Global Bounded Controlled Consensus of Networked Multi-Agents Systems with Non-Identical Dynamical Agents

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Abstract: This paper investigates the global bounded controlled consensus problem of networked Multi-Agent Systems (MAS) exhibiting nonlinear, non-identical node dynamics with communication time-delays. Globally bounded controlled consensus conditions based on pinning control method and adaptive pinning control method are derived. The proposed consensus criteria ensure that all agents eventually move along desired trajectories in terms of boundedness. The proposed controlled consensus criteria generalize the case of identical agent dynamics to the case of non-identical agent dynamics, and many related results of other researches in this area can be viewed as special cases of the above results. Finally, the effectiveness of the theoretical results is demonstrated by means of a numerical simulation.

1. INTRODUCTION

Networked Multi-Agent Systems (NMAS) investigations deal with the study of how network architecture and interactions between network components influence global control goals. This has attracted much attention due to the broad applications of NMAS in many areas. How to design appropriate protocols and algorithms such that the set of agents can realize common objective, such as consensus, is a critical problem, especially for the case of unreliable information exchange and communication delays, and some relevant important contributions have been made in recent years Zampieri [2008], Desai et al. [2001], Ren et al. [2004], Porfiri et al. [2007], Cortés [2009].

The consensus problem requires an agreement to be reached that depends on the state of all agents. The topic has been studied across many fields of science and engineering Liu et al. [2009], Hong et al. [2006], Xiao et al. 2008, Li et al. 2008, Kazerooni et al. 2008, Li et al. 2009,olfati-saber et al. 2004, arack 2007, Chen et al. 2009, Tian et al. 2009, Bliman et al. 2008, Cortés [2008]. It is noted that the agent dynamics in most existing works are often restricted to linear and identical ones. Obviously, in practice, this is not always the case. The controlled consensus problem of NMAS with nonlinear agent dynamics and communication delay are more complicated and just a few results have been made Hill et al. 2008. In addition, most research in consensus problems usually assume that the final consensus value is a constant, which may not be the case in the sense that the information state of each agent may be dynamically evolving in time according to some inherent dynamics. It is interesting to study controlled consensus problems where the final consensus value evolves with time or as a function of environmental dynamics.

The present paper will focus on the global consensus problems of NMAS based on pinning control methods, and the proposed controlled consensus property is formulated in terms of certain boundedness of state errors. Compared with existing related results, this paper make two significant advances. One is that we generalize the related results for the case of identical agent dynamics to the case of non-identical agent dynamics, and the other is we introduce pinning controllers to the selected agents.

The rest of this paper is organized as follows. A controlled continuous-time NMAS model with communication time-delay is presented in Section 2. The main results including pinning control and adaptive pinning control bounded consensus criterion are derived in Section 3 and 4 respectively. Section 5 gives a numerical simulation example to verify the effectiveness of the proposed results, followed by conclusions in Section 6.

2. PROBLEM DESCRIPTION

Let $G=(V,A)$ be a graph of order $N$ consisting of a set of vertices $V = \{ v_1,v_2,\ldots,v_N \}$ and a set of edges $A \subseteq V \times V$. An edge $(v_j,v_i)$ in graph $G$ means that agent $v_i$ sends some information to agent $v_j$. The set of neighbors of agent $v_i$ is denoted by $N_i = \{ v_j \in V : (v_j,v_i) \in A \}$.

An NMAS consisting of $N$ non-identical agents with communication delay is considered here:

$$\dot{x}_i = f_i(x_i) + c \sum_{j \in N_i} a_{ij}x_j(t-\tau), \ i=1,2,\ldots,N, \quad (1)$$

where $x_i = (x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^T \in \mathbb{R}^n$ are the state variables of the agent $v_i$, $f_i(x_i) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ are continuously differentiable mappings with Jacobian $Df_i$, representing the self-dynamics of the agent $v_i$, $c > 0$ denotes the coupling strength, $\Gamma = (\gamma_{ij}) \in \mathbb{R}^{n \times n}$ is the inner coupling matrix, and where $\gamma_{ij} \neq 0$ means two connected agents are linked via their $i$th and $j$th state variables, respectively. The adjacency matrix $A = (a_{ij}) \in \mathbb{R}^{n \times n}$
$R^{N \times N}$ (which is symmetric and irreducible) represents the communication topology relation of the NMAS, and is defined by $a_{ij} = a_{ji} = 1 (i \in \mathcal{N}_i, j \in \mathcal{N}_j)$ and $a_{ii} = - \sum_{j \neq i} a_{ij}$, $\tau$ is a constant coupling delay which reflects the reality that the agent $v_i$ can’t obtain information from agent $v_j$ instantaneously.

