Decoupling Smith Control for Multivariable System with Time Delays

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Abstract: In order to solve the decoupling control problem of multivariable system with time delays, a new decoupling Smith control method for multivariable system with time delays was proposed. Firstly, the decoupler based on the adjoint matrix of the multivariable system model with time delays was introduced. And the decoupled models were reduced to first-order plus time delay models by analyzing the amplitude-frequency and phase-frequency characteristics. Secondly, according to the closed-loop characteristic equation of Smith predictor structure, PI controllers were designed following the principle of pole assignment for Butterworth filter. Finally, using small-gain theorem and Nyquist stability criterion, sufficient and necessary conditions for robust stability were analyzed with multiplicative uncertainties which encountered frequently in practice. The illustrative example was given to show the superiority of response speed and load disturbance rejection performance.

1. INTRODUCTION

Effective control of multivariable with time delays processes is a difficult issue in the context of process control. Input-output loops in a multivariable plant usually have different time delays, and for a particular loop its output could be affected by all the inputs through different time delays. Such a plant can be represented by a multivariable transfer function matrix with multiple time delays around an operating point. The researches of control method for this kind of multivariable plants with multiple time delays have received considerable attention (Daniel et al., 2005; Dominique et al., 2009).

For multivariable coupling system, the controller was designed for each sub-loop by analyzing the dominant pole and amplitude ratio (Jietae et al., 2006). This method led to a large number of controllers design, and the controller regulating of one loop affected the other loops even the performance of the system. Decoupling control is an effective method for multivariable system to eliminate the interactions of sub-loops. By decomposing the system to independent sub-loops, the controller is designed based on independent sub-loop. Static decoupler and dynamic decoupler at the input port were introduced to construct unit feedback closed-loop decoupling control (Astrom et al., 2002; Saeed et al., 2006). But the above method can be used only in two-input two-output process, it can not be extended to the processes with more inputs and outputs. The expected diagonalization transfer matrix was devised for the closed-loop control system, and then the realizable decoupler matrix was proposed by matching nominal system transfer matrix (Wang et al., 2002; Shen et al., 2010). Since the form of transfer matrix was set, the calculation of decoupler was complicated, and the structure of decoupler was too complex to implement.

Therefore, it is necessary to do more researches on decoupling control for multivariable systems.

For single-input-single-output processes with delay, Smith suggested a compensation scheme which can remove the delay from the closed-loop characteristic equation and thus eases feedback control design and improves set-point response greatly. This controller has been extended to multivariable systems with multiple time delays (Liu et al., 2005; Wang et al., 2000). Decoupler was introduced to eliminate the system coupling, and the controller was derived from using the design method of single variable Smith controller (Wang et al., 2000). So the problem of multivariable Smith decoupling is simplified to single variable Smith predictor design, it is a useful way to solve the problem of multivariable system with multiple time delays.

In this work, a new approach to the multivariable Smith predictor controller design is proposed for decoupling and stabilizing of multivariable process with multiple time delays. A decoupler is first introduced based on the adjoint matrix of the multivariable system model with time delays. To achieve better performance, additional delay items are added to diagonal elements of the decoupler matrix. With this decoupler, the multivariable Smith predictor controller is simplified to multiple single-loop Smith predictor controller design. After reducing the decoupled single-loop models to first-order plus time delay (FOPDT) models, PI controllers are designed by using the poles distribution rule of Butterworth filter. Numerical example of multivariable system with time delays validates the efficiency of the proposed method.
2. SMITH CONTROL FOR MULTIVARIABLE SYSTEM WITH TIME DELAYS

The multivariable Smith predictor control structure is shown in Fig. 1, where \( R(s), Y(s), C(s), K(s) \) are the input, output, controller, and decoupler. \( G(s) \) represents the process which is stable nonsingular matrix, i.e., \( \det(G(0)) \neq 0 \).

\[
H(s) = G(s)K(s), \quad H_0(s) \text{ is the same as } H(s) \text{ except with no delays.}
\]

![Multivariable Smith predictor structure](image)

