Reliable Fault-Tolerant Control Design for LPV Systems using Admissible Model Matching

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Abstract: In this paper, an approach to design a Reliable Admissible Model Matching (AMM) Fault Tolerant Control (FTC) for LPV systems is proposed. The suggested strategy is an active FTC strategy that reconfigures the controller on-line taking into account changes due to the faults. The proposed FTC design approach allows to maintain, by design, a certain reliability level in spite of the faults, guaranteeing best achievable performance with some pre-established level of degradation. In particular, the reliability analysis is developed for actuator faults. The effectiveness and performance of proposed approach have been illustrated using a two degree of freedom helicopter.

Keywords: Fault-Tolerant Control, Admissible Model Matching, Reliability, Linear Parameter Varying, $H_2$ Performance, Linear Matrix Inequality.

1. INTRODUCTION

Fault Tolerant Control (FTC) has been consolidated as an important research topic in the control applications during last years (Blanke et al., 2006). The objective of an FTC approach is to maintain desirable closed-loop performance or an acceptable degradation and preserve stability conditions in the presence of component and/or instrument faults. Accommodation capability of a control system depends on many factors such as severity of fault, the robustness of the nominal system and mechanisms that introduce redundancy in sensors and/or actuators.

Fault accommodation has been addressed in the literature considering different control objectives and solution techniques. The interested reader can see Zhang and Jiang (2008) for a recent review. In model matching FTC approaches, the goal of the fault accommodation is defined in terms of similarity between the closed-loop system and nominal system (optimal behavior). In the Pseudo-Inverse Method (PIM) (Ostrov, 1985), a model matching formulation provides a solution that minimizes a performance indication based on the distance (using some norm) between the accommodated and the nominal closed-loop system matrices. In particular cases, an exact model matching can be obtained, but in the general case the optimality of the obtained solution does not guarantee stability. Alternatively, the Admissible Model Matching (AMM) approach was proposed in Staroswiecki (2005a) and later extended in Staroswiecki (2005b). The main idea is to accommodate the controller such that the system closed-loop behavior is guaranteed to be in the set of admissible behaviors. In the original AMM formulation, the set of admissible behaviors is defined in terms of several inequality constraints relating the closed-loop system matrix coefficients (Staroswiecki, 2005b). In this paper, the AMM approach is extended for non-linear systems using Linear Parameter Varying (LPV) techniques (see Apkarian et al. (1995)), recently applied to Fault Detection and Isolation (FDI) and FTC (Bokor, 2009).

However, as discussed in Khelassi et al. (2009) in order to improve the availability of the system, it is crucial to ensure that mean operating time of the novel configuration selected after the fault is enough to achieve the system goal until the end of the mission. This implies adding a new restriction in the control design problem related to reliability maintenance.

This paper considers the previous concepts to develop a reliable AMM FTC approach for LPV systems subject to parametric faults that allows to specify the set of admissible faults. The controller is able to tolerate the parametric faults with an admissible performance degradation that can be represented using Linear Matrix Inequality (LMI) regions. In this LPV fault representation two types of scheduling variables are considered: the magnitude of parametric fault and change in the operating point. Then, an active FTC strategy can be designed using LPV control theory that requires the fault to be detected, isolated and estimated by the FDI scheme and the controller be redesigned on-line accordingly. To achieve the desired objective, a control design algorithm has to be obtained based on the quadratic $H_2$ performance proposed by (Scherer et al., 1997). On the other hand, reliability performance is achieved by determining the maximum actuator level allowed by imposing limits in the control actions using the results presented in Wu et al. (2000a). Then, the AMM LPV problem is solved through a finite number of algebraic LMIs by approximating the LPV system in a polytopic way (Apkarian et al., 1995, Chilali and Gahinet, 1996).

The remainder of the paper is organized as follows: Section II recalls the AMM approach for FTC. Section III describes the proposed AMM approach for LPV systems, that allows to specify the set of admissible faults satisfying the desired control specifications. Section IV presents the controller design using pole placement in LMI regions. The reliability is introduced in order to estimate the degradation of the desired performance. The state-feedback $H_2$ synthesis with pole placement LMI regions is discussed. Section V demonstrates the effectiveness and performance of this approach using a two degree of freedom helicopter. Finally, Section VI presents the main conclusions.
2. AMM APPROACH FOR FTC

The main idea of AMM FTC approach proposed in Staroswiecki (2005a) is that instead of looking for a controller that provides an exact (or best) matching to a given single behavior after the fault appearance, a family of closed-loop behaviors that are acceptable is specified.

