Stabilization of a Magnetic Suspension by Immersion and Invariance and Experimental Robustness Study

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Abstract: This paper is dealing with the problem of stabilization of a magnetic suspension and the experimental tests of the designed nonlinear controller’s robustness. Up to now, in real world applications, classical controllers are the most implemented ones. In order to increase more and more the systems performance according to industry demand, the investigations devoted to use nonlinear controllers in a relative efficient way are still in progress. For this purpose, we investigate the applicability level of the Immersion and Invariance (I&I) based approach to study this problem. The I&I methodology has proven to be a very effective theoretical tool for the stabilization of nonlinear systems and its main advantage consists in making the closed loop system behave asymptotically like a target system. However, its applicability depends on the solvability of a set of partial differential equations (PDEs). In order to simplify this problem, it was recently proposed to consider these PDEs as parameterized algebraic equations. So, we have used this approach to derive two simple and robust controllers for the magnetic suspension system and the obtained experimental results illustrate the effectiveness of this control strategy in comparison with different additional studied controllers.

Keywords: Magnetic suspension system, Immersion & Invariance, Stabilization, Robustness.

1. INTRODUCTION

Magnetic suspension systems have received much attention as a mean of eliminating Coulomb friction due to mechanical contact. They have become popular in two different types of realizations: high-speed motion and precision engineering. Levitation bearings have been used in rotating machinery, for pumps in nuclear installations, for nanometric servo-position actuators in micro-lithography, vibration isolation in precision scientific instruments as well as for the famous high-speed transport systems “Maglev” or “Transrapid” for instance. Magnetic levitation highlights phenomena like nonlinearities, fast dynamics and actuator saturation. Many control techniques have been successfully implemented on magnetic levitation systems, including for example: feedback linearization control (Charara et al. (1996) and Trumper et al. (1997)), passivity-based control (Ortega et al. (1998)), backstepping design approach (Queiroz and Dawson (1996)) or interconnection and damping assignment (Rodriguez et al. (2000)). Many of these are limited by model relevance as well as its parameter accuracy. Among robust control methods, one can quote nonlinear output regulation (Gentili and Marconi (2002)), adaptive control laws (Yang and Tateishi (2001)), and sliding-mode control (Charara et al. (1996) and Gutierrez and Ro (1998)). Comparisons between linear and non-linear controllers have been reported in (Barie and Chiasson (1996) and Rodriguez et al. (2000) as well).

In this paper, we want to stabilize a magnetic suspension using the approach based on Immersion and Invariance (I&I) and to experimentally show that the obtained controller is robust with regards to the parameter uncertainties. Moreover, in order to emphasize the effectiveness level of this technique, a comparison study with some other non linear controllers has been performed. The method of I&I for stabilizing non-linear systems was originated in Astolfi and Ortega (2003) and was further developed in a series of publications recently summarized in Astolfi et al. (2008). The control objective is to find a command which guarantees that the closed-loop system asymptotically behaves like a target dynamic system. This is formalized by finding a manifold in state-space that can be rendered invariant and attractive, with internal dynamics a copy of the desired closed loop dynamics, and designing a control law that steers the state of the system towards that manifold. The applicability of this approach is conditioned by the solvability of a set of PDEs, and in Acosta et al. (2008) a method for avoiding the explicit resolution of these equations was proposed for a class of mechanical systems. In Sarras et al. (2010) the I&I approach was used to stabilize an inverted pendulum with a proven domain of attraction.

This paper is organized as follows. In Section 2 we introduce a brief review of I&I. The modeling of the magnetic suspension, the application of I&I for stabilization and the proof of the domain of attraction is discussed in Section 3. Finally, in Section 4, we present the experimental results using the magnetic suspension, before summarizing our contribution in Section 5.