Fig. 1. Multivariable Smith predictor structure

Consider the multivariable system with the transfer matrix:

\[
G(s) = \begin{bmatrix}
g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\
g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\
\vdots & \vdots & \ddots & \vdots \\
g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s)
\end{bmatrix}
\]

where \( g_j(s) = g_{j0}(s)e^{-\tau_{ij}}, (i, j = 1, \ldots, n) \) are the transfer function from \( i \)th input to \( j \)th output. \( g_{j0}(s) \) are strictly proper, stable scalar rational functions, and \( \tau_{ij} \) are the time delays associated with \( g_j(s) \). When the model is ideal, i.e., the process and its model is matched, the closed-loop transfer function from \( R(s) \) to \( Y(s) \) in Fig. 1 becomes

\[
\Phi(s) = H(s)C(s)[I + H_0(s)C(s)]^{-1}
\]

2.1 Design of Decoupler

Decoupling is to diagonalize the system transfer function \( G(s) \) to \( H(s) \), where \( H(s) = \text{diag}[h_i(s)], (i = 1, 2, \ldots, n) \), \( h_i(s) \) is regular rational transfer function. In this work, \( A \times \text{adj}(A) = \text{adj}(A) \times A = \det(A) \times I \) is used to design the decoupler as follow

\[
K' = \text{adj}(G)
\]

In order to include less delay terms, \( K' \) multiply time delay matrix as

\[
K' = \text{diag}\{e^{-\tau_i}, i = 1, 2, \ldots, n\}
\]

where \( \tau_i \) is smallest time delay in each column of \( K' \). For the revised decoupler without time delays prediction terms, the design can be realized. Then modify the decoupler coefficient in order to calculate conveniently, the decoupler becomes

\[
K(s) = \frac{K_D(s)K'(s)}{\det(G(0))}
\]

So the decoupled process is

\[
H(s) = G(s)K(s) = \frac{G(s)K_D(s)K'(s)}{\det(G(0))} = \text{diag}\{h_1, h_2, \ldots, h_n\}
\]

2.2 Model Reductive

After decoupling, the interactions are eliminated so that the system is equivalent to sub-loops existing independently. To simplify the decoupled process and ensure a good control system performance, model reduction is introduced to each sub-loop of the decoupled process.

Approximation of high order processes by low order plus dead time models is a common practice. As most industrial processes are inertial and time delays processes, which can be described by FOPDT models, the decoupled processes are reduced to FOPDT models in this work. The FOPDT model often reasonably describes the process gain, overall time constant and effective dead time of such a process. In order to find an approximation FOPDT model for \( h_i(s) \), three unknown parameters in (4), namely \( k_i, \tau_{di}, T_i, (i = 1, 2, \ldots, n) \), should be determined

\[
L(s) = \text{diag}\{l_1, l_2, \ldots, l_n\} = \text{diag}\{k_1e^{-\tau_{d1}T_1s}, k_2e^{-\tau_{d2}T_1s}, \ldots, k_ne^{-\tau_{dn}T_1s}\}
\]

Fitting Nyquist plots of high order a low order models at particular points, (5) is suggested to calculate the values of \( k_i, \tau_{di}, T_i \). This equation indicates that the steady state gain and gain margin are the same for the high order process and POPDT model.

\[
\begin{bmatrix}
l_i(0) = h_i(0) \\
l_i(j\omega_{ai}) = h_i(j\omega_{ai}) \\
\angle\{l_i(j\omega_{ai})\} = \angle\{h_i(j\omega_{ai})\}
\end{bmatrix}
\]

where the crossover frequency \( \omega_{ai} \) of the original system is determined using \( \angle l_i(j\omega_{ai}) = \pi \).

As a result, the parameters of the FOPDT model can be calculated using

\[
k_i = h_i(0)
\]

\[
T_i = \frac{\left|\frac{h_i(0)}{h_i(j\omega_{ai})}\right|^2 - 1}{\omega_{ai}}
\]
\[ \tau_{di} = \frac{\pi - \tan^{-1}(T_i \omega_{ci})}{\omega_{ai}} \]  

(8)

Define the matrix \( L_0(s) \) which is the same as the reduced model matrix \( L(s) \) except with no delays,

\[ L_0(s) = \text{diag}\left[ \frac{k_1}{T_1 s + 1}, \frac{k_2}{T_2 s + 1}, \ldots, \frac{k_n}{T_n s + 1} \right] \]  

(9)

then the closed-loop transfer matrix (1) described as

\[ \Phi'(s) = L(s)C(s)\left[I + L_0(s)C(s)\right]^{-1} \]  

(10)