The average dynamic of all agents is defined by the vector field $\dot{f}(x(t)) = \frac{1}{N} \sum_{k=1}^{N} f_k(x(t))$ with Jacobian $D \dot{f}(x(t))$.

The average state trajectory is chosen as the desired moving trajectory

$$s(t) = \frac{1}{N} \sum_{k=1}^{N} x_k(t). \tag{2}$$

**Definition 1** (Hua et al. [2006]): The solution $x_i(t, t_0, \psi_i)$ of the NMAS model (1) is said to be uniformly ultimately bounded with respect to the bound $\epsilon$ if for each $\delta > 0$ there exists $T = T(\epsilon, \delta) > 0$ independent of $t_0$ such that $\|x_i(t, t_0, \psi_i)\| \leq \epsilon$ for all $t \geq t_0 + T$ when $\|x_i(t_0)\| < \delta$, where $\psi_i$ is the initial value.

**Lemma 1** (Hill et al. [2008]): Let $g(t)$ be a non-negative bounded function defined on $R^+$ and

$$\Omega = \{x(t) \in R^n ||x(t)|| \leq \lim_{t \to \infty} g(t)\}. \tag{3}$$

Suppose there exists a strictly positive definite matrix $P(t) \in \mathcal{P}_{\Omega}^{R_{+} \times n}$ and a constant $\delta > 0$ such that the derivative of $V(x(t), t) = x^T(t)P(t)x(t)$ along the trajectory of the system

$$\dot{x}(t) = f(x(t), t), \quad x(t) \in R^n, t \in [0, \infty) \tag{4}$$

satisfies

$$\dot{V} \leq -\delta \|x(t)\|^2 \quad \text{if} \quad \|x(t)\| \geq g(t). \tag{5}$$

For any $t > 0$, let

$$Q_t = \{x(t) | V(x(t), t) \leq \sup_{y(s) \in \Omega_1 \geq 0} \{V(y(s), s)\}\} \tag{6}$$

and

$$e = \lim_{t \to \infty} (\max\{\|x(t)\| | x(t) \in Q_t\}). \tag{7}$$

Then, $x(t)$ converges to the set

$$M = \{x(t) | \|x(t)\| \leq c\}. \tag{8}$$

In the rest of this paper, $x, s, u, e, w, d_i$ and $V$ denote $x(t), s(t), u(t), e(t), w(t), d_i(t)$ and $V(w(t), t)$, respectively.

3. LINEAR FEEDBACK PINNING CONTROLLER

To achieve the goal, feedback control strategy will be applied on a small fraction $\delta (0 < \delta \leq 1)$ of the agents in system (1). Suppose that nodes $i_1, i_2, \cdots, i_l$ are selected to be under control, where $l = [\delta N]$ is a smaller but nearest integer to the real number $\delta N$. This controlled NMAS can be described as

$$\begin{cases}
\dot{x}_{i_k} = f_{i_k}(x_{i_k}) + \sum_{j \in \mathcal{N}_i} a_{ij} \Gamma x_j(t - \tau) + u_{i_k}, 1 \leq k \leq l, \\
\dot{x}_{i_k} = f_{i_k}(x_{i_k}) + \sum_{j \in \mathcal{N}_i} a_{ij} \Gamma x_j(t - \tau), l + 1 \leq k \leq N.
\end{cases} \tag{9}$$

The local linear negative feedback control law is chosen as follows:

$$u_{i_k} = -d_{i_k}(x_{i_k} - s), 1 \leq k \leq l \tag{10}$$

where the feedback gain $d_{i_k} > 0$.