In order to recall the principle of Admissible Model Matching, consider the following LTI system:

\[ \dot{x}(t) = (A - BK)x(t) = Mx(t) \]

where \( M \) is chosen to be stable. In the AMM approach, a set of system matrices that are acceptable is considered and the FTC controller tries to provide a closed-loop behavior inside the set (Staroswiecki, 2005a). Let \( \mathcal{M}_a \) be a set of the matrices such that any solution of:

\[ \dot{x}(t) = Mx(t), \quad M \in \mathcal{M}_a \]

has an acceptable dynamic behavior. The set of reference models \( \mathcal{M}_r \) is defined off-line by the designer.

Moreover, let assume that for the nominal system operation, a state gain feedback \( K_n \) that satisfies some nominal control specifications has been obtained. Then:

\[ \dot{x}(t) = (A_n - B_n K_n)x(t) = M^*x(t) \]

and \( M^* \) is known as the reference model.

For a given fault \( (A_f, B_f) \), the goal of the fault accommodation is to find a feedback gain \( K_f \) that provides an admissible closed-loop behavior:

\[ \dot{x}(t) = (A_f - B_f K_f)x(t) \in \mathcal{M}_a \]

In Staroswiecki (2005b), a characterization of \( \mathcal{M}_a \) is provided by a set of \( d \) inequality constraints:

\[ \mathcal{M}_a = \{ M : \Phi(m_j, i = 1, \ldots, n, \ j = 1, \ldots, n) \leq 0 \} \]

where \( m_j, i = 1, \ldots, n, \ j = 1, \ldots, n \) are the entries of matrix \( M \), \( \Phi : \mathbb{R}^{n \times n} \rightarrow \mathbb{R} \) is a given vector function, and the constrains are written \( \Phi(M) \) for short. However, these set of inequalities are difficult to be obtained and not systematic procedure is described in Staroswiecki (2005b, 2006). Moreover, some practical specifications are obtained by a set of non-linear equations (see the illustrative example in Staroswiecki (2005b)) and this leads to the formulation of the fault accommodation as a non-convex optimization problem.

3. AMM FTC FOR LPV SYSTEMS

3.1 LPV Modeling

Let us consider that the non-linear system to be controlled can be described by the following LPV representation as follows:

\[ \dot{x}(t) = A(\theta(t))x(t) + B(\theta(t))u(t) \]

\[ y(t) = C(\theta(t))x(t) + D(\theta(t))u(t) \]

where \( u(t) \in \mathbb{R}^m \) is the system input, \( y(t) \in \mathbb{R}^r \) is the system output, \( x(t) \in \mathbb{R}^n \) is the state vector and \( \theta \in \mathbb{R}^n \) is the system vector of time-varying parameters.

The common motivation for LPV representations is the approximation of non-linear dynamics, being the parameter scheduling typically associated to changes in the operating conditions that can be estimated using some measured system variables \( p(t) \). In this work, additionally to the schedule of the parameters with the operating conditions, they are scheduled with the fault estimation \( \hat{f}(t) \) provided by an FDI module (Wu et al., 2000b).

Then, the scheduling of the LPV parameters can expressed as:

\[ \theta \doteq \theta(p(t), \hat{f}(t)) \]

where the fault estimation \( \hat{f}(t) \) is in the set of tolerated fault \( \hat{f}(t) \in \mathcal{F}_f \) and the operating point \( p(t) \) is in range of operating conditions \( p(t) \in \mathcal{P} \).

