An active fault-tolerant control schema for delayed systems

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Abstract: nowadays fault-tolerant control (FTC) strategies become unavoidable to ensure at the same time control and fault diagnosis objectives. Furthermore, delayed system are increasingly encountered in many real-world applications. Therefore, an active fault-tolerant control strategy for delayed system is presented in this paper. The core of the proposed strategy is the simultaneous state and fault estimations provided by an observer known as multi-integral observer. Sufficient conditions for ensuring the control law stability as well as the estimation error convergence of the observer are given employing the LMI framework. The effectiveness of the proposed approach is illustrated via a simulation example.

Keywords: fault-tolerant control, delayed systems, multi-integral observer, LMI

1. INTRODUCTION

Fault-tolerant control (FTC) has received increasing attention since few past decades [Zhang and Jiang, 2008, Ding, 2009, Muenchhof et al., 2009]. Indeed, control and fault diagnosis and isolation (FDI) cannot be considered as two independent tasks to be separately accomplished. On the one hand, a control strategy is often designed to introduce some robustness in the closed-loop with respect to disturbances acting on the system. Hence, from the control point of view, a fault acting on the system is a disturbance whose effects must be cancelled or attenuated to ensure control performances. On the other hand, the same fault is considered from the FDI point of view as signal to be detected and consequently its effects must not be attenuated to enable its detection. Clearly, control and FDI goals are antagonist goals. A TFC strategy can then be employed to cope with control and FDI objectives at the same time.

Fault-tolerant control can be carried out in many manners according to the employed strategy (an extensive bibliographical review is proposed by Zhang and Jiang [2008]). Among them, active FTC is based on an adaptation mechanism in the control law. Recently, Witeczak et al. [2008] and Kheder et al. [2010] have respectively proposed an active FTC strategy for discrete-time and continuous-time linear systems as well as Takagi-Sugeno fuzzy system. In their strategies, a reference model is employed to modelling the nominal system behaviour free of faults and the adaptation mechanism is given by fault estimations provided by an observer. However, these approaches are only valid when the faults taken into account are constant faults (e.g. step signals).

Furthermore, systems with delays are frequently encountered in several real-world processes. Therefore, considerable attention has been paid to this kind of systems. Appropriate theoretical tools, in the frequency and in the time-domain, for analysis, control and state estimation are well established [Bliman, 2001, Sename, 2001, Richard, 2003]. However, much less works are devoted to the FTC of delayed systems. In this work, a fault-tolerant control for this kind of system is proposed.

The proposed FTC strategy is based on a feedback state control law with an additional term. This last is given by an estimation of the actuator faults acting on the system. Thanks to this estimation an adaptation mechanism is introduced in the closed-loop of the system. In this way, the proposed FTC can be considered as an active fault-tolerant control strategy. The on-line estimation of faults is provided via a FDI block composed mainly by a multi-integral observer (details are given in section 2). The use of this observer makes it possible the estimation of a more general kind of faults given by a polynomial form. The constant fault assumption is then relaxed because this kind of fault is a particular case of a polynomial of degree zero.

This paper is organised as follows. The problem statement and details regarding the proposed FTC strategy are presented in section 2. Some preliminaries results needed to stability analyse of the proposed FTC are presented in section 3. The main results are given in section 4. Sufficient conditions to ensure the stability of the suggested FTC are obtained in this section under a LMI form with the help of a Lyapunov-Krasovskii functional. In section 5, the performance of the proposed approach is shown via a simulation example.