Combine (9) and (10) and rearrange the order of the nodes in the network. Let the first $l$ nodes be controlled, and $e_i = x_i - s, i = 1, 2, \cdots, N$. It’s obvious that $\sum_{k=1}^{N} a_{ij} \Gamma x_j(t - \tau) = 0$ and $\sum_{i=1}^{N} e_i = 0$. Then by applying the Newton-Leibniz formula, error systems can be written as

$$\begin{cases}
\dot{e}_i = D_f(s)e_i + \sum_{j \in \mathcal{N}_i} a_{ij} \Gamma e_j(t - \tau) \\
+ \int_{0}^{1} (Df_i(s + \tau e_i) - D\dot{f}(s)) e_i d\tau \\
- \frac{1}{N} \sum_{k=1}^{N} \int_{0}^{1} Df_k(s + \tau e_k) e_k d\tau \\
+ f_i(s) - f(s) - d_i e_i, 1 \leq i \leq l, \\
\dot{e}_i = D_f(s)e_i + \sum_{j \in \mathcal{N}_i} a_{ij} \Gamma e_j(t - \tau) \\
+ \int_{0}^{1} (Df_i(s + \tau e_i) - D\dot{f}(s)) e_i d\tau \\
- \frac{1}{N} \sum_{k=1}^{N} \int_{0}^{1} Df_k(s + \tau e_k) e_k d\tau \\
+ f_i(s) - f(s), l + 1 \leq i \leq N.
\end{cases} \tag{11}$$

The following work will focus on simplifying the error systems (11) by means of a series of transformations using a procedure similar to Hill et al. [2008].

Define the following matrix

$$D = \text{diag}(D_1, D_2, \cdots, D_N) \in R^{nN \times nN},$$

where $D_i = \text{diag}\{-d_i, -d_i, \cdots, -d_i\} \in R^{nN \times nN}$. Let $e = (e_1^T, e_2^T, \cdots, e_N^T)^T$, then (11) becomes

$$\dot{e} = \Sigma(t)e + cA \otimes e(t - \tau) + I(t) e - \frac{1}{N} H(t)e + F(t), \tag{12}$$

where $\Sigma(t) = I_N \otimes D\dot{f}(s) + D, H^T(t) = (H_1^T(t), \cdots, H_N^T(t)), H_i(t) = (\int_{0}^{1} Df_i(s + \tau e_i) d\tau, \cdots, \int_{0}^{1} Df_N(s + \tau e_N) d\tau), F_i^T(t) = (f_i^2(s) - F^2(s), \cdots, f_i^2(s) - F^2(s)), I(t) = \text{diag}\{\int_{0}^{1} (Df_i(s + \tau e_i) - D\dot{f}(s)) d\tau, i = 1, 2, \cdots, N\}.$

Since $A$ is symmetric and irreducible, according to Hill et al. [2008], there exists a unitary matrix $\Phi = (\Phi_i)_{N \times N} = (\Phi_1, \Phi_2, \cdots, \Phi_N)$. This together with $w(t) = (\Phi^T \otimes I_n)e$ gives
\[ \dot{w} = (\Phi^T \otimes I_n) \Sigma(t) (\Phi \otimes I_n) w \\
+ (\Phi^T \otimes I_n) (cA \otimes \Gamma) (\Phi \otimes I_n) w(t - \tau) \\
+ (\Phi^T \otimes I_n) I(t)(\Phi \otimes I_n) w \\
- \frac{1}{N} (\Phi^T \otimes I_n) H(t)(\Phi \otimes I_n) w + (\Phi^T \otimes I_n) F(t). \]  
(13)

Note that $H(t) = \sqrt{N} \sum_{k=1}^{N} (0 \cdots 0 \Phi_k 0 \cdots 0) \otimes \int_0^1 Df_k(s + \tau \epsilon_k) d\tau$, where $\Phi_k$ stands for the matrix with its $k$-th column equal to $\Phi_1$ and the remaining elements are zero. Then we have $\frac{1}{N} (\Phi^T \otimes I_n) H(t)(\Phi \otimes I_n) = \frac{1}{N} \sum_{k=1}^{N} (0 \cdots 0 I_k 0 \cdots 0) \otimes \int_0^1 Df_k(s + \tau \epsilon_k) d\tau (\Phi \otimes I_n)$, where $I_k$ stands for the matrix with its $k$-th column equals $(1 \cdots 0)^T$ and the remaining of its elements are zero.