2.3 Design of PI Controller

In order to design a set of PI tuning formulae for a FOPDT model, the PI parameters \( K_p, T_i \) in (11) should be defined.

\[ C(s) = \text{diag}\{K_p(1 + \frac{1}{T_{ip}s}), K_p(1 + \frac{1}{T_{ip}s}), \ldots, K_p(1 + \frac{1}{T_{ip}s})\} \]  

(11)

The Smith predictor introduced a compensator can remove the delay from the closed-loop characteristic equation. It can be seen from the closed-loop transfer function characteristic equation of FOPDT model as follow

\[ s^2 + \frac{1}{K_p} + \frac{K_p}{T_i} s + \frac{K_p}{T_i T_H} = 0 \]  

(12)

The principle of Butterworth filter is poles assign on unit circle symmetrically in the left half of complex plane by using real axis as symmetry axis (Li et al., 2006). In this case, the poles distribute uniformly on the Butterworth circle. For the ideal pole positions, Butterworth filter has the performance of low pass filter. The Butterworth filter is defined as

\[ g(s) = \frac{\omega_0^n}{s^n + \beta_{n-1} \omega_0 s^{n-1} + \ldots + \beta_1 \omega_0 s + \omega_0^n} \]

According to the principle of Butterworth, the expected transfer function characteristic equation for second-order system is designed. Distributing the poles on the position of the angle \( \theta = 45^\circ \) to imaginary axis, the coefficient can be determined by \( \beta_0 = 1, \beta_1 = 1.414 \). The parameters of PI controller are determined as follows:

\[ K_p = \frac{\beta_0 \omega_0 T_i - 1}{k_i} \]

(13)

\[ T_i = \frac{\beta_0 \omega_0 T_i - 1}{T_i \beta_0 \omega_0^2} \]

(14)

where \( \omega_0 \) is cut off frequency.

3. ROBUST STABILITY ANALYSIS

The multivariable Smith system with uncertain process is shown in Fig. 2. \( \Delta(s) \) are uncertainties. Let the nominal stable process transfer function be \( G(s) \) and the real process be described by the family:

\[ \Pi = \left\{ G'(s) = G(s) + \Delta(s) \right\} \]

where the perturbation \( \Delta(s) \) is stable.

Here, the robust stability analysis is focused on the process additive, multiplicative input and output uncertainties, which are commonly encountered in engineering practice. Usually, the process additive uncertainties can be viewed as parameter perturbation \( \Delta(s) = G(s) \Delta_s(s) \) to the process transfer matrix, and the actual process family may be described as \( \Pi_s = \left\{ G'(s) = G(s) + \Delta_s(s) \right\} \), where \( \Delta_s(s) \) is assumed to be stable. The process multiplicative input uncertainties can be loosely interpreted as the process input actuator uncertainties \( \Delta(s) = G(s) \Delta_i(s) \), and the actual process family may be described as \( \Pi_j = \left\{ G'(s) = G(s)(I + \Delta_j(s)) \right\} \), where \( \Delta_{2j}(s) \) is assumed to be stable. The process multiplicative output uncertainties can be practically viewed as the process output measurement uncertainties \( \Delta(s) = \Delta_{2o}(s)G(s) \), and the actual process family may be described as \( \Pi_{2o} = \left\{ G'(s) = (I + \Delta_{2o}(s))G(s) \right\} \), where \( \Delta_{2o}(s) \) is assumed to be stable.

![Fig. 2. Multivariable Smith system with uncertain process](image)

The transfer function \( F(s) \) from \( \dot{Z} \) to \( \dot{U} \) in Fig.2 is described as

\[ F(s) = -KC[I + GKC - (H - H_o)C]^{-1} \]  

(15)

According to the equivalent relationship between the small gain theorem and the multivariable spectral radius stability criterion, the necessary and sufficient condition of robustly stable for multivariable system with multiple time delays in Fig.1 is (Ricardo et al., 2009):

\[ \rho(F(s)\Delta(s)) < 1, \forall \omega \in [0, \infty) \]  

(16)

Correspondingly, the spectrum stability constrains shown in (16) can be checked graphically by observing whether the
magnitude plots of the left sides of (16) fall below the unity for all \( \omega \in [0, \infty) \).