In this paper, the kind of LPV systems considered are the ones such that the time-varying parameter vector \( \theta(t) \) varies within a polytope, known as polytopic LPV systems (Apkarian et al., 1995). In particular, the state-space matrices range in a polytope of matrices defined by the convex hull of a finite number of matrices \( N \). That is,

\[ \begin{pmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{pmatrix} \in \text{Co} \left( \sum_{j=1}^{N} \alpha_j(\theta) \begin{pmatrix} A_j(\theta) & B_j(\theta) \\ C_j(\theta) & D_j(\theta) \end{pmatrix} \right) \]

with \( \alpha_j(\theta) \geq 0, \sum_{j=1}^{N} \alpha_j(\theta) = 1 \) and \( \theta^i = \theta(p^i, f^i) \) is the vector of parameters corresponding to \( p^i \) model. Each \( p^i \) model is called a vertex system.

Consequently, the LPV fault representation (8) can be expressed as follows:

\[ \begin{align*}
\dot{x}(t) &= \sum_{j=1}^{N} \alpha^i(\theta) A_j(\theta(t))x(t) + B_j(\theta(t))u(t) \\
y(t) &= \sum_{j=1}^{N} \alpha^i(\theta) C_j(\theta(t))x(t) + D_j(\theta(t))u(t)
\end{align*} \]

where \( A_j, B_j, C_j \) and \( D_j \) are the state space matrices defined for \( p^i \) model. Notice that, the state space matrices of system (11) are equivalent to the interpolation between LTI models, that is:

\[ A(\theta) = \sum_{j=1}^{N} \alpha^i(\theta) A_j(\theta(t)) \quad \text{and analogously for } B(\theta(t)), C(\theta(t)) \quad \text{and } D(\theta(t)). \]

The polytopic system is scheduled through functions \( \alpha^i(\theta(t)) \) that lie in a convex set

\[ \Psi = \left\{ \alpha^i(\theta) \in \mathbb{R}^N, \alpha^i(\theta) = [\alpha^1(\theta), \ldots, \alpha^N(\theta)]^T, \right\} \]

\[ \alpha^i(\theta(\cdot)) \geq 0, \sum_{j=1}^{N} \alpha^i(\theta(t)) = 1 \] \( \alpha^1(\theta(t)) = 1 \).

There are several ways of implementing (10) depending on how \( \alpha^i(\theta(t)) \) functions are defined. Here, the function \( \alpha^i(\theta(t)) \) is defined via barycentric combination of vertexes as suggested by Apkarian et al. (1995).

3.2 Motivation

The goal of the AMM FTC is to maintain acceptable control performances under the presence of the pre-established set of faults. In case that AMM approach is combined with an active strategy, once a fault has appeared, its magnitude will be estimated by the FDI module and the controller will be adapted accordingly (accommodation), trying to maintain acceptable performance. This leads to the control structure shown in Fig. 1 that can be viewed equivalent to a gain-scheduling control structure where the fault \( f \) is the scheduling variable and the FDI module is the parameter estimation algorithm. This suggests that gain-scheduling LPV theory can be used for the design of active AMM FTC. Moreover, this approach allows to be applied to non-linear systems whose behavior can be represented by a LPV representation that includes the faults (8).
Then, the controller must be adapted according to faults and operating conditions. Considering the system to be controllable and reliable, the relationship between the actuator load increase, to keep desired performance, and reliability is the LPV fault representation (11), a state feedback control law is written as follows:

$$u(t) = -K(\theta)x(t) = \sum_{j=1}^{N} a_j(\theta) [K_j(\theta^j) x(t)]$$

(13)

such the closed-loop behavior satisfies

$$M(\theta_j) = \sum_{j=1}^{N} \alpha_j(\theta_j) M_j(\theta^j) \in M$$

(14)

where: $M(\theta^j) = A_j(\theta^j) - B_j(\theta^j)K_j(\theta^j)$. Then, the following definition follows:

**Definition 1. Admissibility using LPV fault representation:**

The set of admissibility $M_\alpha$ using LPV fault representation can be defined as:

$$M_\alpha = \{A(\theta), B(\theta), K(\theta)\} : f \in F \cup \Phi M(\{A(\theta), B(\theta), K(\theta)\}) \leq 0 \}$$

(15)

where $\Phi M$ is the set of constraints that guarantee the condition (14). The set $M_\alpha$ contains the set of admissible models $(A(\theta_j), B(\theta_j), K(\theta_j))$ such that the closed-loop system can be achieved with the control law (13).

