Smooth Switching $H_\infty$ PI Controller for Local Traffic On-ramp Metering, an LMI Approach

Antoine Lemarchand∗ Damien Koenig∗ John J. Martinez∗

∗ Control system department, GIPSA-lab, UMR 5216, Grenoble-INP, France.

Abstract: This paper deals with the design of a bank of robust $H_\infty$ PI controllers for local traffic control. Traffic flow is modeled by the Uncertain Cell Transmission Model (UCTM) which is an affine switched model. It switches between different operating modes according to the traffic state (state dependent switching) and the uncertain parameters of the system. A robust PI controller is calculated for each mode of the UCTM. Since the switching law contains uncertainties we use a convex combination of controllers when the operating mode is uncertain. This smooth switching rule is motivated by statistical analysis of field data. The global stability of the closed loop system is proven by computing a PieceWise Quadratic (PWQ) Lyapunov function. A LMI is given to calculate the controllers. The approach is illustrated and validated in a study-case simulation.

Keywords: Traffic control, switching theory, uncertain linear systems

1. INTRODUCTION

On-ramp metering is one of the most investigated way to increase the efficiency of freeways. The on-ramp metering aims at the improving of traffic condition by controlling the inflow of freeways.

We focus on the synthesis of robust local controllers. Some solutions are already been proposed such as ALINEA in Haj-Salem et al. [2001] and others such as Sun and Horowitz [2005]. In this paper, we assume that the optimal density profile to apply on the road is known. Theses optimal references can be computed by solving a linear optimization problem as proposed in Jacquet [2008], Jacquet et al. [2008], Gomes and Horowitz [2006]. The local controllers designed in this paper can be used as the local controllers of the hierarchical control schemes proposed in Stephanedes and Chang [1993] or Kotsiados and Papageorgiou [2005]. The references computed by the optimization layer are off-line calculation based on a nominal model. Due to parametric uncertainties and disturbances, the optimal solution computed by this algorithm can’t be applied directly on the system. The aim of local controllers is to ensure that the system really tracks optimal references. The Cell Transmission Model (CTM) proposed in Daganzo [1994] has been extended with parametric uncertainties in Lemarchand et al. [2010b]. Using this model, a bank of robust switched PI controller can be designed as proposed in Lemarchand et al. [2010a]. These PI controllers guarantee the attenuation of the $H_\infty$ norm of the transfer function between disturbance and tracking error. In this paper we propose to use a convex combination of different controllers when the operating mode is uncertain. The weights applied to the controllers are motivated by the probability density functions of the uncertain parameters. The global stability of the proposed approach is proven using a PieceWise Quadratic (PWQ) Lyapunov function similar to Johansson et al. [1999].

The paper is organized as follows. A brief presentation of the uncertain UCTM is provided in Section 2. Section 3 presents the study case. Section 4 presents the state space partition, and the associated probability functions. Section 5 presents the design of robust switched PI controllers. The stability of the closed loop system is proven with a PWQ Lyapunov function. The $H_\infty$ controllers are calculated by solving a LMI optimization problem. Some simulation results are provided in Section 6. In Section 7, some concluding remarks end the paper.

2. UNCERTAIN CTM

The CTM is a discrete first order traffic model. It considers the road section divided into elementary cells. In each cell, the density of vehicles $\rho(k)$ is considered homogenous. The CTM is constituted of cells, junctions, on-ramps and off-ramps. These elements are depicted in Fig. 1. The CTM is characterized by a conservation equation for vehicles and a flow calculation equation.

![Cell Transmission Model](image)

Fig. 1. Cell Transmission Model.

2.1 Conservation Equation

For each cells,
\( \rho_i(k+1) = \rho_i(k) + \frac{T}{l_i} (\phi_i(k) + u_i(k) - \phi_{i+1}(k) - \phi_{\text{out}i+1}(k)), \)

where \( \rho_i(k) \) (veh/km) is the density of vehicles in cell \( i \), \( \phi_i(k) \) and \( \phi_{i+1}(k) \) (veh/h) are respectively the flow in junction \( i \) and \( i+1 \), \( u_i(k) \) and \( \phi_{\text{out}i+1}(k) \) (veh/h) are respectively the flow entering and leaving cell \( i \) via on/off-ramps, \( l_i \) is the length of cell \( i \) and \( T \) is the period of discretized time.