Thus, a simple calculation gives $\frac{1}{N} (\Phi^T \otimes I_n) H(t)(\Phi \otimes I_n) = \frac{1}{N} \sum_{k=1}^{N} \left( \frac{\gamma_k}{\tau} \right) \otimes \int_0^1 Df_k(s + \tau \epsilon_k) d\tau$, where $\gamma_k \in R^{i \times i}$ and $\tau \in R^{(N-1) \times N}$. Therefore, $\dot{w} = \Sigma(t) w + cA \otimes \Gamma \omega(t - \tau) (\Phi \otimes I_n) F(t)$.

If $w_i(t) \equiv 0$, only $w_{2i}, w_{3i}, \ldots, w_{Ni}$ need to be considered. Rewriting in the component form we have $\dot{w}_i = \Sigma_i(t) w_i + cA_i \Gamma_i \omega_i(t - \tau) + (\Phi_i^T \otimes I_n) I(t)(\Phi \otimes I_n) w_i$,

\[ + (\Phi_i^T \otimes I_n) I(t)(\Phi \otimes I_n) w, \quad i = 2, 3, \ldots, N, \]  
(14)

where $\Sigma_i = DF_i(s) + D_i$.

So far, the consensus problem of system (1) has been transferred to the stability problem of the $N - 1$ of $n$-dimensional systems.

**Theorem 1** Suppose there exist positive definite matrices $P_i(t) \in PC_{+}^{n \times n}, Q_i$, and constants $\zeta > 0, \gamma > 0, a > 0$ and $b > 0$ such that

\[ a\|x\|^2 \leq x^T P_i(t) x + \int_{t-\tau}^{t} w_i^T(\alpha) Q_i w_i(\alpha) d\alpha \leq b\|x\|^2, \]

\forall t \in R^+, \quad x \in R^n, \quad i = 2, 3, \ldots, N,  
(15)

\[ \dot{P}_i(t) + P_i(t) \Sigma_i(t) + \Sigma_i^T(t) P_i(t) + Q_i \\
+ c^2 \lambda_i^2 P_i(t) \Gamma_i Q_i^{-1} \Gamma_i^T P_i(t) + \zeta I \leq 0, \quad i = 1, 2, \ldots, N, \]

\[ ||I(t)|| \leq \gamma, \quad i = 1, 2, \ldots, N. \]  
(16)

Let

\[ \mu(t) = ||F(t)|| \]

be bounded and

\[ \beta = \left( \sum_{i=2}^{N} ||P_i(t)||^2 \right)^{\frac{1}{2}}, \]

(19)

if $\zeta > 2\gamma \beta$, then system (12) converges to the set

\[ M = \{ e ||e|| \leq \frac{2b \beta \lambda_i m_i}{a \zeta - 2\gamma \beta - \delta} \}, \]

(20)

for any fixed time delay $\tau > 0$, namely, $e(t) = x_i(t) - \frac{1}{N} \sum_{k=1}^{N} x_k(t) \rightarrow 0$ as $t \rightarrow \infty$, where $\delta > 0$ is any constant satisfying $\delta < \zeta - 2\gamma \beta$, and then the NMAS (1) achieves bounded consensus for any fixed time delay $\tau > 0$.

**Proof.** Choose the following Lyapunov-Krasovskii functional as

\[ V = \sum_{i=2}^{N} V_i, \]

(21)

\[ V_i = w_i^T P_i(t) w_i + \int_{t-\tau}^{t} w_i^T(\alpha) Q_i w_i(\alpha) d\alpha. \]

(22)

Differentiating (23) along the trajectory of (14) gives

\[ \dot{V}_i = w_i^T (\dot{P}_i(t) + P_i(t) \Sigma_i(t) + \Sigma_i^T(t) P_i(t) + Q_i) w_i \\
+ 2w_i^T P_i(t)(\Phi_i^T \otimes I_n) I(t)(\Phi_i \otimes I_n) w_i \\
+ 2w_i^T P_i(t)(\Phi_i^T \otimes I_n) F(t) + 2w_i^T(c\lambda_i P_i(t) \Gamma_i) w_i(t - \tau) \\
- w_i^T(t - \tau) Q_i w_i(t - \tau). \]

(23)