The constraints $M(\theta_j), B(\theta_j), K(\theta_j))$ can be defined via LMI’s including performance, stabilization and pole placement specifications.

4. ADMISSIBLE CLOSED-LOOP BEHAVIOR THROUGH LMI REGIONS

4.1 LMI regions

According to Chilali and Gahinet (1996), a subset $\mathcal{D}$ of the complex plane is called an LMI region if there exists a symmetric matrix $\rho = [\rho_{ij}] \in \mathbb{R}^{n_{x,m}}$ and a matrix $\eta = [\eta_{ij}] \in \mathbb{R}^{n_{x,m}}$ such that

$$\mathcal{D} = \{z \in \mathbb{C} : f_D(z) < 0 \}$$

(16)

where the characteristic function $f_D(z)$ is given by $f_D(z) = [\rho_{ij} + \eta_{ij}z + \eta_{ij}z^2]_{0 \leq k < m}$ ($f_D(z)$ is valued in the space of $m \times m$ Hermitian matrices and that “< 0” stands for negative definite).

An interesting LMI region for control purposes is the set $\mathcal{D}(\rho, r, \theta)$ of complex number $x + jy$ such that:

$$x - \rho < 0, \quad |x + jy| < r, \quad \text{and} \quad \theta_1 x < -|y|$$

(17)

Confining the closed-loop poles to this region ensures a minimum decay rate $\rho$, a minimum damping ratio $\zeta = \cos \theta$, and a maximum undamped natural frequency $\omega_d = r \sin \theta$.

This region is the intersection of three elementary LMI regions: an $\rho$ stability region, a disk of radius $r$ and the conic sector $\theta$.

4.2 Active AMM FTC Admissibility using LMI Regions

Tornil-Sin et al. (2008) proposes a set admissible behaviors $\mathcal{M}$ using LMI regional pole placement. This method locates the poles in particular convex regions called $\mathcal{D}$-regions. The fault accommodation is formulated in terms of several LMI problems. Using this approach, the admissibility condition (6) can be defined using the set admissible behaviors $\mathcal{M}$ proposed by Tornil-Sin et al. (2008) and LMI regions pole placement as follows:

**Definition 2. Admissibility of active AMM FTC using LMI regions:**

The set of admissible behaviors $\mathcal{M}_\alpha$ can be described by:

$$M_\alpha = \{A(\theta) - B(\theta)K(\theta) : \sigma(\{A(\theta) - B(\theta)K(\theta)\}) \in \mathcal{D}_\alpha \}$$

(18)

where $\mathcal{D}_\alpha = \mathcal{D}(\rho, r, \theta)$ is a desired LMI region included in complex left half plane.

Thus, following Chilali and Gahinet (1996), admissibility condition (18) can be expressed using LMIs considering (14). The following set of LMI regions should be solved if and only if a symmetric matrix $X_j$ exists for all $j \in \{1, \ldots, N\}$ such that:

$$A_j - B_jK_j X_j + X_j A_j^T - K_j^T B_j^T + 2\rho X_j < 0$$

$$X_j A_j^T - L_j^T B_j + r X_j < 0$$

Thus, the controller $K(\theta)$ satisfying LMIs (19)-(21) is then determined as follows: $K(\theta) = \sum_{j=1}^{N} K_j(\theta_j)$. Thus, the active AMM FTC control law is given by (13).

Note that the solutions of the LMIs (22)-(24) is done off-line and the evaluation of (13) just requires the computation of $\alpha_j(\theta)$. Due to the simplicity of this computation, the real-time implementation of the controller reconfiguration is possible.