Notations: the following notations will be used all along this paper. $P > 0$ ($P < 0$) denotes a positive (negative) definite matrix $P$; $X^T$ denotes the transpose of matrix $X$, $I$ is the identity matrix of appropriate dimension. Symmetric terms in symmetric matrices are denoted by $(\ast)$ e.g. $\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} = \begin{bmatrix} A & B \\ (\ast) & C \end{bmatrix}$.
2. PROBLEM STATEMENT

Consider the following linear delayed system corrupted by an unknown additive fault $f(t)$ acting on the actuator:

$$
\dot{x}_f(t) = Ax_f(t) + A_d x_f(t - \tau) + B(u(t) + f(t)) \quad (1a)
$$

$$
y_f = C x_f(t) \quad (1b)
$$

$$
x_f(\theta) = \phi(\theta) \quad \theta \in [-\tau, 0] \quad (1c)
$$

where $x_f \in \mathbb{R}^n$ is the state vector, $y_f \in \mathbb{R}^p$ the output of the system, $u \in \mathbb{R}^m$ the input, $f \in \mathbb{R}^n$ the unknown actuator fault and $\tau$ the known delay state. The continuous initial conditions are given by the vector function $\phi$. The matrices $A \in \mathbb{R}^{n \times n}$, $A_d \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$ are known and appropriately dimensioned.

The goal of this paper is to seek a control law to ensure the closed-loop stability of the system (1) as well as the actuator fault detection and isolation. This goal can be well accomplished by introducing the following control law:

$$
u(t) = -K \hat{x}(t) - \hat{f}(t) \quad (2)
$$

where $\hat{x}$ and $\hat{f}$ are respectively the state and the fault estimations and $K \in \mathbb{R}^{m \times s}$ the feedback gain to be found. The estimations needed in the control law (2) are computed with the help of the following FDI block:

$$
\dot{\hat{x}}(t) = A \hat{x}(t) + A_d \hat{x}(t - \tau) + Bu + B \hat{f} \quad (3a)
$$

$$
\dot{\hat{f}}(t) = K_{i,0}(y_f(t) - \hat{y}_f(t)) + \hat{f}_1(t) \quad (3b)
$$

$$
\dot{\hat{f}}_1(t) = K_{i,1}(y_f(t) - \hat{y}_f(t)) \quad (3c)
$$

$$
\hat{y}_f(t) = C \hat{x}(t) \quad (3d)
$$

where $\hat{x} \in \mathbb{R}^n$ is the estimated state vector, $y_f(t) \in \mathbb{R}^p$ the output system vector, $\hat{y}_f(t) \in \mathbb{R}^p$ the estimated output vector, $K_p \in \mathbb{R}^{p \times p}$ and $K_{i,i} \in \mathbb{R}^{m \times p}$ for $i = 0, 1$ are the observer gains to be determined.

It can be noted that the proposed FDI block (3) is able to provide simultaneously the state and the fault estimations. The observer (3) is known as multi-integral observer (see Orjuela et al. [2009] for an extended review of multi-integral observers) which can be considered as a generalisation of the classic proportional-integral-observer [Busawon and Kabore, 2001, Söllker et al., 1995]. Indeed, the $K_p$ gain in (3a) ensures a proportional correction according to the output estimation error $y_f(t) - \hat{y}_f(t)$. The $K_{i,i}$ gains provide an integral correction in the loops given by the equations (3b) and (3c). Hence, the core of the FDI block is the multi-integral recursive equations (3b) and (3c).

Here, for the sake of simplicity, the actuator fault $f(t)$ acting on the system is considered as a polynomial of degree one signal. Consequently, the FDI block enables the fault estimation when the considered faults are piece-wise polynomial of degree one or degree zero because only two integral actions are considered (see remark 1).

**Hypothesis 1.** The actuator fault $f(t)$ takes a polynomial form of degree one, i.e. $f(t) = q_1 t + q_0$ where $t$ is the time and the terms $q_1$ and $q_0$ are unknown.

**Remark 1.** The assumption 1 is not restrictive and it can easily be relaxed considering supplementary integral actions. Indeed, faults $f(t)$ given by a polynomial of degree $k$ can be estimated by the multi-integral observer (3) using $k+1$ integral recursive loops in the observer, i.e. $K_{i,(k+1)}$ gains must be employed. Here, only two integral actions are considered according to the assumption 1. Three integral actions will be required for a polynomial of degree two and so on.