Applying the Young Inequality to the equality (23) results in

\[ V_i \leq w_i^T (\dot{P}_i(t) + P_i(t) \Sigma_i(t) + \Sigma_i^T(t) P_i(t) + Q_i) w_i \\
+ c^2 \lambda_i^2 P_i(t) \Gamma_i Q_i^{-1} \Gamma_i^T P_i(t) w_i + 2w_i^T P_i(t)(\Phi_i^T \otimes I_n) F(t) + 2w_i^T P_i(t)(\Phi_i^T \otimes I_n) I(t)(\Phi_i \otimes I_n) w_i. \]

(24)

Condition (16) implies that the first term on the right hand side of (24) satisfies

\[ w_i^T (\dot{P}_i(t) + P_i(t) \Sigma_i(t) + \Sigma_i^T(t) P_i(t) + Q_i) w_i \\
+ c^2 \lambda_i^2 P_i(t) \Gamma_i Q_i^{-1} \Gamma_i^T P_i(t) w_i \leq -\zeta ||w_i||^2. \]

(25)

The second term on the right hand side of (24) satisfies

\[ 2w_i^T P_i(t)(\Phi_i^T \otimes I_n) I(t)(\Phi_i \otimes I_n) w_i \leq 2\gamma ||P_i(t)|| ||w_i|| ||w_i||. \]

(27)

\[ V = \sum_{i=2}^{N} V_i, \]

then

\[ \dot{V} = \sum_{i=2}^{N} \dot{V}_i \]


\[ = -\zeta ||w||^2 + 2(2\gamma ||w|| + \mu(t)) \sum_{i=2}^{N} ||w_i|| ||P_i(t)|| \\
\leq -\zeta ||w||^2 + 2(2\gamma ||w|| + \mu(t)) ||w|| \left( \sum_{i=2}^{N} ||P_i(t)||^2 \right)^{\frac{1}{2}}. \]

(28)

Thus, when

\[ ||w|| \geq \frac{2\beta \lambda_i m_i}{a \zeta - 2\gamma \beta - \delta}, \]

(29)

gives

\[ \dot{V} \leq -\delta ||w||^2. \]

(30)

Applying Lemma 1 completes the proof.
4. ADAPTIVE PINNING CONTROLLER

In this section, globally consensus criteria will be derived via direct adaptive pinning control method. Without loss of generality, still assume that the first $l$ agents are selected as pinned agents with the adaptive controllers:

\[
\begin{aligned}
\dot{u}_i &= -d_i(x_i - s), \quad 1 \leq i \leq l, \\
\dot{d}_i &= h_i c^T_i P_t(t) e_i, \\
\dot{u}_i &= 0, \quad l + 1 \leq i \leq N, \\
\end{aligned}
\]

(31)

where constant $h_i > 0$ and positive definite matrix $P_t(t) \in R^{n \times n}$. Applying Newton-Leibniz formula, then the error NMAS can be rewritten as

\[
\begin{aligned}
\dot{e}_i &= Df(s)e_i + c \sum_{j \in N_i} a_{ij} \Gamma e_j(t - \tau) \\
&\quad + \int_0^1 (D\dot{f}(s + \tau e_i) - Df(s)) e_i d\tau \\
&\quad - \frac{1}{N} \sum_{k=1}^N \int_0^1 Df_k(s + \tau e_k) e_k d\tau \\
&\quad + f_i(s) - \bar{f}(s) - d_i e_i, \quad 1 \leq i \leq l, \\
\dot{e}_i &= Df(s)e_i + c \sum_{j \in N_i} a_{ij} \Gamma e_j(t - \tau) \\
&\quad + \int_0^1 (D\dot{f}(s + \tau e_i) - Df(s)) e_i d\tau \\
&\quad - \frac{1}{N} \sum_{k=1}^N \int_0^1 Df_k(s + \tau e_k) e_k d\tau \\
&\quad + f_i(s) - \bar{f}(s), \quad l + 1 \leq i \leq N. \\
\end{aligned}
\]

(32)

Repeating a similar procedure to the previous subsection, the controlled consensus problem of system (1) is equivalent to the stability problem of the following $N - 1$ of $n$-dimensional systems.

\[
\begin{aligned}
\dot{w}_i &= Df(s(t))w_i - d_i w_i + c \lambda_i \Gamma w_i(t - \tau) \\
&\quad + (\Phi^T_i \otimes I_n) l(t)(\Phi^T_i \otimes I_n)w \\
&\quad + (\Phi^T_i \otimes I_n) F(t), \quad 2 \leq i \leq l, \\
\dot{d}_i &= h_i w_i P_t(t) w_i, \\
\dot{w}_i &= D\bar{f}(s)w_i + c \lambda_i \Gamma w_i(t - \tau) \\
&\quad + (\Phi^T_i \otimes I_n) l(t)(\Phi^T_i \otimes I_n)w \\
&\quad + (\Phi^T_i \otimes I_n) F(t), \quad l + 1 \leq i \leq N, \\
\end{aligned}
\]

(33)

where $w_i$, $w$, $\Phi$, $\Phi_i$, $l(t)$ and $F(t)$ are the same as the previous subsection.