4.3 Reliability admissibility condition

When a fault occurs, the control law is modified in order to recover the closed loop performance from the fault impact on the system behavior. As explained in Guenab et al. (2006), the value of actuator failure rate changes because the control action should be increased in order to compensate the fault effect. In this case, the energy consumption increases and the value of failure rate becomes higher due to the actuator load increase. Thus, there is an interplay between maintaining closed-loop performance and reliability. The relationship between the actuator load increase, to achieve desired performance, and reliability
can be established. One of the most used relations between both is based on assuming that actuator fault rates changes with the load through the following exponential law:
\[ \lambda_i = \lambda_i^0 e^{\beta_i u_i} \]  
(25)
where \( \lambda_i \) represents the baseline failure rate (nominal failure rate) while \( u_i \) is the control action for the \( i^{th} \) actuator. Parameter \( \beta_i \) is a fixed factor that depends on the actuator characteristics. Thus, the reliability of the actuator can be expressed in terms of its load as follows:
\[ R_i(t) = e^{-\lambda_i t} = e^{\lambda_i^0 e^{\beta_i u_i} t} \]  
(26)
Let us consider that a predefined reliability threshold \( R_{i,h} \) should be maintained until the end of the system mission at time \( t_{end} \). This threshold defines the minimal value of the acceptable reliability value in the degraded faulty mode. The aim is to translate this threshold to a load threshold that can be applied to the actuator. This actuator load threshold can be derived from (26) as follows:
\[ |u_{i,h}| = \frac{1}{\beta_i} \ln \left( \frac{\ln R_{i,h}}{\lambda_i^0 t_{end}} \right) \]  
(27)
Then, in the AMM FTC control design the following restriction in the \( i^{th} \) actuator control action should be added:
\[ u_i \in [-u_{i,h}, u_{i,h}] \]  
(28)
This restriction could be added following the idea proposed by Wu et al. (2000a) that allows to take into account constraints in the control action when designing an LPV controller introducing extra LPV parameters, that take into account the non-linearity induced by these constraints, as follows:
\[ \theta_i(u_i) = \frac{\sigma_i(u_i)}{u_i}, \text{ for } i = 1, 2, \cdots, n_u \]  
(29)
where
\[ \sigma_i(u_i) = \begin{cases} u_i & \text{if } |u_i| < u_{i,h} \\ \text{sign}(u_i)u_{i,h} & \text{if } |u_i| \geq u_{i,h} \end{cases} \]  
(30)
For this study, the actuator fault is considered in the polytopic LPV system (11) as follows:
\[ \dot{x}(t) = \sum_{j=1}^{n_k} a_j(p^i, f^j)x(t) + B_j(p^i, f^j)u(t) \]  
(31)
where the matrix \( B_j(p^i, f^j) \) can be written in relation of the nominal matrix \( B_j(p^i) \) and the actuator fault effect \( f_{j,U}^i, i = 1, \cdots, n_u \), as:
\[ B_j(p^i, f^j) = B_j(p^i)(I - \Gamma) \]  
(32)
where \( f_{j,U}^i = 0 \) denotes that the actuator without fault. When \( 0 < f_{j,U}^i < 1 \), a partial actuator fault is considered.

When the actuator fault is presented, the input control should be increased in order to compensate the fault effect and using (32) is possible to establish the following relation:
\[ u_{i,h}^{nom} = (1 - f_{j,U}^i)^{-1}u_i \]  
(33)
where \( u_{i,h}^{nom} \) is the nominal control input to obtain the desired value. According to (30), \( u \leq u_{i,h} \). Then, the maximum fault can be expressed as:
\[ f_{j,U}^{max} = 1 - \frac{u_{i,h}}{u_{i,h}^{nom}} \]  
(34)

4.4 AMM LPV FTC design using \( H_2 \) Performance

Any controller \( K(\theta) \) satisfying (18) is an admissible solution to the AMM problem. But, it is still necessary to establish some performance criteria to select a particular gain in order to accommodate the control loop minimizing performance degradation.

According to (Scherer et al., 1997), a \( K(\theta) \) achieves \( H_2 \) performance, when,
\[ K_{H_2}^D(j\omega) = \| T_{\nu} \|_2 = \sup_{\nu \in \nu} H_{\nu}(j\omega) < \nu \]  
(35)
if only if \( D = 0 \) and exists \( X = X^T > 0 \) and \( W = W^T > 0 \) such that (22)-(24) and
\[ X_1 + X_2A_{\nu}X_1(j\omega)^T + B_1(j\omega)X_2 + L_1^jB_1(j\omega)^T \leq 0 \]  
(36)
for all \( j\nu \) models.