The FTC design problem can be stated as follows: we seek the control gain $K$ as well as the observer gains $K_P$ and $K_{i,1}$ such as the closed-loop stability of the system (1) and the actuator fault detection and estimation are simultaneously satisfied.

3. PRELIMINARIES

The closed-loop of the system (1), under control law (2), is given by:

$$
\dot{x}_f(t) = (A - BK)x_f(t) + A_d x_f(t - \tau) + BK\dot{x}(t) + Bf(t) + BK(\hat{x}(t) - \hat{x}(t)) + BK\hat{f}(t)
$$

$$
y_f = C x_f(t) \quad (4b)
$$

Equations (4) are obtained by adding and subtracting the term $BK\dot{x}(t)$.

The state estimation error is:

$$
e_x(t) = x_f(t) - \hat{x}(t) \quad (5)
$$

and the fault estimation errors are given by:

$$
e_f(t) = f(t) - \hat{f}(t) \quad (6)
$$

$$
e_{f,1}(t) = \dot{f}(t) - \hat{f}_1(t) \quad (7)
$$

It can be point out, from (6) and (7), that the first integral loop (3b) in the multi-integral observer provides an estimation $\hat{f}(t)$ of the fault $f$. Whereas the second integral loop (3c) provides an estimation $\hat{f}_1(t)$ of the first derivative of the fault $f$.

The closed-loop system (4), by taking into account (6) and (7), becomes:

$$
\dot{x}_f(t) = (A - BK)x_f(t) + A_d x_f(t - \tau) + BK\dot{x}(t) + BK\hat{x}(t) + BK\hat{f}(t)
$$

$$
y_f = C x_f(t) \quad (8b)
$$

It can be remarked, from these last equations, that an appropriate choice of the control gain $K$ improves the dynamic performance of the closed-loop system providing that the convergence towards zero of $e_x(t)$ and $e_f(t)$ is guaranteed. Consequently, their convergence towards zero must be ensured.

The time-evolution of the state estimation error (5) can be investigated by considering equations (8) and (3a):

$$
e_x(t) = (A - K_p C)e_x(t) + A_d e_x(t - \tau) + B e_f(t) \quad (9)
$$

The time-evolution of the fault estimation errors (6) and (7) are respectively given by:

$$
\dot{e}_x(t) = (A - K_p C)e_x(t) \quad (10)
$$

$$
\dot{e}_f(t) = f(t) - \hat{f}(t) \quad (11)
$$

$$
\dot{e}_{f,1}(t) = \frac{1}{\tau} \int_{t-\tau}^{t} (f(s) - \hat{f}_1(s)) ds \quad (12)
$$
\[ \dot{e}_f(t) = \dot{f}(t) - \dot{\hat{f}}(t) = e_{f,1}(t) - K_{I,0}C e_x(t) \]  \hspace{1cm} (10)
\[ \dot{e}_{f,1}(t) = \ddot{f}(t) - \dot{\hat{f}}_1(t) = 0 - K_{I,1}C e_x(t) \]  \hspace{1cm} (11)

Note that \( \ddot{f}(t) = 0 \) according to the assumption 1. Hence, the fault estimation errors are coupled because the fault estimation is performed in a recursive manner by means of the integral loops.

The equations (8a), (9), (10) and (11) can finally be gathered as follows:

\[ \dot{X}_{aug}(t) = A_{aug}X_{aug}(t) + \dot{\hat{A}}_{aug}X_{aug}(t - \tau) \]  \hspace{1cm} (12)

where

\[
X_{aug}(t) = \begin{bmatrix} x_f^T(t) & e_{f,1}^T(t) \\ e_f^T(t) & e_{f,1}^T(t) \end{bmatrix}^T \in \mathbb{R}^{2(n+m)}
\]  \hspace{1cm} (13)

\[
A_{aug} = \begin{bmatrix} A - BK & BK & B & 0 \\ A - K_p C & B & 0 & 1 \\ -K_{I,1} C & 0 & 1 & 0 \\ 0 & -K_{I,0} C & 0 & 0 \end{bmatrix}
\]  \hspace{1cm} (14)

\[
\dot{\hat{A}}_{aug} = \begin{bmatrix} A_d & 0 & 0 & 0 \\ 0 & A_d & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\]  \hspace{1cm} (15)