2.2 Flow Calculation

The flow between two cells is the minimum of three quantities C. F. Daganzo [1995].

\[ \phi_i(k) = \min(v_{i-1} - \rho_{i-1}(k) - \phi_{\text{out}i+1}(k), \phi_M, \]

\[ w_i. (\rho_{\ell i} - \rho_i(k)) - u_i(k). \]

(2)

with \( v_i \) (km/h) the free flow speed, \( w_i \) (km/h) the backward congestion propagation speed, \( \rho_{\ell i} \), (veh/km) the jam density (i.e. maximal density), and \( \phi_{\ell i} \), (veh/h) the maximum flow that can travel from upstream to downstream cell.

Junction modes:

From (2) one can identify the three possible modes of the junction: A free mode (F) where the flow is proportional to the upstream cell concentration, a decoupled mode (D) where the flow is equal to the maximal flow, and a congested mode (C) where the flow is proportional to the remaining space in downstream cell. A graphical representation of (2) is provided in Fig. 2. It is called the fundamental diagram.

2.3 Uncertain Fundamental Diagram

The nominal parameters of the fundamental diagram can be computed using the calibration methods described in Munoz et al. [2004] with experimental data\(^1\). The obtained diagram contains uncertainties. They can be modeled as the following parametric uncertainties Lemarchand et al. [2010a]:

\[ v_{i-1}(k) = v_{0i-1} + \delta v_{i-1}(k), \]

\[ \phi_M(k) = \phi_M + \delta \phi_M(k), \]

\[ w_i(k) = w_0 + \delta w_i(k), \]

where \( v_0 \), \( \phi_M \), and \( w_0 \) are respectively the nominal free flow speed, maximum flow, and backward congestion propagation speed, and \( \delta v(k), \delta \phi_M(k) \) and \( \delta w(k) \) the corresponding uncertainties. The parametric uncertainties of the system are bounded, i.e.

\[ |\delta v_{i-1}(k)| \leq \Delta v_{i-1}, \]

\[ |\delta \phi_M(k)| \leq \Delta \phi_M, \]

\[ |\delta w_i(k)| \leq \Delta w_i. \]

The uncertain fundamental diagram is depicted in Fig. 2.

2.4 Compact Matrix Form

For a given section of road, the CTM can be written as a discrete uncertain affine switched system Lemarchand et al. [2010a]. Define

\[ \alpha(k) := [\alpha_1(k), \cdots, \alpha_{N+1}(k)] \]

\( \alpha(k) \) is defined by (5). We notate \( \Omega \) the set of cases. In case 1, all the junctions of the section are in free mode (F). Just before congestion appears (case 2) junction 4 switches to decoupled mode (D). Then in case 3, 4, 5 and 6, the congestion propagates backward (i.e. junction 5 to 2 switch to congested mode (C)).

4. PROBABILITY FUNCTIONS

Since there are uncertain parameters on the switching rules, \( \alpha(k) \) can’t be obtained directly from \( \rho(k) \). Consider

\[ \rho_i(k+1) = \rho_i(k) + \frac{T}{l_i} (\phi_i(k) + u_i(k) - \phi_{i+1}(k) - \phi_{\text{out}i+1}(k)), \]

where \( \rho_i(k) \) (veh/km) is the density of vehicles in cell \( i \), \( \phi_i(k) \) and \( \phi_{i+1}(k) \) (veh/h) are respectively the flow in junction \( i \) and \( i+1 \), \( u_i(k) \) and \( \phi_{\text{out}i+1}(k) \) (veh/h) are respectively the flow entering and leaving cell \( i \) via on/off-ramps, \( l_i \) is the length of cell \( i \) and \( T \) is the period of discretized time.

1 Real time measurements realized on D383 road (near Lyon, France)
a junction without off/on-ramps. We take the following notations:

\[ v_{Mi} = v_{0i} + \Delta v_i, \quad v_{mi} = v_{0i} - \Delta v_i \]
\[ w_{Mi} = w_{0i} + \Delta w_i, \quad w_{mi} = w_{0i} - \Delta w_i \]

Then, from (2) junction \( i \) is Free if

\[ v_{mi} \leq w_{mi} (\rho_{i+1} - \rho_i) \]

congested if

\[ v_{mi} > w_{mi} (\rho_{i+1} - \rho_i) \]

On the partition of state space where both (9) and (10) does not hold, the mode of junction \( i \) depends on uncertain parameters values. Therefore we deduce the sub-space

![Fig. 4. Mode subspaces for junction \( i \)](image)

where junction \( i \) is in Free mode (\( \Psi_{F} \)), in Congested mode (\( \Psi_{C} \)) in Decoupled mode (\( \Psi_{DC} \)), either Free or Congested mode (\( \Psi_{FC} \)), either Free or Decoupled mode (\( \Psi_{FD} \)), either Decoupled or Congested mode (\( \Psi_{DC} \)). These sub-spaces are depicted in Fig. 4.