When performance criteria \( H_2 \) are used to design the AMM FTC, additionally to check if the closed-loop behavior is inside the set of admissible behaviors, the control performance index in faulty situation \( J_{\text{fault}} \) can be compared with the one \( J_{\text{nofault}} \) obtained in non-faulty situation. Then, it is reasonable to define the performance admissibility condition of the control as follows
\[ J_{\text{fault}} < (1 + \tau)J_{\text{nofault}} \]  
(38)
where \( \tau > 0 \) specifies the overcost (i.e. the performance degradation) that is acceptable. Defining an acceptable overcost \( \tau \), condition (38) allows to assess if a given fault is admissible or not regarding performance degradation.

5. APPLICATION EXAMPLE

5.1 Description of Twin-Rotor MIMO System

The TRMS is a laboratory setup developed by Feedback Instruments Limited for control experiments. The system is perceived as a challenging engineering problem due to its high non-linearity, cross-coupling between its two axes, and inaccessibility of some of its states through measurements. The TRMS mechanical unit has two rotors placed on a beam together with a counterbalance whose arm with a weight at its end is fixed to the beam at the pivot and it determines a stable equilibrium position (Fig. 2). The TRMS consists of a beam pivoted on its base in such a way that it can rotate freely both in the horizontal and vertical planes. At both ends of the beam there are rotors (the main and tail rotors) driven by DC motors. The system input is \( u = [u_t, u_m] \) where \( u_t \) denotes the voltage of the tail rotor and \( u_m \) is the voltage of the main rotor. The system states are
\[ x = (l_{th}, \omega_t, \Omega, \theta_0, l_{av}, \omega_{av}, \Omega, \theta_0)^T \]
where \( l_{th/av} \) is the current of tail/main rotor, \( \omega_t/\omega_{av} \) is the rotational velocity of the tail/main rotor, \( \Omega \) is the angular velocity around the horizontal/vertical axis and \( \theta_0 \) is the azimuth/pitch angle of beam.

### 5.2 The TRMS LPV model

The mathematical model of the TRMS is given by a set of the non-linear differential equations that can be found in Rahideh and Shaheed (2008). From this non-linear model, a LPV representation can be obtained as follows:

\[
\begin{align*}
\dot{x}(t) &= A(\theta(t))x(t) + B(\theta(t))u(t) \\
y(t) &= Cx(t)
\end{align*}
\]

(39)

where:

\[
A(\theta) = \begin{bmatrix}
a_{11} & a_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{21} & a_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{32} & a_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{43} & a_{44} & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{54} & a_{55} & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{65} & a_{66} & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{76} & a_{77} & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{87} & a_{88} & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
B(\theta) = \begin{bmatrix}
b_{11}(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b_{21}(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b_{32}(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b_{43}(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b_{54}(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b_{65}(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b_{76}(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b_{87}(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(39)

where \( \theta = [\theta_1, \ldots, \theta_8]^T = [\alpha_2(\theta_1), \alpha_3(\theta_2), \alpha_34(\theta_3), \alpha_35(\theta_4), \alpha_35(\theta_5), \alpha_35(\theta_6), \alpha_37(\theta_7), \alpha_33(\theta_8), b_{11}(\theta), b_{22}(\theta)]^T \).

Parameters in the LPV model are scheduled using the available measurements outputs, i.e., \( \theta(t_1) \) and \( \theta(t_2) \). The rest of states needed for control are estimated using an state observer.

To implement the AMM LPV FTC in the TRMS using the proposed approach, it is necessary to obtain the polytopic representation (11) of the system (39). The simplest polytopic approximation relies on bounding each LPV parameter by an interval. This approximation is known as bounding box approach (Aapkarian et al., 1995). An alternative approach known as small hard is based on finding the smaller polytope that contains the LPV parameters. This is the approach applied to this application example.

### 5.3 FTC design

To illustrate the proposed approach, is applied to this case study considering that the desired values of the angles are \( \theta_l = 0.4 \) and \( \theta_l = 0.02 \). To achieve the control objective, nominal control input is defined as: \( u_{nom} = [0.425, 0.278]^T \).