4. SIMULATION

In order to show the effectiveness of the proposed design method, the following system is considered.

\[
G = \begin{bmatrix}
1.986e^{-0.71s} & -5.24e^{-60s} & -5.984e^{-2.24s} \\
66.7s + 1 & 400s + 1 & 14.9s + 1 \\
-0.0204e^{-0.59s} & 0.33e^{-0.68s} & -2.38e^{-0.42s} \\
(7.14s + 1)^2 & (2.38s + 1)^2 & (1.43s + 1)^2 \\
-0.374e^{-7.75s} & 11.3e^{-3.79s} & 9.811e^{-1.95s} \\
22.2s + 1 & (21.74s + 1)^2 & 11.36s + 1
\end{bmatrix}
\]

The decoupler is designed using (2)

\[
K(s) = \begin{bmatrix}
k_{11} & k_{12} & k_{13} \\
k_{21} & k_{22} & k_{23} \\
k_{31} & k_{32} & k_{33}
\end{bmatrix}
\]

where

\[
k_{11} = 0.0591e^{-0.09s} + 0.4910e^{-2.03s} \\
(2.38s + 1)^2(11.36s + 1) + (1.43s + 1)(21.74s + 1)^2,
\]

\[
k_{12} = 0.9386e^{-59.29s} + 1.2346e^{-3.73s} \\
(400s + 1)(11.36s + 1) + (1.43s + 1)(21.74s + 1)^2,
\]

\[
k_{13} = 0.2277e^{-59.29s} + 0.0361e^{-1.79s} \\
(400s + 1)(1.43s + 1)^2 + (1.49s + 1)(2.38s + 1)^2,
\]

\[
k_{21} = 0.0037 + 0.0163e^{-5.99s} \\
(11.36s + 1)(7.14s + 1)^2 + (22.2s + 1)(1.43s + 1)^2,
\]

\[
k_{22} = 0.3558 - 0.0409e^{-7.69s} \\
(11.36s + 1)(66.7s + 1) - (22.2s + 1)(14.9s + 1),
\]

\[
k_{23} = 0.0863 - 0.0022e^{-1.7s} \\
(1.43s + 1)^2(66.7s + 1) - (1.43s + 1)^2(14.9s + 1),
\]

\[
k_{31} = -0.0004e^{-2.2s} + 0.0023e^{-6.25s} \\
(21.43s + 1)^2(7.14s + 1)^2 + (22.2s + 1)(2.38s + 1)^2,
\]

\[
k_{32} = -0.0099e^{-2.2s} + 0.0358e^{-65.45s} \\
(21.43s + 1)(66.7s + 1) + (22.2s + 1)(400s + 1),
\]

\[
k_{33} = 0.012e^{-0.06s} - 0.002e^{-59.46s} \\
(2.38s + 1)^2(66.7s + 1) - (1.43s + 1)^2(400s + 1).
\]

After revising the time delays and the coefficient, the decoupled processes are expressed as

\[
h_1 = \frac{0.1174}{Q_1} e^{-0.85s} + \frac{0.9755}{Q_2} e^{-2.74s} - \frac{0.0194}{Q_3} e^{-60s} - \frac{0.0854}{Q_4} e^{-65.99s} + \frac{0.0251}{Q_5} e^{-4.44s} - \frac{0.0138}{Q_6} e^{-8.48s},
\]

\[
h_2 = \frac{0.1174}{Q_1} e^{-0.68s} + \frac{0.9755}{Q_2} e^{-2.62s} - \frac{0.0191}{Q_3} e^{-59.88s} - \frac{0.0852}{Q_4} e^{-65.87s} + \frac{0.0252}{Q_5} e^{-4.32s} - \frac{0.0135}{Q_6} e^{-8.37s},
\]

\[
h_3 = \frac{0.1177}{Q_1} e^{-1.65s} + \frac{0.9751}{Q_2} e^{-3.79s} - \frac{0.0196}{Q_3} e^{-61.05s} - \frac{0.0851}{Q_4} e^{-67.04s} + \frac{0.0248}{Q_5} e^{-5.49s} - \frac{0.0135}{Q_6} e^{-9.54s},
\]

where

\[
Q_1 = (66.7s + 1)(2.38s + 1)^2(11.36s + 1),
\]

\[
Q_2 = (66.7s + 1)(1.43s + 1)^2(21.74s + 1)^2,
\]

\[
Q_3 = (400s + 1)(11.36s + 1)(7.14s + 1)^2,
\]

\[
Q_4 = (400s + 1)(22.2s + 1)(1.43s + 1)^2,
\]

\[
Q_5 = (14.9s + 1)(21.74s + 1)^2(7.14s + 1)^2,
\]

\[
Q_6 = (14.9s + 1)(22.2s + 1)(2.38s + 1)^2,
\]