2. REVIEW OF IMMERSION AND INVARIANCE

In this section we briefly recall the I&I approach as was introduced in Astolfi and Ortega (2003). The major result is synthesised by the following theorem.
Theorem 1. Consider the system\(^1\)
\[
\dot{x} = f(x) + g(x)u,
\]
with state \(x \in \mathbb{R}^n\) and control \(u \in \mathbb{R}^m\), with an equilibrium point \(x_e \in \mathbb{R}^n\) to be stabilized. Let \(p < n\) and assume we can find the mappings
\[
\alpha(\cdot): \mathbb{R}^p \to \mathbb{R}^p, \quad \pi(\cdot): \mathbb{R}^p \to \mathbb{R}^n, \quad c(\cdot): \mathbb{R}^p \to \mathbb{R}^m,
\]
\[
\phi(\cdot): \mathbb{R}^n \to \mathbb{R}^{n-p}, \quad \psi(\cdot): \mathbb{R}^{n(n-p)} \to \mathbb{R}^m,
\]
such that the following hold.

(H1) (Target system). The system
\[
\dot{\xi} = \alpha(\xi),
\]
with state vector \(\xi \in \mathbb{R}^p\), has an asymptotically stable equilibrium at \(\xi_e \in \mathbb{R}^p\) and \(x_e = \pi(\xi_e)\).

(H2) (Immersion condition). For all \(\xi \in \mathbb{R}^p\)
\[
f(\pi(\xi)) + g(\pi(\xi))c(\pi(\xi)) = \frac{\partial \pi}{\partial \xi} \alpha(\xi).
\]

(H3) (Implicit manifold). The set identity
\[
\{x \in \mathbb{R}^n | \phi(x) = 0\}
= \{x \in \mathbb{R}^n | x = \pi(\xi) \text{ for some } \xi \in \mathbb{R}^p\}.
\]
holds.

(H4) (Manifold attractiveness and trajectory boundedness). All trajectories of the system
\[
\dot{z} = \frac{\partial \phi}{\partial x} [f(x) + g(x)\psi(x,z)],
\]
\[
\dot{x} = f(x) + g(x)\psi(x,z),
\]
are bounded and satisfy
\[
\lim_{t \to \infty} z(t) = 0.
\]
Then \(x_e\) is an asymptotically stable equilibrium of the closed loop system
\[
\dot{x} = f(x) + g(x)\psi(x, \phi(x)).
\]

3. APPLICATION OF IMMERSION AND INVARIANCE

3.1 Magnetic suspension system

We consider the magnetic suspension depicted in fig. 1, roughly speaking it is composed of a rotor and an actuator. The rotor axis is in line with the acceleration due to the gravity and is subject to the force generated by the actuator thus causing vertical displacement. The actuator is consisting of two coils and two voltage controlled current sources. Its state space model (8) is obtained by applying Newton’s law for the mechanical subsystem and Faraday’s and Kirchhoff’s laws for the electrical subsystem, for more details one may see e.g. (Woodson and Melcher (1968))

\[\begin{align*}
\dot{x} &= v \\
\dot{v} &= \frac{b^2}{4mc} - g \\
\dot{\lambda} &= -R \frac{(x_0 - x_e - \xi)}{2c} + u.
\end{align*}\]

The state space vector is \(X = [\xi \ v \ \lambda]^T\). The equilibrium position to be stabilized, \(x_e\) is the equilibrium position to be stabilized, \(v\) is the velocity of the pendulum and \(\lambda\) is the magnetic flux in the core of the control coil. \(m, c, g, R, x_0\) are constants denoting the mass of the pendulum, the constant of the electromagnetic force, the gravitational acceleration, the electrical resistance of the electromagnet’s coil, and the distance between the tip of the pendulum and the tip of the electromagnet when the pendulum is in the rest position, respectively.

\[\begin{align*}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= -V'(\xi_1) - R(\xi_1, \xi_2)\xi_2,
\end{align*}\]

where \(V\) denotes the free potential and \(R\) the (possibly nonlinear) damping function, that for generality is depicted here as a function of \(\xi_1\) and \(\xi_2\). In order to ensure the local asymptotic stability of the equilibrium \(\xi_e = 0\), we must make the following assumptions.