The set of admissible closed loop behaviors is defined as:

\[
M = \{ M : e^{igt(M)} \in D(0.5, 1.5, \pi/2) \}
\]

(40)

and the performance degradation of the performance is \( \tau = 0.1 \)

The TRMS has two actuators, one related to the tail rotor denoted as \( \alpha_i \) and other related to the main rotor denoted as \( \alpha_{av} \). The values of the nominal failure associated to the actuators are: \( \alpha_1 = 8.9e - 12 \) and \( \alpha_2 = 1.53e - 10 \) for tail and main rotor, respectively. The parameter \( \beta_1 = 6 \) and \( \beta_2 = 15 \) are defined according with the main and tail rotor characteristics.

To select the reliability threshold \( R_{1,h} \) and actuator load threshold \( u_{th} \), a evaluation of reliability \( R_t \) (26) is presented in Figs. 3(a)-(3(b)). Additionally, the maximum tolerable fault (34) can be obtained.

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**Fig. 3. Reliability evaluation for fault in the tail rotor \( f_{tan} \).**

Two design can be considered (see Figs. 3(a)-(3(b))) depending if the emphasis is put on either in reliability or performance. In the first design, the emphasis is on the actuator reliability:

- \( R_{1,th} = 95\% \) \( u_{th} = 1.23 \) and \( f_{max}^{th} = 0.66 \)
- \( R_{1,th} = 90\% \) \( u_{th} = 0.35 \) and \( f_{max}^{th} = 0.2 \)

while the second design is focused on the performance improvement:

- \( R_{2,th} = 90\% \) \( u_{th} = 1.35 \) and \( f_{max}^{th} = 0.69 \)
- \( R_{2,th} = 80\% \) \( u_{th} = 0.4 \) and \( f_{max}^{th} = 0.3 \)

The controller is designed to tolerate the actuator faults defined previously, keeping the admissible closed-loop behavior (40) and using the procedure described in Section 4.4. These faults have been incorporated in the LPV model as scheduling variables. Then, a polytopic model with \( N = 420 \) vertices is obtained for each case.

### 5.4 Fault scenario

In this scenario, an actuator fault is introduced in the main rotor. The fault is defined as follows:

\[
f_{tan}(\theta) = \begin{cases} 
0, & t < 10s \\
0.3, & t \geq 10s
\end{cases}
\]

(43)

**Fig. 4. Azimuth and pitch angle with an actuator fault \( f_{tan} = 30\% \)**
(42), the reliability is degraded to improve the performance. Therefore, the maximum admissible fault is bigger in this case as shown in Fig. 3(b). Moreover, the set-point corresponding to $\theta_1$ is inside the desired interval (see Fig. 3(a)) and $\theta_1$ is stabilized in 1.8s (see Fig. 3(b)).

Finally, Fig. 5 shows that the control inputs are within their respective thresholds defined in the reliability (41) and performance (42) designs. In case of the reliability design (41), the control input $u_m$ (see Fig. 5(b)) is saturated and consequently does not achieve the desired value as shown in Fig. 4.

6. CONCLUSIONS

In this paper, an approach to design the Reliable AMM FTC for LPV systems have been proposed. The suggested approach reconfigures the controller on-line by means of varying parameters that can change with the operating point and the fault. As a particular case, faults in actuators have been considered. The advantage of this approach is that allows the controller design to be defined by a set of admissible faults. When the fault is in this admissible interval the system can be recovered with the desired performance.

Another advantage of the proposed FTC design approach is that allows to define the reliability level in spite of the faults. Then, it is possible to establish in the design the reliability level guaranteeing best achievable performance with some pre-established level of degradation and to define a set of admissible faults.

The controllers have been designed such that it can stabilize the faulty plant using LMI pole placement. The advantage of a LMI region is that can locate the poles of the closed-loop system in a subregion of the complex plane. To select a solution, it has been used the notion of quadratic $H_2$ performance described by a finite number of algebraic LMI. The potential and performance of the approach has been demonstrated in a illustrative application to a two degree of freedom helicopter.

As a further research, the computation of fault estimation will be proposed including the effect of model uncertainties. Also, the influence of delay introduced by FDI module will be analyzed.

REFERENCES