Stability of the time-delay augmented system (12) must be ensured in order to guarantee the fault-tolerant control goals. In this work, stability conditions of (12) are established to allow the simultaneous determination of the gains \( K, K_p \) and \( K_{I,i} \) for \( i = 0, 1 \).

4. MAIN RESULTS

In this section, the stability of the augmented delayed system (12) will be investigated. Several candidates Lyapunov functional can be found in the literature to proof the stability of delayed systems. A review of these Lyapunov functional has been recently proposed by Kharitonov [2010]. In this section, delay-independent conditions are obtained using the Lyapunov-Krasovskii approach. The proposed delay-independent conditions take finally a LMI form [Boyd et al., 1994].

The stability of the system (12) is investigated with the help of the Lyapunov-Krasovskii functional proposed by Mondiá and Kharitonov [2005]:

\[
V(t) = X_{aug}(t) \dot{X}_{aug}(t) + \int_{-\tau}^{0} X_{aug}(t + \theta)e^{2\alpha \theta}Q X_{aug}(t + \theta)d\theta \]  \hspace{1cm} (16)

where \( P \) and \( Q \) are symmetric, positive definite matrices and \( \alpha \) is the decay rate ensuring exponential convergence velocity towards zero of the Lyapunov-Krasovskii functional.

Exponential stability of the system (12) is guaranteed if there exists a Lyapunov-Krasovskii functional (16) such that the following inequality holds [Boyd et al., 1994]:

\[
\dot{V}(t) + 2\alpha V(t) < 0
\]  \hspace{1cm} (17)

In order to obtain stability conditions under a LMI form the following lemma will be needed.

Lemma 1. For any constant real matrices \( X \) and \( Y \) with appropriate dimensions then the following property holds for any positive matrix \( H \):

\[
X Y + Y^T X^T \leq X H^{-1} X^T + Y^T H Y
\]

Theorem 1. Consider the delayed system (1), the multi-integral observer (3) and assumption 1. There exists a fault-tolerant control (2) if, for a given positive scalar \( \alpha \) symmetric, there exists positive definite matrices \( \hat{P} \), \( R \) and \( Q \) of appropriated dimensions, two matrices \( K \) and \( G_{aug} \) solution of the constrained optimisation problem, such that:

\[
\begin{bmatrix}
\Omega & BK & 0 \\
\Psi & \Psi^T & \Delta \\
\Psi & \Psi^T & 1 - e^{-2\alpha \tau} \bar{Q}
\end{bmatrix} < 0
\]

where

\[
\begin{bmatrix}
\Omega & BK & 0 \\
\Psi & \Psi^T & \Delta \\
\Psi & \Psi^T & 1 - e^{-2\alpha \tau} \bar{Q}
\end{bmatrix} = \begin{bmatrix}
AR + RA^T + 2\alpha R \\
\hat{P} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\Delta = MP \hat{A}_{aug} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
\hat{A}_{aug} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \hat{P} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tilde{K}_{obs}
\]\n
and \( \hat{A}_{aug} \) defined by (12). \( K \) is the state feedback gain in the control law (2). The observer gains need in (3) are given by \( \hat{K}_{obs} = \hat{P}^{-1} G_{aug} \) where \( \hat{K}_{obs} = [K^T_0, K^T_{I,0}, K^T_{I,1}]^T \).