Assumption 1. The free potential \(V(\xi_1)\), satisfies \(V'(0) = 0\) and \(V''(0) > 0\), where \(V'\) denotes the gradient of \(V\).

Assumption 2. The damping function \(R(\xi_1, \xi_2)\) satisfies \(R(0,0) > 0\).

The choice of the target dynamics suggests a natural choice of the mapping \(\pi\) as...
Thus, immersion condition (3) takes the form of a set of two PDEs
\begin{align}
\pi_3^2(\xi_1, \xi_2)/4mc - g &= -V'(\xi_1) - R(\xi_1, \xi_2)\xi_2, \\
\frac{\partial \pi_3}{\partial x_1} \xi_2 - \frac{\partial \pi_3}{\partial \xi_2} (-V'(\xi_1) - R(\xi_1, \xi_2)\xi_2) &= \frac{R}{2c}(x_0 - x_1 - \xi_1)\pi_3 + c(\pi(\xi)).
\end{align}

From the first equation we obtain
\begin{equation}
\pi_3(\xi_1, \xi_2) = 2\sqrt{mc}g - V'(\xi_1) - R(\xi_1, \xi_2)\xi_2. 
\end{equation}

The remaining conditions of Theorem 1 must now be verified. The definition of the implicit manifold of (H3) is straightforward
\begin{equation}
\phi(x) = -\lambda - \pi_3(\bar{x}, v) = 0.
\end{equation}

We are also proposing another definition of $\phi(x)$ which is preferred because it uses $\pi_3$ instead of $\pi_3^2$
\begin{equation}
\phi(x) = \frac{1}{2}(\lambda^2 - \pi_3^2(\bar{x}, v)) = 0.
\end{equation}

We must study now the distance between the system trajectories and the manifold, defined as $z = \phi(x)$ and called the off-the-manifold coordinate. If we choose $\phi(x)$ as in (14) the dynamics of $z$ are given as
\begin{equation}
\dot{z} = -\frac{R}{2c}(x_0 - x_1 - \bar{x})\lambda + \psi(X, \bar{x}) - \pi_3(\bar{x}, v).
\end{equation}

By imposing $\dot{z} = -\gamma_1z$, $\gamma_1 > 0$ in order to ensure exponential convergence of $z$ towards 0, we obtain the following control law
\begin{equation}
\psi_1(X, \phi(x)) = \pi_3(\bar{x}, v) + \frac{R}{2c}(x_0 - x_1 - \bar{x})\lambda - \gamma_1(\lambda - \pi_3(\bar{x}, v)).
\end{equation}

Using choice (15) for the off-the-manifold coordinate, its dynamics are given by:
\begin{equation}
\dot{z} = \lambda \psi(X, z) - \frac{R}{2c}(x_0 - x_1 - \bar{x})\lambda^2 - \frac{1}{2}\frac{d(\pi_3^2(\bar{x}, v))}{dt}.
\end{equation}

It immediately follows that the controller
\begin{equation}
\psi_2(X, \phi(x)) = \frac{1}{2\lambda} \frac{d(\pi_3^2(\bar{x}, v))}{dt} - \gamma_2(\lambda^2 - \pi_3^2(\bar{x}, v)) + \frac{R}{2c}(x_0 - x_1 - \bar{x})\lambda,
\end{equation}
fixes the off-the-manifold dynamics to $\dot{z} = -\gamma_2z$, $\gamma_2 > 0$, hence ensuring exponential convergence towards zero. For this controller special care must be taken when $\lambda$ crosses zero. In order to avoid this problem, we propose to saturate the magnetic flux
\begin{equation}
\lambda_F = \begin{cases} 
\lambda, & |\lambda| \leq \epsilon \\
\frac{\lambda}{s} & |\lambda| > \epsilon
\end{cases}.
\end{equation}

**Remark 1.** We have not expanded the time derivatives of $\pi_3(\bar{x}, v)$ and $\pi_3^2(\bar{x}, v)$, because, for this system, we can construct the command in a nested loop fashion, where the inner-loop controller calculates $\pi_3(\bar{x}, v)$ or $\pi_3^2(\bar{x}, v)$ and the outer-loop controller calculates $\psi_1(X, \phi(x))$ utilising the numerical derivatives of $\pi_3$ or $\pi_3^2$.

