimisation problem:

\[ \min_{x} J = \min_{x} \| (S_{des}(j\omega) - S_k(j\omega, x)) \|_2 + \lambda(1 - \lambda) \| (C_s(j\omega) \cdot S_k(j\omega, x)) \|_2 \]  \hspace{1cm} (25)

where \( \lambda \in [0 \hspace{0.2cm} 1] \) is a parameter to be chosen. The controller resulting from the optimisation problem is called the Optimised-RRD with Control Effort Weight (OPT-RRD-CEW).

5. RESULTS

The controllers designed in (24) and (25) have been applied to the non-linear 3DOF ship’s model used as benchmark in Perez et al. (2006). Controller’s performances can be derived for different encounter angles and forward speed considering the Reduction of Roll, \( RR(\phi) = 100 \cdot (1 - \text{rms}(\phi_{ol})/\text{rms}(\phi_{\text{ref}})) \), which quantifies the controller damping performances for a given wave scenario. As a comparison the performances of a MV-CLOC derived in Perez (2005) were also computed. The performances of the controllers are summarised in Table 1 for the following conditions: \( M_3 = 0.3 \) at \( \omega_s = 1.1315 \text{ [rad/s]} \), \( M_1 = 0.5 \) for \( \omega \in [0.75\omega_s, 1.25\omega_s] \). The vessel is sailing with \( U = 15 \text{ [kts]} \) in slight waves (sea state 3). A wave disturbance for a sea
Table 1. Control system performances

<table>
<thead>
<tr>
<th>RR(φ) [rms]</th>
<th>χ_e = 45°</th>
<th>χ_e = 90°</th>
<th>χ_e = 135°</th>
</tr>
</thead>
<tbody>
<tr>
<td>MV-CLOC</td>
<td>18.50%</td>
<td>20.79%</td>
<td>22.96%</td>
</tr>
<tr>
<td>Opt-RRD</td>
<td>20.4%</td>
<td>51.4%</td>
<td>57.23%</td>
</tr>
<tr>
<td>Opt-RRD-CEW</td>
<td>40.34%</td>
<td>56.73%</td>
<td>60.98%</td>
</tr>
</tbody>
</table>

Results show a significant roll reduction for the OPT-RRD and OPT-RRD-CEW with respect to the MV-CLOC. The introduction of the weighting term substantially modifies the closed loop performances at quartering seas, see Fig. 4. The control scheme is valid for different ship loads and speed, but the controller, based on ship motion model linearisation should be recomputed.

6. CONCLUSIONS

This paper outlined the main issues related to the linear design of a rudder-roll damping controller, presenting a novel approach for quantifying the limitations imposed by the non minimum phase dynamics. An rudder-roll damping controller was derived shaping the output sensitivity function. The controller expression was included in a constraint optimisation problem that was used to limit the disturbance amplification outside the attenuation bandwidth. The optimisation problem also allows to weight the control effort to prevent saturation on the rudder machinery. The closed loop performances were evaluated and simulation results showed a significant enhancement in roll reduction capability when compared with a Minimum Variance Cheap Limiting Optimal Controller approach.

ACKNOWLEDGEMENTS

We are grateful to Dr. Roberto Galeazzi, from the Technical University of Denmark for his support in developing this work.

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