We perform a statistical study of experimental data to characterize the sub-space \( \Psi_{FC} \). The normalized cumulate probability density function depicted in Fig. 5 can be calculated by doing a convolution product between the probability density functions for uncertainties \( \delta v_i \) and \( \delta w_i \). This function represent the normalized probability to be in congested mode in the sub-space \( \Psi_{FC} \).

The approximation function (12) is given in Fig. 5.

\[ \mu(x) = \frac{\arctan((\frac{x-a}{b})-0.5) \cdot \beta}{2 \cdot \arctan(\beta/2)} + 1 \]

where \( a \) and \( b \) are the bound calculated thanks to (9) and (10), and \( \beta \) a tuning parameter. The comparison between the normalized cumulate convolution product and the approximation function (12) is given in Fig. 5.

The probability for junction \( i \) to be in Free, Decoupled or Congested mode are respectively denoted \( \mu_F(\rho_i), \mu_D(\rho_i), \mu_C(\rho_i) \). From (9),(10) and (12) we write:

\[ \mu_C(\rho_i) = \begin{cases} 0, & \text{if } \rho_i < \rho_{mi-1} \\ 1, & \text{if } \rho_i > \rho_{mi-1} \\ \frac{\arctan((\rho_i-\rho_{mi-1}) \cdot \beta/2)}{2 \cdot \arctan(\beta/2)} + 1, & \text{otherwise} \end{cases} \]

![Fig. 5. Normalized cumulate convolution product, approximation function \( (\beta = 8, a = -1, b = 1) \)](image)

\[ \rho_{mi-1} = \frac{w_{mi-1}}{v_{mi-1}} (\rho_{Ji} - \rho_i) \]

\[ \rho_{MI-1} = \frac{w_{MI-1}}{v_{MI-1}} (\rho_{Ji} - \rho_i) \]

The Fig. 6 depicts \( \mu_F(\rho) \), \( \mu_C(\rho) \). Thus, we can build the subspaces for each case as an intersection of the subspaces defined in Fig. 4. We define \( \Psi_{Case3} \) as the subspace where the subsystem 3 is active, \( \Psi_{Case4} \) as the subspace where the subsystem 4 is active and \( \Psi_{Case34} \) as the subspace where the subsystem 3 or 4 may be active. They are defined as follows:

\[ \Psi_{Case3} = \psi_{F1} \cap \psi_{F2} \cap \psi_{D1} \cap \psi_{C1} \cap \psi_{D6} \]
\[ \Psi_{Case4} = \psi_{F1} \cap \psi_{F2} \cap \psi_{F3} \cap \psi_{C4} \cap \psi_{C1} \cap \psi_{D6} \]
\[ \Psi_{Case34} = \psi_{F1} \cap \psi_{F2} \cap \psi_{F3} \cap \psi_{FC1} \cap \psi_{C1} \cap \psi_{D6} \]

We notate I the set of subspaces, \( I_0 \) is the set of partitions containing the origin, \( I_1 = I \setminus I_0 \). We notate \( \kappa(\Psi_i) \) the set of subsystem that may be active in subspace \( \Psi_i \). The probability to be in Case \( n \) is called \( \mu_n(\rho) \). It can be easily defined as a product of corresponding individual probabilities for each junctions. Remark that:
\[ \mu_n(\rho) = 0, \rho \in \Psi_i, n \not\in \kappa(\Psi_i) \]  
\[ \mu_n(\rho) \neq 0, \rho \in \Psi_i, n \in \kappa(\Psi_i) \]  
\[ \sum_{n \in \kappa(\Psi_i)} \mu_n(\rho) = 1 \]

5. REGULATOR DESIGN

We propose to design a smooth switching PI controller. A PI controller is designed for each case in \( \Omega \). Then a convex combination of these PI controllers is applied on the system. The weights for each controller are calculated with respect to the probabilities defined in the previous section.