Proof. The time-derivative of the functional (16) evaluated via the Leibniz-Newton formula gives:

\[
\dot{V}(t) = \dot{X}_{aug}(t)^T P X_{aug}(t) + X_{aug}(t)^T \dot{P} X_{aug}(t) \hspace{1cm} (18)
\]

\[
\dot{V}(t) = -2\alpha \int_{-\tau}^{0} X_{aug}(t + \theta)e^{2\alpha \theta}Q X_{aug}(t + \theta)d\theta
\]

which becomes by considering the dynamics of the delayed augmented system (12):

\[
\dot{V}(t) = -2\alpha \int_{-\tau}^{0} X_{aug}(t + \theta)e^{2\alpha \theta}Q X_{aug}(t + \theta)d\theta \hspace{1cm} (19)
\]

\[
\dot{V}(t) = \begin{bmatrix} X_{aug}(t) \\ X_{aug}(t-\tau) \end{bmatrix}^T \begin{bmatrix} \tilde{A}_{aug}^T P + P \hat{A}_{aug} Q \hspace{1cm} \tilde{A}_{aug}^T \tilde{K}_{obs} Q \end{bmatrix} \begin{bmatrix} X_{aug}(t) \\ X_{aug}(t-\tau) \end{bmatrix}
\]

It should be noted that the Lyapunov-Krasovskii functional (16) can be rewritten as follows:

\[
V(t) = \begin{bmatrix} X_{aug}(t) \\ X_{aug}(t-\tau) \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_{aug}(t) \\ X_{aug}(t-\tau) \end{bmatrix} \hspace{1cm} (20)
\]

\[
+ \int_{-\tau}^{0} X_{aug}(t + \theta)e^{2\alpha \theta}Q X_{aug}(t + \theta)d\theta
\]
Consequently, the inequality (17), ensuring the stability of the FTC, is guaranteed if the following inequality holds:

\[
\begin{bmatrix}
A^T_{aug}P + PA_{aug} + Q + 2\alpha P + \frac{\dot{\lambda}_{aug}}{e^{-2\alpha t}Q}
\end{bmatrix} < 0
\] (21)

This inequality is obtained by taking into account (19) and (20) in (17) and noting that the obtained inequality takes a quadratic form in \( [x_{aug}(t) x_{aug}(t-\tau)]^T \). However, the inequality (21) is not under a LMI form in \( P, Q, K, K_P \) and \( K_{I,i} \). Indeed, these parameters are hardly related in matrices \( A_{aug} \). Consequently, this inequality cannot be solved using standard numerical algorithms. However, the inequality (21) can be written under a LMI form by applying some transformations as follows.

Two particular matrices \( P \in \mathbb{R}^{2(n+m) \times 2(n+m)} \) and \( M \in \mathbb{R}^{2(n+m) \times 2(n+m)} \) are introduced:

\[
P = \begin{bmatrix}
P_1 & 0 & 0 & 0 \\
0 & \tilde{P} & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\] and

\[
M = \begin{bmatrix}
P^{-1}_1 & 0 & 0 & 0 \\
0 & 0 & 0 & I
\end{bmatrix}
\] (22)

with \( P_1 = P_T^T > 0 \in \mathbb{R}^{n \times n} \) and \( \tilde{P} = \tilde{P}_T^T > 0 \in \mathbb{R}^{2m+n \times 2m+n} \) of appropriate dimensions. The inequality (21) can now be pre and post multiplied by the matrix \( \Phi = diag(M, I) \) as follows:

\[
\begin{bmatrix}
M(A^T_{aug}P + PA_{aug} + Q + 2\alpha P)M & MP\dot{\lambda}_{aug} \\
(+) & (-)
\end{bmatrix} < 0
\] (23)

which becomes using the Schur complement [Boyd et al., 1994]:

\[
\begin{bmatrix}
\Gamma & MP\dot{\lambda}_{aug} \\
(+) & (-)
\end{bmatrix} < 0
\] (24)

with

\[
\Gamma = A^T_{aug}P + PA_{aug} + 2\alpha P
\] (25)