All that is left to be proven is the boundedness of trajectories of the closed-loop system. If we consider system (8) in closed loop with controller (17), after changing the coordinate system to $(x, v, \eta, z)$, with $\eta = \lambda - \pi_3(\bar{x}, v)$ we obtain the following (extended) closed loop dynamics:
\begin{equation}
\begin{aligned}
\dot{x} &= v \\
\dot{v} &= \pi_3(\bar{x}, v) + \eta^2/4mc - g \\
\dot{\eta} &= -\gamma_1z \\
\dot{z} &= -\gamma_1z.
\end{aligned}
\end{equation}

From the last equation of (20), it is obvious that $z$ exponentially converges towards zero, thus $\eta(t)$ is bounded for all $t$. The subsystem formed by the first two equations of (20) with input $\lambda^2 = \pi_3^2(\bar{x}, v)$ represents the target dynamics and is asymptotically stable (via the choice of $V$ and $R$) thus all its trajectories are bounded. The only assumption that has to be made is that this subsystem remains stable for $\lambda^2 = (\pi_3(\bar{x}, v) + \eta(t))^2$, in other words, that it is robust with respect to a bounded input disturbance. For second control (19), the closed loop dynamics, after the partial coordinate transformation $(x, v, \eta, z)$ with $\eta = \lambda - \pi_3(\bar{x}, v)$, are:
\begin{equation}
\begin{aligned}
\dot{x} &= v \\
\dot{v} &= \pi_3^2(\bar{x}, v) + \eta/4mc - g \\
\dot{\eta} &= -\gamma_2z \\
\dot{z} &= -\gamma_2z.
\end{aligned}
\end{equation}

It is obvious once again that $z(t)$ approaches zero exponentially fast and thus $\eta(t)$ is bounded. The subsystem formed by the first two equations of (21) takes the following form
\begin{equation}
\begin{aligned}
\dot{x} &= v \\
\dot{v} &= -V'(\bar{x}) - R(\bar{x}, v)v + \eta/4mc
\end{aligned}
\end{equation}

which is nothing more than the target dynamics plus the additive bounded perturbation term $\eta/4mc$. Since the target dynamics are asymptotically stable (by choice), it follows immediately that the trajectories of (21) are bounded and satisfy $\lim_{t \to +\infty} x(t) = 0$, thus condition (H4) is verified.

**Remark 2.** It is straightforward to check that the controls satisfy $\psi_1(X, 0) = c(\pi(\xi))$ and $\psi_2(X, 0) = c(\pi(\xi))$, with $c$ defined by (12), thus the Immersion condition (H3) is satisfied.

The following proposition summarises the stability result from the application of the aforementioned controllers.

**Proposition 1.** For any function $\pi_3$ satisfying (14), with the functions $V$ and $R$ chosen such that assumptions A.1 and A.2 hold, the zero equilibrium of magnetic suspension system (8)
in closed loop with controller (17) or controller (19) and with \( r_1, r_2 > 0 \), is locally asymptotically stable.

The design procedure is concluded by choosing the functions \( V \) and \( R \). For simplicity, we select:

\[
\begin{align*}
R(\xi_1, \xi_2) &= k_R \\
V'(\xi_1) &= k_V \xi_1,
\end{align*}
\]

which gives the very simple form for \( \pi_3 \)

\[
\pi_3(\bar{x}, \bar{v}) = 2\sqrt{m_c g - k_V \bar{x} - k_R \bar{v}}.
\]

Moreover, we can state the following:

**Proposition 2.** Controllers (17) or (19), with choice (23) of \( \pi_3 \) or \( \pi_3^2 \), ensure that the equilibrium point \( \bar{X}_* = [0 0 \bar{X}]^T \) of the closed loop system is asymptotically stable with domain of attraction containing the set \(( -\infty, x_0 - x_* ) \times \mathbb{R} \times \mathbb{R} \).