5.1 Extended system

First, the system is extended with an integrator (Section 5.1) as in Lemarchand et al. [2010a]. To keep the stabilizability property of the system, we can only extend one state of the system with an integrator. For the case 1 and 2, we extend the system with an integrator on \( \epsilon_5(k) \) (where \( \epsilon_5(k) = \rho_i(k) - \rho_j^c(k) \)). For the case 3 to 6, we extend the system with an integrator on the density of the cell where the congestion front stands (respectively \( \epsilon_4(k) \) to \( \epsilon_1(k) \)). So, the new state vector becomes

\[ X(k) = \begin{pmatrix} \rho(k) \\ z(k) \end{pmatrix} \]  

with:

\[ z(k + 1) = z(k) + \begin{cases} \\ \epsilon_5(k), & \text{if case 1 or 2} \\ \epsilon_4(k), & \text{if case 3} \\ \epsilon_3(k), & \text{if case 4} \\ \epsilon_2(k), & \text{if case 5} \\ \epsilon_1(k), & \text{if case 6} \end{cases} \]

The extended system is described in a compact form as follows:

\[
X(k + 1) = \begin{pmatrix} (A_{\alpha(k)} + E_{\alpha(k)} \Delta(k) G_{\alpha}) X(k) \\ + a_{\alpha(k)} + B_{\alpha(k)} w(k) \end{pmatrix} \\
Z(k) = C_{\alpha(k)} X(k)
\]

where \( Z(k) \) is the tracking error of the cell where congestion front stands.

5.2 PWQ Lyapunov Function

To ensure that the system is globally stable, we have to find a candidate Lyapunov function. Since a global Lyapunov function can’t be found, we build a piecewise quadratic Lyapunov function. On this purpose, we have to ensure continuity at each bound of subspaces defined in (13). These bounds are represented in continuous lines in Fig. 4.

To follow the approach proposed in Johansson et al. [1999], we introduce the following notations:

\[ \tilde{X}(k) = \begin{pmatrix} X(k) \\ 1 \end{pmatrix} \]

Proposition 1. Choose a candidate Lyapunov function of the form:

\[ V(X) = \begin{pmatrix} X^T P X, \forall X \in \Psi_i, i \in I_0 \\ X^T P \bar{X}, \forall X \in \Psi_i, i \in I_1 \end{pmatrix} \]  

with \( I_0 \) the set of partitions containing the origin, \( I_1 = I \setminus I_0 \) and where \( P_{\Psi} \) is chosen as follows:

\[ P_{\Psi} = \begin{cases} \\ L_i^T T L_i, & \forall X \in \Psi_i, i \in I_0 \\ L_i^T T L_i, & \forall X \in \Psi_i, i \in I_1 \end{cases} \]

In addition, If

\[ \begin{align} \\
\bar{L}_i \bar{X} = \bar{L}_i \bar{X}, & \forall X \in \Psi_i \cap \Psi_j, i, j \in I \\
\bar{L}_i = \begin{pmatrix} L_i \\ 0 \ldots 0 \end{pmatrix}, & \forall i \in I_0 
\end{align} \]

Then \( V(X) \) is continuous along all bounds of the partitioned state space.

The problem is now to compute matrices \( \bar{L}_i, i \in I \). An efficient approach to compute these matrices is presented in Johansson et al. [2000]. According to Fig. 4, the partitioned state space for a junction contains 7 bounds. Depending of the junction characteristics, some of these bounds may not be crossed. Ensuring the continuity at each bounds may be conservative and may increase the problem size.

We present the method for junction with no on/off-ramps. In this case the system can only cross 2 bounds (\( \Psi_{F_i} \cap \Psi_{FC_i} \) and \( \Psi_{FC_i} \cap \Psi_{CI_i} \)). A typical trajectory of the system in this case (\( \tau_{FC} \)) is represented in Fig. 4. We start by computing \( \bar{L}_1 \Psi_{F_i} \) and \( \bar{L}_1 \Psi_{FC_i} \) for junction \( i \) such that

\[ \bar{L}_1 \Psi_{F_i} \left( \frac{\rho_{i-1}}{1} \right) = \bar{L}_1 \Psi_{FC_i} \left( \frac{\rho_{i-1}}{1} \right) \]

\[ \forall \frac{\rho_{i-1}}{1} \in \Psi_{F_i} \cap \Psi_{FC_i} \]

This condition will ensure continuity of the Lyapunov function at the bound \( \Psi_{F_i} \cap \Psi_{FC_i} \) defined by the equation:

\[ u_{M-1} \rho_{i-1} + w_m \rho_i = w_m \rho_i \]

The computational method for this matrix is detailed in Johansson et al. [1999].