Reducing the decoupled system model, the crossover frequencies of \( h_1(s), h_2(s), h_3(s) \) are \( \omega_1 = 0.048 rad/s \), \( \omega_{x2} = 0.053 rad/s \), and \( \omega_3 = 0.046 rad/s \). According to (6), (7), and (8), the parameters of FOPDT models are calculated. So the reduced multivariable transfer matrix is

\[
L(s) = \begin{bmatrix}
e^{-32.44s} & 0 & 0 \\
139.5s + 1 & 0 & 0 \\
0 & 140.6s + 1 & 0 \\
0 & 0 & e^{-36.72s}
\end{bmatrix}
\]

Fig. 3 shows the Nyquist plots of the element of \( h_1(s) \), and their FOPDT approximation model \( l_i(s) \). The Nyquist plot of reduced model and process model matched approximately at low frequency, so the model reduction is effective.

Using (13) and (14), the parameters of PI controller are

\[
K_{P_1} = 13.1, T_{I_1} = 86, K_{P_2} = 15.2, T_{I_2} = 83, K_{P_3} = 20.5, T_{I_3} = 64.3
\]

Output responses to unit step function in the first input are shown in Fig. 4. For the simulation, a 0.2 step disturbance has been applied at the process input at \( t = 200s \). It is clearly seen that there is no overshoot in the setpoint responses by using the proposed method, and the process output responses are almost decoupled from each other. Moreover, obviously improved response speed and load disturbance rejection performance comparing with Wang’s method are obtained.
To demonstrate robustness of the proposed method, all the static gains of each element in the process transfer matrix are actually 40% larger, and in another case, all the time constants of each element in the process transfer matrix are assumed 40% larger to introduce the un-modeled dynamics.

According to the robust stability analysis given in section 3, the magnitude plots of spectral radius for identifying robust stability of the corresponding systems are shown in Fig. 6. It can be seen that both of the peak values (dotted and dash dot lines) are much less than the unity, indicating that the proposed control system facilitates good robust stability.

In order to further test the robust stability of the system, multiplicative input uncertainties $\Delta_{x0}$ and multiplicative output uncertainties $\Delta_{y}$ are supposed to the simulation.
Fig 6 has shown the corresponding magnitude plots of spectral radius based on the assumed multiplicative output uncertainties $\Delta_{2\theta}$, multiplicative input uncertainties $\Delta_{2\iota}$, gain uncertainty, and time constant uncertainty, which indicate that the proposed control system control preserve robust stability well. The corresponding perturbed system responses are shown in Fig. 5.

$$
\Delta_{2\theta} = \begin{bmatrix}
    s + 1 & 1 & 1 \\
    2s + 1 & s + 0.2 & 5s + 1 \\
    1 & 4s + 1 & 2 \\
    s + 1 & s + 1 & s + 1 \\
    1 & 1 \\
    s + 1 & s + 2 & 2s + 1
\end{bmatrix},
$$

$$
\Delta_{2\iota} = \begin{bmatrix}
    s + 0.2 & 1 & 1 \\
    s + 1 & 4s + 1 & 2s + 1 \\
    s + 2 & s + 5 & 2 \\
    3s + 1 & s + 1 & s + 1 \\
    1 & 1 & 1 \\
    s + 1 & s + 2 & 2s + 1
\end{bmatrix}
$$

Fig. 6. Spectral radius magnitude curve of perturbed system

2. CONCLUSION

(1) A new multivariable Smith predictor scheme for decoupling and stabilizing multivariable processes with multiple time delay is proposed, which can be used by high-dimensional processes.

(2) With the adjacent matrix approach to the decoupler design and the model reduction of the decoupled process, the multivariable Smith predictor design is simplified to multiple single-loop Smith predictor design.

(3) Based on the pole assignment of Butterworth filter, the PI controller for each decoupled loop is designed. Robust stability analysis is presented using the equivalence relation of small-gain theorem and Nyquist stability criterion. The spectrum stability constrains can be checked graphically by observing the magnitude plots of spectral radius. Numerical example has been given to illustrate the approach with which significant performance improvement over the existing methods.

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