**Theorem 2** Suppose there exist positive definite matrices $P_t(t) \in \mathcal{P}_{+}^{n \times n}$, $Q_1$ and constants $\zeta > 0$, $\gamma > 0$, $a > 0$ and $b > 0$ such that

\[
\begin{aligned}
|a| |x|^2 &\leq x^T P_t(t) x_i + \int_0^t x^T(\alpha) Q_1 x_i(\alpha) d\alpha \\
&\quad + \frac{(d_i - d)^2}{h_i} \leq b |x|^2, \forall t \in R^+, \; x \in R^n, \; i = 2, 3, \cdots, N, \\
\end{aligned}
\]

(34)

\[
\begin{aligned}
P_t(t) + P_t(t) D\bar{f}(s) + (D\bar{f}(s))^T P_t(t) + Q_1 - 2dP_t(t) + c^2 \lambda_i^2 P_t(t) \Gamma Q_1^T P_t(t) + \zeta I \preceq 0, \quad i = 1, 2, \cdots, N, \\
\end{aligned}
\]

(35)

(17) and $\zeta > 2\gamma/b$ are satisfied, then the system (12) converges to the set (20) for any fixed time delay $\tau > 0$, where $\mu(t)$ and $\beta$ are the same as in (18) and (19), respectively, $\delta > 0$ is any constant satisfying $\delta < \zeta - 2\gamma/b$, and then the NMAS (1) achieves bounded consensus for any fixed time delay $\tau > 0$.

**Proof.** Construct the following Lyapunov-Krasovskii functional as

\[
V = \sum_{i=2}^N V_i + \sum_{i=2}^N \frac{(d_i - d)^2}{h_i}, \quad 2 \leq i \leq l, \\
V_i = w_i^T P_t(t) w_i + \int_{t-\tau}^t w_i^T(\alpha) Q_1 w_i(\alpha) d\alpha \\
&\quad + \frac{(d_i - d)^2}{h_i}, \quad 2 \leq i \leq l, \\
V_i = w_i^T P_t(t) w_i + \int_{t-\tau}^t w_i^T(\alpha) Q_1 w_i(\alpha) d\alpha, l + 1 \leq i \leq N, \\
\]

(36)

(37)

where $d$ is a positive constant to be determined.

Differentiating (37) along the trajectory of (33) gives

\[
\begin{aligned}
\dot{V}_i &= w_i^T (\dot{P}_t(t) + P_t(t) D\bar{f}(s) + (D\bar{f}(s))^T P_t(t) + Q_1 \\
&\quad - 2dP_t(t)) w_i + 2w_i^T P_t(t) (\Phi^T_i \otimes I_n) l(t)(\Phi^T_i \otimes I_n) w \\
&\quad + 2w_i^T P_t(t) (\Phi^T_i \otimes I_n) F(t) + 2w_i^T (c\lambda_i P_t(t)^T) w_i(t - \tau) \\
&\quad - w_i^T (t - \tau) Q_1 w_i(t - \tau). \\
\end{aligned}
\]

(38)

The remaining part of the proof is similar to that of Theorem 1, so is therefore omitted here. This completes the proof.