At this stage, the block \( \Gamma M \) in (24) is not under a LMI form. By taking into account the definition (22) of \( P \) and \( M \), the block \( \Gamma M \) in (24) can be written as:

\[
\begin{bmatrix}
P^{-1}_1(A-BK) + (A-BK)P^{-1}_1 + 2\alpha P^{-1}_1 BK B & 0 \\
(+) & (*)
\end{bmatrix} \Psi + \Psi^T
\] (26)

where

\[
\Psi = \tilde{P} \begin{bmatrix}
[0 0 0 0] - [K_P^T K_{I,0}^T K_{I,1}^T] \\
[0 0 0 0]
\end{bmatrix}
\] (27)

The change of variables \( G_{aug} = \tilde{P} [K_P^T K_{I,0}^T K_{I,1}^T]^T \) enables the transformation of the block \( \Psi + \Psi^T \) in a LMI form in \( \tilde{P} \) and \( G_{aug} \). Unfortunately, a similar change of variable, e.g. \( \delta = K_P^{-1} \), does not allow to transform the first block in (26) into a LMI form in \( P_1^{-1} \). \( K \) and \( \delta \) because these parameters are coupled by the block \( BK \). To avoid this problem, the lemma 1 can be applied.

Hence, the matrix (26) can be upper bonded, with the help of the lemma (1), by:

\[
M \Gamma M \leq \begin{bmatrix}
P^{-1}_1(A-BK) + (A-BK)P^{-1}_1 + 2\alpha P^{-1}_1 BK B & 0 \\
(+) & (*)
\end{bmatrix} \Psi + \Psi^T
\] (28)

with the particular choice \( H = I \). Taking into consideration the obtained inequality (28) in (24) and applying two times the Schur complement in (24), we obtain finally:

\[
\begin{bmatrix}
\Omega & BK & B & 0 \\
K & (*) & (*) & (*)
\end{bmatrix} \Psi + \Psi^T
\] (29)

where \( R = P^{-1}_1 \) and

\[
\Omega = RA^T + AR + 2\alpha R
\] (30)

Hence, the exponential stability of the fault-tolerant control is ensured by this LMI.

5. SIMULATION EXAMPLE

In this section, the following delayed system with two inputs, two outputs and three states is considered:

\[
\begin{align*}
\dot{x}_f(t) &= Ax_f(t) + A_d x_f(t - \tau) + B(u(t) + f(t)) \\
y_f &= C x_f(t)
\end{align*}
\]

where

\[
A = \begin{bmatrix}
-1.0 & 0.5 & 0.6 \\
-0.3 & -0.9 & 0.0 \\
-1.3 & 0.6 & -0.2
\end{bmatrix}, \quad A_d = \begin{bmatrix}
-0.1 & -0.5 & 0.6 \\
-0.3 & -0.1 & 0.2 \\
0.0 & -0.2 & -0.8
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
1.0 & 2.0 \\
0.2 & 0.5 \\
0.5 & 0.35
\end{bmatrix}, \quad C = \begin{bmatrix}
0.7 & 0.3 & 0.1 \\
0.3 & 0.5 & -0.2
\end{bmatrix}
\]

The constant time-delay \( \tau \) acting on the system state is given by \( \tau = 1 \).

The following results are obtained from theorem 1 using Yalmip interface [Löfberg, 2004] and SeDuMi solver [Sturm, 1999]:

\[
K = \begin{bmatrix}
-0.055 & 0.160 & 0.190 \\
0.015 & -0.060 & -0.060
\end{bmatrix}
\]

\[
K_P = \begin{bmatrix}
-15.6299 & 29.9112 \\
-9.2427 & 9.2260 \\
24.1958 & -18.5761
\end{bmatrix}
\]

\[
K_{I,0} = \begin{bmatrix}
58.5134 & -46.3019 \\
-37.1006 & 48.1319
\end{bmatrix}
\]

\[
K_{I,1} = \begin{bmatrix}
6.8704 & -3.6454 \\
-7.4446 & 12.2209
\end{bmatrix}
\]
for a decay-rate $\alpha = 0.45$.