*Proof:* From Proposition 1, we have established exponential convergence to the manifolds \( z = \lambda - \pi_3 \) or \( z = \frac{1}{2}(\lambda^2 - \pi_3^2) \) and proved the boundedness of trajectories of the extended system \((x,z)\), it only remains to examine the target dynamics on the manifold in order to determine the domain of attraction of the equilibrium. The restriction on the position \( \xi_1 \) is due to the fact that \( \lambda \) tends to infinity as \( x \) approaches \( x_0 \). For choice (22) of \( V \) and \( R \), the explicit expression of the target dynamics is

\[
\begin{align*}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= -k_V \xi_1 - k_R \xi_2.
\end{align*}
\]

Now, consider the following Lyapunov function candidate which represents the scaled energy of the system

\[
H(\xi_1, \xi_2) = \frac{1}{2} \xi_1^2 + V(\xi_1) = \frac{1}{2} \xi_2^2 + \frac{1}{2} k_V \xi_1^2,
\]

which is clearly positive definite for \( [\xi_1, \xi_2] \in ( -\infty, x_0 - x_* ) \times \mathbb{R} \) and take its derivatives along the trajectories of (24), more specifically

\[
\dot{H}(\xi_1, \xi_2) = -k_R \xi_2^2 \leq 0.
\]

Since \( H(\xi_1, \xi_2) \) is radially unbounded, it follows that \( \xi_1, \xi_2 \) are bounded. Since \( \dot{H} \) is only negative semi definite, we cannot directly conclude on the asymptotic stability.

Let us define the set

\[
\Omega = \{ \xi_1 \in ( -\infty, x_0 - x_*), \xi_2 \in \mathbb{R} \mid \dot{H} = 0 \} = \{ \xi_1 \in ( -\infty, x_0 - x_*), \xi_2 = 0 \}.
\]

But from target dynamics (24), restricted to the set \( \Omega \), we have the following implications

\[
\xi_2 = 0 \Rightarrow \dot{\xi}_2 = 0 \Rightarrow \dot{\xi}_1 = 0.
\]

And thus, the largest invariant set contained in \( \Omega \) is just the zero equilibrium. We can now apply LaSalle’s invariance principle to conclude on the asymptotic stability of the zero equilibrium, with the domain of attraction containing the set \(( -\infty, x_0 - x_* ) \times \mathbb{R} \times \mathbb{R} \).

4. EXPERIMENTAL RESULTS

The control laws obtained in the previous section were implemented and tested on a magnetic suspension available in our department of automatic control and described in Section 2. The main characteristics are \( m = 0.084 \, \text{kg}, c = 2.6 \times 10^{-5} \frac{\text{N} \cdot \text{m}}{\text{s}^2}, x_0 = 2.5 \times 10^{-3} \, \text{m}, R = 10 \, \Omega \). The digital controllers were implemented through a Simulink scheme and the communication with the process was established using Real Time Windows Target on the software level and on the hardware level using a data acquisition board DT2801. The resolution of the ADC/DAC interface is 12 bits and the sampling period chosen is \( T_s = 3 \, \text{ms} \). The total mechanical displacement of the pendulum in the vertical direction is 5 mm which is mapped to \([-10,10]\) Volts by the position sensor. The command signal is the voltage across the electromagnet’s coil and it is limited to the interval \([-10,10]\) Volts. For control (17), the values chosen for the controller gains \( k_V, k_R \) and the rate of convergence \( \gamma_1 \) are

\[
k_V = 8000, k_R = 450, \gamma_1 = 100.
\]

In order to test the behaviour of the closed loop system, we studied the position step response of the pendulum. The initial position is 1.5 mm and the final one is 3.5 mm. The position response is shown in Fig. 2 along with the other state variables, namely the velocity of the pendulum and the current through the coil.

![System response with control (17) + Integrator.](image)

The overshoot of the position response is very small, around 4\%, the response time obtained is 92 ms and the command effort is small.