5. EXAMPLES

To demonstrate the theoretical results obtained above, an NMAS consisting of 12 agents is constructed and is described as follows

\[
\dot{x}_i(t) = f_i(x_i(t)) + c \sum_{j \in N_i} a_{ij} \Gamma x_j(t - \tau), \\
\]

(39)

where $f_i(x_i(t)) = B_i x_i(t) + g(x_i(t))$, $B_i(i = 1, 2, \cdots, 6)$ and $B_i(i = 7, 8, \cdots, 12)$ are chosen as follows:

\[
\begin{aligned}
&\begin{cases}
-10 + 0.1 \times (i - 1) & 10 - 0.1 \times (i - 1) \\
1 & -1 \\
0 & -15 - 0.1 \times (i - 1) \\
1 & -1 \\
0 & -15 - 0.1 \times (i - 6) \\
1 & -1 \\
0 & -15 - 0.1 \times (i - 12) \\
1 & -1 \\
\end{cases}, \\
\end{aligned}
\]

and

\[
g(x_i(t)) = -9.5 \sin\left(\frac{\pi x_{i1}(t) + \pi}{3.2}\right), \quad i = 1, 2, \cdots, 12. \\
\]

The communication coupling matrix $C = (C^T_1 C^T_2 \cdots C^T_{12})$,$C_1 = (-8 1 1 0 1 1 1 0 1 1 1 1 1 0), \; C_2 = (1 - 8 1 1 0 1 0 1 1 1 0), \; C_3 = (1 1 - 7 1 0 0 0 1 0 1 1 1), \; C_4 = (0 1 1 1 6 0 1 0 0 1 0 1 1), \; C_5 = (1 1 - 6 0 1 1 1 1 1 0), \; C_6 = (1 1 0 0 1 0 5 1 0 1 1 1), \; C_7 = (1 1 0 1 1 1 7 1 0 1 0 1), \; C_8 = (0 1 0 1 0 1 1 - 6 0 1 1 1 1), \; C_9 = (1 1 0 0 1 1 0 0 - 7 1 1 1 1), \; C_{10} = (1 1 1 1 1 1 1 1 - 10 1), \; C_{11} = (1 1 1 0 0 0 0 1 1 1 1 - 7 1), \\
\]

(40)
\( C_{12} = (0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ -5) \), \( \Gamma = \text{diag\{2, 2, 2\} \), respectively, where the matrix \( A \) is produced by means of the Scale-Free network program.

Design the following controllers

\[
\begin{align*}
    u_{ik} &= -d_k (x_{ik} (t) - s(t)), & i_k &= 1, 2 \text{ and } 10, \\
    u_{ik} &= 0, & \text{else},
\end{align*}
\]

with \( d_1 = 0.5, d_2 = 0.5, d_{10} = 0.5 \) and

\[
\begin{align*}
    u_{ik} &= -d_k (t) (x_{ik} (t) - s(t)), & i_k &= 1, 2 \text{ and } 10, \\
    d_k (t) &= h_k e_k^T P_k u_k (t) e_k, \\
    u_{ik} &= 0, & \text{else},
\end{align*}
\]

with \( h_1 = 0.1, h_2 = 0.2, h_{10} = 0.3, s(t) \) can then be evaluated by simulation.

Given the initial values of 12 agents as \((10 \ 5 \ -10)^T, (12 \ 6 \ -12)^T, (14 \ 7 \ -14)^T, (16 \ 8 \ -16)^T, (18 \ 9 \ -18)^T, (20 \ -10 \ -20)^T, (-18 \ 11 \ 18)^T, (-16 \ 12 \ 16)^T, (-14 \ 13 \ 14)^T, (-12 \ 14 \ 12)^T, (-10 \ 15 \ 10)^T, (-8 \ 16 \ 8)^T \) respectively and \( P_k (t) = I_3, d_1 (0) = 1, d_2 (0) = 1, d_{10} (0) = 1 \). We may verify the conditions of Theorem 1 and Theorem 2 readily. This demonstrates the bounded consensus of the NMAS is achieved for any time delay \( 0 < \tau \leq 0.06 \). Simulation results are depicted in Fig.1 to Fig.8 for \( \tau = 0.06 \) and \( \epsilon = 1 \).

6. CONCLUSION

In this paper, the controlled consensus problems of NMAS with different agent dynamics have been investigated. The derived criteria are verified via theoretical analysis and numerical simulation. The consensus for the NMAS is achieved based on pinning control and adaptive pinning control methods. It should be noted that the conditions are still restrictive and all the delays are the same. Further investigations will focus on relaxing these limitations.

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REFERENCES


Fig. 5. All agent error dynamics under pinning control.

Fig. 6. All agent error dynamics under adaptive pinning control.

Fig. 7. Adaptive gain curves.

Fig. 8. Adaptive pinning controllers curves.