Figure 1 shows the time-evolution of the outputs considering a classic control law given by $u(t) = -K \dot{x}(t)$ and a fault vector $f = [f_1, f_2]^T$ given by:

$$f_1(t) = \begin{cases} 
0 & 0 \leq t \leq 160 \\
1 & 160 < t \leq 250 \\
0 & 250 < t 
\end{cases}$$

$$f_2(t) = \begin{cases} 
0 & 0 \leq t \leq 20 \\
0.01t & 20 < t \leq 120 \\
1 & 120 < t 
\end{cases}$$

It can be noted that this classic control law is not truly effective because the system outputs are far away from the equilibrium point. The robustness of this control law can be improved but the FDI becomes then very difficult.

![Fig. 1. Time-evolution of the system outputs without FTC](image)

The proposed FTC will be evaluated in the following two sections. Two cases will be considered:

1. the faults $f(t)$ take a polynomial form of degree one according to the assumption 1,
2. the faults are not polynomial of degree one but slowly time-varying signal.

This last case, is investigate to assess the fault estimation performances provided by the observer (3) when the assumption 1 is not satisfied.

### 5.1 Results with polynomial of degree one faults

The state estimation error is plotted in figure 2 and the measured and the estimated outputs are shown in figure 3. The error around the origin time is due to the differences between the initial conditions of the system and those of the observer.

The simulation results show the effectiveness of the proposed FTC (see figures 1 and 3). Note in particular the fault estimation provided by the multi-integral observer. As it can be seen from figure 4, the fault estimation error is null for polynomial faults of degree one (see the time interval $20 < t < 120$). The proposed FTC enables the real-time estimation of unknown faults acting at the same time and consequently FDI goals are accomplished.

Indeed, fault estimations can be directly employed as residual signals for fault detection in a FDI strategy. Besides, control goals are also satisfied because the system outputs remain globally around the equilibrium point even if a fault acts on the system.

![Fig. 2. Time-evolution of the state error $e_x = x_f - \hat{x}$](image)

![Fig. 3. Time-evolution of $y_f$ and its estimated $\hat{y}$](image)

![Fig. 4. Time-evolution of the fault $f$ and its estimated $\hat{f}$](image)

### 5.2 Results without polynomial of degree one faults

Now, the proposed FTC can be assessed when one of the faults acting on the system is not a polynomial of degree one but a slowly time-varying signal:

$$f_1(t) = \begin{cases} 
0 & 0 \leq t \leq 160 \\
-0.75 \sin(0.02t + \pi) & 160 < t 
\end{cases}$$
In the simulation, an additional disturbance acting on the system outputs is also considered to show the noise impact on the estimation. This disturbance is a normally distributed random signal which amplitude lies in the interval $[-0.1, 0.1]$.

Figures 5 and 6 show respectively the time-evolution of the fault and the state estimations in this case. As clearly seen from these two pictures, the proposed scheme provides satisfactory performance even if the assumption 1 concerning the fault modelling is not respected. It is also important to recognize that the provided estimation is deteriorated for faults given by time-varying signals of high frequencies. However, the proposed FTC provides very satisfying results for slowly time-varying faults.

Fig. 5. Time-evolution of the faults $f$ and its estimated $\hat{f}$.

Fig. 6. Time-evolution of the states $x$ and its estimated $\hat{x}$.

6. CONCLUSION

This paper has proposed an active fault-tolerant control strategy for delayed system. The core of this strategy is the simultaneous state and fault estimations provided by an observer known as multi-integral observer. Sufficient conditions for ensuring the control law stability as well as the estimation error convergence of the observer are derived employing the LMI framework. This strategy has been tested in simulation and encouraging results are obtained.

The extension of the proposed results to take into consideration multiple delays is actually investigated. The eigenvalues placement must also be investigated in order to improve the transient response of the control law. On the other hand, the use of less conservative Lyapunov-Krasovskii functional to provide dependant-delay conditions is an interesting perspective for further research.

REFERENCES