**Remark 3.** We added in parallel with the I&I controller a discrete integrator in order to cancel the steady state error. One may note that its addition does not change the stability result shown earlier if the integrator has an anti-windup mechanism implemented.

The gain of the integrator was set to \( K_i = 0.5 \) for all measurements. The convergence of the closed loop trajectories towards the invariant manifold \( z = \lambda - \pi_3 = 0 \) and once close to the manifold, the convergence towards the equilibrium points is shown in Fig. 3.
It is also apparent, from Fig. 3, that the system response when ascending is asymmetrical with respect to the one when descending, due to the fact that only one electromagnet is used to control the position of the pendulum. Under the same test conditions, the closed loop system with command (19), offers the response depicted in Fig. 4. The values of the gains selected for this controller are

\[ k_V = 8000, k_R = 500, \gamma_2 = 90. \]

For this version, the overshoot is around 10\%, the response time is 80 ms and we can observe that the control signal reaches the upper limit of 10 V, but the electrical current well remains within the physical bounds. Fig. 5 shows the attractiveness of the implicit manifold \( z = \frac{1}{2} x^2 - \frac{1}{2} R_3^2 = 0 \) and the convergence of the trajectories towards the equilibrium points. Again, we notice the asymmetry of the system but it is less pronounced in this case.

Finally, we have tested the robustness of control (17) with respect to perturbations and parameter’s uncertainties. Fig. 6, shows the system behaviour when the parameters of the model, more specifically \( m, c, \) and \( R \) were subjected to a variation of \( \pm 50\% \).

Fig. 4. System response with control (19) + Integrator.

For this version, the overshoot is around 10\%, the response time is 80 ms and we can observe that the control signal reaches the upper limit of 10 V, but the electrical current well remains within the physical bounds. Fig. 5 shows the attractiveness of the implicit manifold \( z = \frac{1}{2} x^2 - \frac{1}{2} R_3^2 = 0 \) and the convergence of the trajectories towards the equilibrium points. Again, we notice the asymmetry of the system but it is less pronounced in this case.

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Fig. 5. Attractivity of the invariant manifold for control (19).

It comes out from the experimental results that the controller is robust with respect to parameter uncertainties, the performance, compared with the nominal one, is preserved. Furthermore, in order to compare the results obtained with some other designed nonlinear controllers, we have tested two popular control strategies, namely backstepping (Figs. 8-9) and nested-loop passivity control (Figs. 10-11). From the figures below we observe that the nominal performance is similar to the I&I controller but their robustness is weaker with regards to parameter uncertainties. For shorten, the position behaviors have been reported only.

Fig. 7 displays the response of the closed loop system with control (17) when the pendulum is disturbed from its steady state regime to the positive limit. We observe that the controller stabilizes the system and the command effort is small.

Fig. 6. Closed loop response in the presence of parameter uncertainties.

For this version, the overshoot is around 10\%, the response time is 80 ms and we can observe that the control signal reaches the upper limit of 10 V, but the electrical current well remains within the physical bounds. Fig. 5 shows the attractiveness of the implicit manifold \( z = \frac{1}{2} x^2 - \frac{1}{2} R_3^2 = 0 \) and the convergence of the trajectories towards the equilibrium points. Again, we notice the asymmetry of the system but it is less pronounced in this case.

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5. CONCLUSION

This paper reports the experimental results of the application of two I&I controllers for the magnetic suspension system and provides a proof of the local asymptotic stability and the domain of attraction of the closed loop system. The controllers derived have simple expressions thus more convenient for the practical implementation. The closed loop dynamics converge to the target dynamics in a smooth fashion, requiring moderate control effort, thus achieving asymptotic model matching. Moreover, it was shown that the closed-loop system is robust with respect to perturbations induced as displacements of the pendulum and with respect to parameter uncertainties as well. Finally, one may conclude that the performance is shown to be superior to those of backstepping and nested-loop passivity based approaches, in terms of robustness with regards to parameter uncertainties and disturbances.

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