Using the same procedure, we are able to compute \( \bar{L}_2 \Psi_{FC_i} \) and \( \bar{L}_2 \Psi_{CI_i} \) such that

\[ \bar{L}_2 \Psi_{FC_i} \left( \frac{\rho_{i-1}}{1} \right) = \bar{L}_2 \Psi_{CI_i} \left( \frac{\rho_{i-1}}{1} \right) \]

\[ \forall \frac{\rho_{i-1}}{1} \in \Psi_{FC_i} \cap \Psi_{CI_i} \]

Then, taking:

\[ \bar{L}_{\Psi_{F_i}} = \begin{pmatrix} \bar{L}_1 \Psi_{F_i} \\ \bar{L}_2 \Psi_{F_i} \end{pmatrix} \]

\[ \bar{L}_{\Psi_{FC_i}} = \begin{pmatrix} \bar{L}_1 \Psi_{FC_i} \\ \bar{L}_2 \Psi_{FC_i} \end{pmatrix} \]

\[ \bar{L}_{\Psi_{CI_i}} = \begin{pmatrix} \bar{L}_1 \Psi_{CI_i} \\ \bar{L}_2 \Psi_{CI_i} \end{pmatrix} \]

Condition (22) holds at all bounds crossed by the system trajectories \( \tau_{FC} \).
For a junction containing an on-ramp, the continuity of the Lyapunov function has to be ensured at 3 bounds ($\Psi_{Fi} \cap \Psi_{FDi} \cap \Psi_{DCi} \cap \Psi_{Ci}$). The constraint matrices for a system with several junctions can be easily computed from the matrices (27-27)

5.3 LMI Formulation

**Proposition 2.** if $\exists T = T^T > 0$ and $e_n > 0, U_n, n \in \Omega$ such that:

(27) holds for all $\Psi_i \in I_0, n \in \kappa(\Psi_i)$,
(28) holds for all $\Psi_i \in I_1, n \in \kappa(\Psi_i)$,
The LMI's (27) and (28) are defined at the top of the next page.

Then:

- The extended system (19) is stable under state feedback $u(k) = - \sum \mu_i(X(k))K_nX(k), K_n = U_nT$,
- The $H_\infty$ norm of the transfer function between $w(k)$ and $Z(k)$ is bounded by $\gamma$

**Proof** We write the proof for (27) i.e. for $\Psi_i \in I_0, k \in \kappa(\Psi_i)$. We write the following $H_\infty$ attenuation criteria:

$$X(k + 1)^T P_k X(k) + X(k)^T P_k X(k) + Z(k)^T Z(k) - \gamma^2 w(k)^T w(k) < 0$$

(29)

Taking $A_{a_n} - B_{a_n}K_n = \Gamma_n$, (29) is equivalent to

$$\begin{pmatrix}
\Pi_{11} & \Pi_{12} \\
E_{a_n} & -\gamma^2 I
\end{pmatrix} < 0$$

(30)

$$\Pi_{11} = (\Gamma_n + F_{a_n}\Delta(k).G_a)^T L_i^T TL_i(\Gamma_n + F_{a_n}\Delta(k).G_a)$$
$$\Pi_{12} = (\Gamma_n + F_{a_n}\Delta(k).G_a)^T L_i^T TL_iE_{a_n}$$

Applying Schur complement, we obtain the following inequality:

$$\begin{pmatrix}
-L_i^T TL_i + C_n^T C_n & 0 & (\Gamma_n + F_{a_n}\Delta(k).G_a)^T L_i^T \\
\ast & -\gamma^2 I & E_{a_n}^T L_i^T \\
\ast & \ast & -T_i^T
\end{pmatrix} < 0$$

Using majoration lemma, (5.3) holds if $\exists e_n > 0$ such that:

$$\begin{pmatrix}
-L_i^T TL_i + C_n^T C_n & 0 & 1^T L_i^T \\
\ast & -\gamma^2 I & E_{a_n}^T L_i^T \\
\ast & \ast & -T_i^T + e_n L_i F_{a_n}^T F_{a_n} L_i^T
\end{pmatrix} < 0$$

Multiplying on both side by $\begin{pmatrix} T_i & 0 \\ 0 & I \end{pmatrix}$ and applying twice Schur complement, the following inequality is obtained:

$$\begin{pmatrix}
-T^{-1} L_i^T TL_i T^{-1} & 0 & T^{-1} \gamma^2 T_i L_i^T - T_i T^{-1} C_n^T T_i - I G_{a_n}^T \\
\ast & -\gamma^2 I & E_{a_n}^T L_i^T \\
\ast & \ast & \Pi_{33}^T
\end{pmatrix} < 0$$

$$\Pi_{33} = -T_i^T + e_n L_i F_{a_n}^T F_{a_n} L_i^T$$

Since $-T_i^T L_i^T TL_i T^{-1} \leq -T_i^T L_i^T L_i T^{-1} + T^{-1}$, then the previous inequality is true if

$$-T_i^T L_i^T L_i T^{-1} - L_i T^{-1} + T^{-1} 0 \\
\ast & -\gamma^2 I & E_{a_n}^T L_i^T \\
\ast & \ast & \Pi_{33}^T
\ast & \ast & \ast & \ast & \ast & -\gamma^2 I$$

$$\begin{pmatrix}
-T_i^T L_i^T L_i T^{-1} - L_i T^{-1} + T^{-1} 0 \\
\ast & -\gamma^2 I & E_{a_n}^T L_i^T \\
\ast & \ast & \Pi_{33}^T
\ast & \ast & \ast & \ast & \ast & -\gamma^2 I
\end{pmatrix} < 0$$

Taking $U_n = K_nT_i^{-1}$, LMI (27) is obtained. Then using Theorem 1 in Johansson et al. [1999], the proof is completed. The proof is similar for (28) with:

$$\begin{pmatrix}
\bar{A}_{a_n} & 0 \\
\bar{B}_{a_n} & 0
\end{pmatrix}, \bar{C}_{a_n} = (C_{a_n} 0)$$

6. SIMULATION RESULTS

For simulation, we take the on-ramp neighborhood depicted in Fig. 3. We choose the following cell parameters (cells are homogenous):

- Cell length $l = 0.3km$
- Free flow speed $v = 80km/h \pm 5\%$
- Backward congestion speed $v = 35km/h \pm 15\%$
- Maximum flow $\phi_M = 7000km/h \pm 8\%$

These parameters values and uncertainties ranges are chosen according to experimental data.

To test the efficiency of the proposed local regulation, we generate density profile to apply on the road. These references fits the nominal system (6) with $\Delta(k) = 0$. In the chosen scenario, the system follow a congestion front that propagates backward then forward throw all the cases considered in this study. So we are able to validate the behavior of the system on each operating mode and throw each switching surface. Fig. 7 depicts the results of the simulation using the switched PI controller presented in Lemarchand et al. [2010a]. Fig. 8 depicts the results of the simulation using the smooth switched PI controller. The first plot depicts in dashed line the density profile to follow, and in full line the behavior of the system. The
second plot depicts the active controller and the third plot the on-ramp flow.

![Image of controller and on-ramp flow plots]

Fig. 8. Smooth Switched PI controller

The proposed approaches ensure a good tracking of density profiles, despite parametric uncertainties and disturbances. With the smooth switched PI controller, the fast variation of the on-ramp flow and the chattering effect are eliminated.

7. CONCLUSIONS

In this paper, we designed a smooth switched controller for a switching system that contains uncertainties in the switching rule. The controller applied on the system is a convex combination of different controllers. The weights for each controller are calculated thanks to the probability to be in a particular operating mode. The proposed controller shows good results while following density profiles, and limits the chattering effect at switching surface. The computation of the PWQ Lyapunov function and of the controllers may be complex and the proposed LMI conservative. This approach may be hard to implement on a larger study case, despite this the smooth switching law seems to be a nice implementation trick that should be investigated further.

REFERENCES


A. Lemarchand, J. J. Martinez, and D. Koenig. Hierarchical coordinated freeway on-ramp metering using switching system theory. In IFAC Symposium on Structured System Control, Ancona, Italy, September 2010b.

